Density Gradients in a Rotating Stratified Fluid: Experimental Evidence for a New Instability

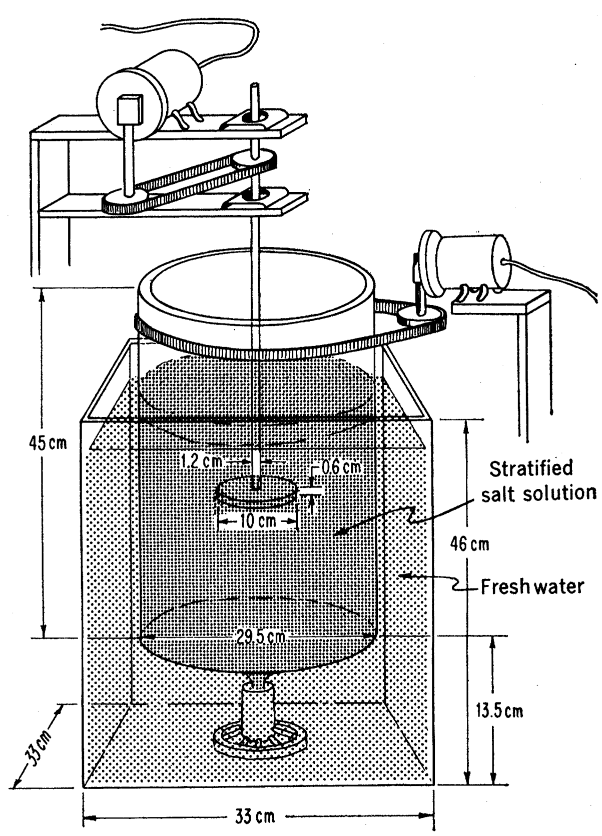
Abstract. *Velocity gradients induced in a rapidly rotating, density-stratified saltwater solution by a slowly rotating disk produce sharp vertical gradients of density, which appear as regularly spaced, curved horizontal sheets when the ratio of angular velocities exceeds a critical value. The existence of the sheets is apparently a finite amplitude manifestation of a recently proposed viscous instability.*

Fluid dynamical instabilities can manifest themselves in unexpected ways. In the experiment reported here, a rotating cylinder is filled with a fluid whose density is a linear function of depth; motion relative to the cylinder is induced by a submerged rotating disk. Large relative motion produces a remarkable change in the density field: sharp vertical gradients, which appear as curved horizontal sheets. The sheets also appear near the bottom of the container when its speed is changed markedly. The experimental results suggest that the mechanism of formation of the sheets is the "viscous overturning" instability recently proposed by McIntyre (*1*).

In the course of the experiment, the cylinder (Fig. 1) is filled with a linearly stratified (*2*) aqueous NaCl solution and is brought with the disk to the desired rotation speed slowly enough so that excessive mixing does not occur (in particular, so that no evidence of the present instability appears). The disk speed (ω) relative to the cylinder speed (Ω) is then varied, and shadowgraphs of the density field, produced by shining parallel light through the apparatus onto a screen, are photographed.

No perturbations in the density field are observed when the ratio of angular velocities ω/Ω (Rossby number) is much less than 1. As the Rossby number is increased, sharp vertical gradients of the density begin to appear as curved horizontal sheets in the region of the disk. At first, single sheets appear above and below the disk; for higher Rossby numbers, more sheets appear, farther from the disk. The axial symmetry of the pattern is evident in Fig. 2, a typical shadowgraph of the density field for Rossby number -1. The vertical distance between the sheets is several times the thickness of the sheets and is remarkably uniform. With the rotation speeds of disk and cylinder held constant, the pattern maintains itself for at least several days. Overturning motion between the sheets has been directly observed on the shadowgraph screen; potassium permanganate particles dropped through the fluid reveal a steplike pattern in the basic velocity field. When the relative disk speed is reduced to zero, the sheets slowly disappear. Sheets have been observed for both positive and negative Rossby number and for three different ratios of disk to cylinder diameter: 27.8/27.8, 10.1/27.8, and 10.1/13.8 (in centimeters).

The overall stratification S $[(1/\rho)(d\rho/dz)]$ is measured before and after the experiment by extracting samples of



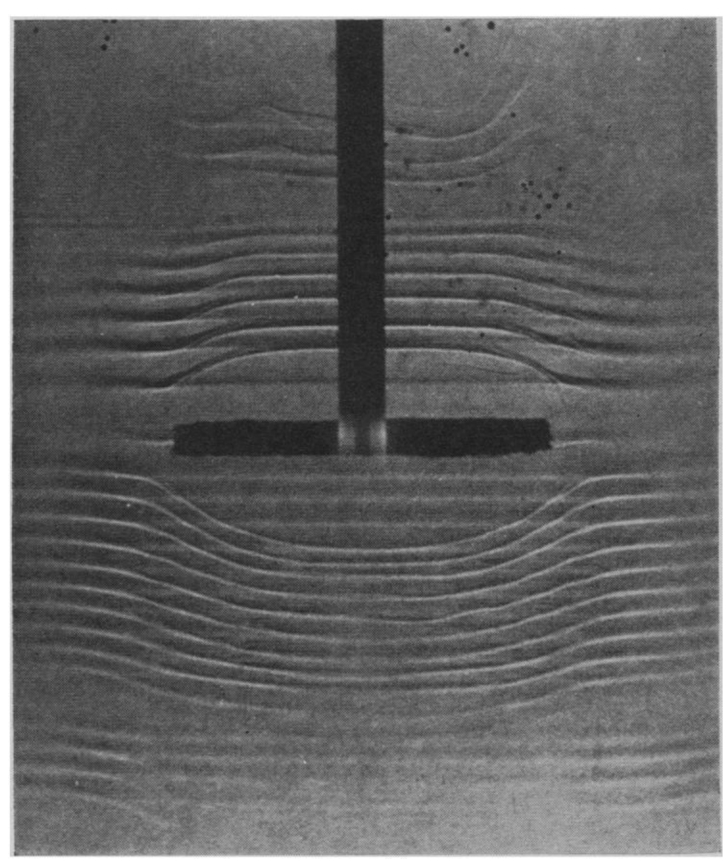


Fig. 1 (left). Schematic diagram of apparatus. The cylinder is placed in the water-filled box in order to minimize distortion. Fig. 2 (right). A shadowgraph of the density field in a rotating stratified fluid. The bright lines indicate regions of rapidly changing density. Here the Rossby number is -1, cylinder angular velocity is 1.05 rad/sec, cylinder diameter is 27.8 cm, disk diameter is 10.1 cm, $S = 0.0017 \pm 0.0002$ cm^{-1}, $T = 20°$C, and $\sigma = 700$. The rod holding the disk is 1.27 cm in diameter.

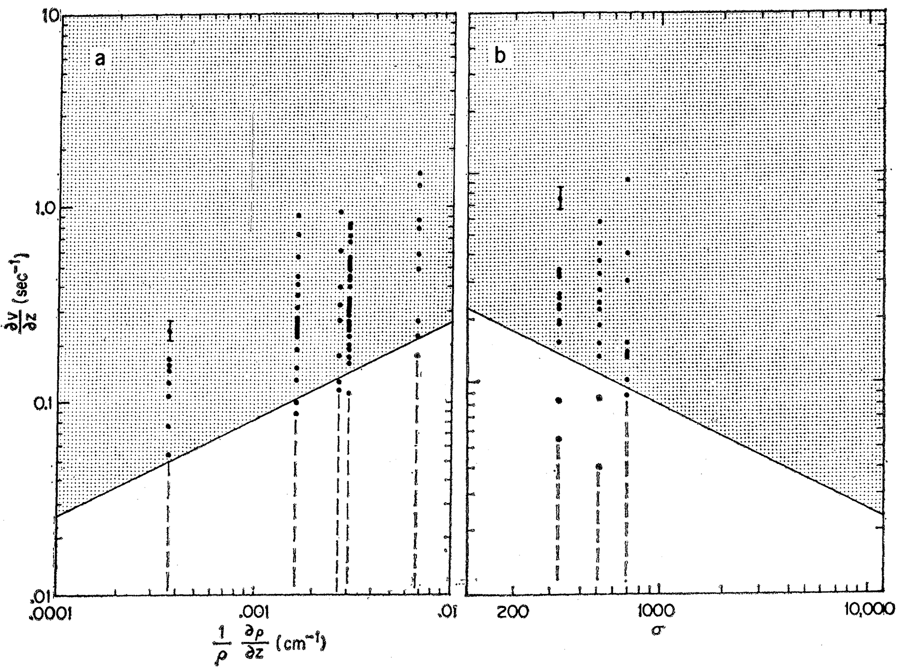


Fig. 3. (a) Velocity gradient against stratification parameter. The large dots represent the parameters of observed sheets; the dashed lines indicate values of the parameters for which no sheets were observed. A representative experimental error is indicated. Here $\Omega = 0.99$ rad/sec, $\sigma = 700$, $T = 20°C$, cylinder diameter is 13.8 cm, and disk diameter is 10.1 cm. (b) Velocity gradient against Schmidt number. Here $\Omega = 0.98$ rad/sec, and $S = 0.0017 \pm 0.0002$ cm^{-1}.

fluid at discrete heights (z) and measuring their indices of refraction (proportional to density ρ). The stratification typically is linear to within 5 percent and varies less than 10 percent over the time of the experiment. The parameters for the experiment are such that the vertical gradient of the mean velocity at any point is proportional to the radial derivative of the mean density field (a "geostrophic" balance). We assume that the slope of a given sheet represents the slopes of the lines of constant density in that region and infer a velocity gradient for each sheet. The parameters for the sheet of minimum curvature (the first visible sheet from the bottom of the container) determine the critical velocity gradient. This procedure is accurate to 10 percent for a Rossby number of 0.1; for larger Rossby numbers, nonlinear effects have been included in the calculations.

McIntyre (*1*) has shown that a shear flow in approximate geostrophic and hydrostatic balance (the baroclinic circular vortex) can be unstable for both large and small Prandtl number or Schmidt number σ (the ratio of diffusion coefficients of momentum and heat or salt). Moreover, he has shown (*3*) that the instability process can be identified in numerical calculations of certain flows in a laterally heated rotating annulus. The observed density sheets in the present experiment can be interpreted as a finite amplitude manifestation of his "viscous overturning" at large Schmidt number. The physical mechanism depends on viscous torques and the horizontal density gradients produced by the relative motion in the rotating system. When the diffusion of momentum is much larger than the diffusion of density, the viscous torques bring a particle displaced radially outward into momentum equilibrium at the new radius; the relative absence of density diffusion causes the particle to rise vertically. Similarly, a particle moved inward will tend to sink. The resultant overturning is the source of vorticity for the instability. The instability thus produced by counter gradients of density and angular momentum is exactly analogous to that produced by counter gradients of heat and salt (*4*).

McIntyre's calculations yield a critical gradient Richardson number Ri [defined as $gS/(\partial v/\partial z)^2$] for the onset of instability at large Schmidt number. In terms of $\partial v/\partial z$, the instability criterion is

$$\frac{\partial v}{\partial z} > \left[\frac{4gS}{\sigma}\left(1 + \frac{1}{2\Omega}\frac{\partial v}{\partial r}\right)\right]^{1/2} \quad (1)$$

where g is the acceleration of gravity, Ω is the rotation rate of the fluid, and $\partial v/\partial r$ is the radial derivative of the mean velocity field.

In order to compare the parameter dependence of the observed pattern with the instability criterion, we varied S and σ while keeping the other parameters constant. The results are summarized in Fig. 3, where the maximum velocity gradient for each observed sheet is plotted against S and σ. The range of velocity gradients for which no sheets were observed is shown by the dashed lines. The shaded regions in the figure delineate the instability area. Of the observed sheets, 88 percent fall in the area of predicted instability; the magnitudes of velocity gradients for which no sheets were observed also agree reasonably well with the predicted stable regime. Since the observed density perturbations are of finite amplitude, the comparison of theory and experiment at the onset of instability must be viewed with some caution. However, Fig. 3a shows reasonably good agreement with the instability criterion (Eq. 1); the data presented in Fig. 3b are not inconsistent with the theory.

The observed distance between the sheets was found to be independent of velocity gradient $\partial v/\partial z$ over the range 0.1 to 0.75 sec^{-1} and to be equal to 0.51 ± 0.06 cm for $\Omega = 0.93$ rad/sec, $S = 0.0030 \pm 0.0003$ cm^{-1}, and $\sigma = 700$. The wavelength of the maximally growing disturbance of the theory is close to this distance. At $\partial v/\partial z = 0.1$ sec^{-1}, the theory predicts a value of 0.45 cm for this quantity. The finite amplitude nature of the observation and the simpler geometry of the theory may preclude this identification.

A growth rate is estimated by increasing the disk speed and measuring the e-folding time for the appearance of each new pattern. The decay rate is estimated by measuring the time for the disappearance of the pattern upon decrease of the disk speed. For $\Omega = 0.96$ rad/sec, $S = 0.0010 \pm 0.0001$ cm^{-1}, and $\sigma = 700$, the theoretical growth rate yields a doubling time of 22.5 seconds, which is smaller but not in disagreement with the observed time for the full development of a sheet (about 750 seconds) which should be several times larger.

The experimental results thus appear to be consistent with the predictions of the viscous instability theory. Although the parameters in the present system do admit other instabilities, both symmetric and nonsymmetric, the possibility appears remote. The symmetric nature of the observed pattern precludes explanations based on nonsymmetric instability; the length scales predicted for this experiment from classic symmetric instabilities (*5*) are much larger than the ones observed.

We have also observed sheets in an arrangement where the linear gradient lies below a layer of homogeneous density. In that case, the sheets appear at the top of the gradient, below the homogeneous layer. Density sheets also appear near the bottom of the container when the cylinder speed is changed markedly, and then they decay and disappear as the fluid reaches the new equilibrium rotation speed. Their role in the process of nonlinear, stratified unsteady flow is not yet understood.

The existence of horizontal sheets of density gradient in the laboratory immediately raises the question of the presence of the proposed mechanism in nature. McIntyre (*1*) has suggested that the process could be operative in frontogenesis in the atmosphere, and R. W. Stewart (*6*) has suggested that some mechanism based on the difference between the turbulent transfer coefficients of momentum and density is responsible for the density microstructure observed in freshwater lakes. The present instability would be operative on a scale determined by molecular diffusion, should the molecular coefficients and gradient Richardson number satisfy the instability criterion (Eq. 1); moreover, should the small-scale turbulence in nature result in transfer of momentum and density characterized by constant turbulent transfer coefficients, an eddy Prandtl number could be used in the criterion and the length scales would be increased.

For molecular diffusion, the experiment has demonstrated that sheets appear when σ is greater than 310. Since the proposed instability mechanism is the same for all σ greater than 1, we infer that sheets will appear for all σ between 1 and 310 whenever the critical velocity gradient is exceeded. For example, in a freshwater lake, $\sigma = 7$. The typical temperature gradients of about 0.01°C per centimeter observed in regions of lake microstructure (*6*) lead to instability according to the present theory, if the basic flow is geostrophic and the velocity gradient $\partial v / \partial z > 0.06$ sec^{-1}, a value which is not uncommon (*7*) and which has been observed in regions of microstructure in the Mediterranean Sea (*8*).

Turbulent diffusion coefficients in the instability requirement indicate that the gradient Richardson number must be less than one-fourth the eddy Prandtl number, a criterion that is often met (*9*). Simultaneous measurements of gradient Richardson number and transfer coefficients in the regions of microstructure are required to test the hypothesis that the mechanism exists on these larger scales.

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10. I thank M. Stern who first pointed out to me the relevance of the viscous instability to the present experiment, P. H. Stone and F. H. Abernathy for valuable discussions, and J. Duffee, E. Engelmann, and C. Hamaker, of Antioch College, and M. Egan and R. Weller, of Harvard University, for help in the experiments. Supported by a grant NOOO-14-67-A-0298-0011 from the Office of Naval Research to Harvard University.

11 March 1971

Lead Suppression of Mouse Resistance to Salmonella typhimurium

Abstract. *Mice were treated with subclinical doses of lead nitrate for 30 days. Lead-treated mice showed greater susceptibility to challenge with* Salmonella typhimurium *than controls which received no lead. This result confirms the hypothesis that treatment with lead reduces the resistance of mice to bacterial infection.*

According to Chisolm (*1*), "Among the natural substances that man concentrates in his immediate environment, lead is one of the most ubiquitous." The scientific literature is replete with reports on the toxic manifestations, diagnosis, and treatment of lead poisoning. There are also suggestions of subclinical influences of lead on the well-being of man and animals. However, little attention has been directed toward determining the effects and influence of lead on the resistance to bacterial invasion and immunologic reactivity of susceptible hosts. Williams *et al.* (*2*) suggested that lead may inactivate antibodies and thereby interfere with mechanisms whereby man and animals resist infectious disease. Their conclusions were based on the study of an acute, fatal illness in a 23-month-old child with a history of eating paint. Neuropathologic findings were so similar to those seen in acute septicemia that, in spite of concentrations of lead of 0.348 mg per 100 ml of blood, it was assumed that death was due to bacterial agents that were able to grow uninhibited as a result of lead-mediated antibody inactivation. The influence of various environmental pollutants on resistance to disease has been suggested in a number of reports (*3–5*). These include the study of Friend and Trainer (*3*) who described the interaction of an organochlorine pollutant and viral infection in mallard ducklings. Selye *et al.* (*4*) discussed the effect of lead acetate on the susceptibility of rats to bacterial endotoxins, and similar studies have more recently been reported in chicks (*5*).

The study described herein was designed to determine the influence of exposure to subclinical doses of lead on the resistance of mice to bacterial infection. The results confirm the hypothesis that exposure to low concentrations of lead in mice leads to reduced resistance to bacterial infection.

Seventy-five white mice (Swiss-Webster strain) were divided into three groups of 25 each and housed five per cage. The mice were of uniform size and age, weighing 18 to 20 g. Each mouse was given a daily intraperitoneal injection of either soluble lead nitrate or saline for 30 days according to the following paradigm: group I, 100 μg of lead nitrate in 0.5 ml of saline solution; group II, 250 μg of lead nitrate in 0.5 ml of saline solution; and group III (control group), 0.5 ml of sterile saline solution. During the 30-day period of exposure to subclinical doses of lead nitrate (*6*) and saline, no signs of toxicity were observed. Five mice died during this period from other causes: one in group