

page 170, where there are eight typographical errors. Perhaps some of the other puzzling statements one finds in the book are also printer's errors.

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Limit and Convergence

The Development of the Foundations of Mathematical Analysis from Euler to Riemann. I. GRATTAN-GUINNESS. M.I.T. Press, Cambridge, Mass., 1970. xvi, 186 pp., illus. \$10.

This compact volume is an austere critique, addressed to those with background in advanced calculus, of certain specific problems truly pertinent to foundational questions in analysis. It is as difficult to read as it is rewarding, for it offers few facile generalities, concentrating instead on the details of deep theorems. Topics treated include definitions (for functions of a real variable) of limit, continuity, the derivative and the integral, and the convergence of infinite series. The account opens abruptly with a controversy of the mid-18th century on the general solution of the "wave equation" for vibrating strings. D'Alembert insisted that a solution should be differentiable, Euler held that it need be merely continuous, and Daniel Bernoulli expressed it as an infinite series of sines. When in the early 19th century infinite series of trigonometric functions again arose in Fourier's work on heat diffusion, Cauchy questioned their validity. The crux of the matter was a theorem enunciated in Cauchy's *Cours d'analyse* of 1826:

When the terms of a series are continuous functions of x in the vicinity of a particular value x_0 for which the series is convergent, the sum of the series is also a continuous function of x in the vicinity of x_0 .

Cauchy had been one of the early analysts to give attention to conditions for the convergence of infinite series, and his *Cours* included the first batch of tests, several of which still bear his name. Nevertheless, not distinguishing adequately between series of constant terms and series of functions, he failed to achieve the concept of uniform convergence. Abel, who studied the behavior of a series of functions at the end points of its range of convergence, suspected that Cauchy's theorem ad-

mitted of exceptions; and in a paper of 1848 Philip Seidel, a student of Dirichlet, corrected the theorem through the introduction of what he called infinitely slow convergence, an idea hit upon independently by Stokes, who published it in 1849. More precise and comprehensive were the views expressed by Weierstrass in his lectures during the 1850's in which he specifically introduced uniform convergence through the delta-and-epsilon technique. Thus the "Age of Rigor" came to maturity in the "Weierstrassian analysis" which his students presented to the world.

The penetrating arguments by the author make a stark distinction between what he calls the "limit-achieving" concept of the 18th century and the "limit-avoidance" of the 19th. The latter view, akin to the later Weierstrassian "epsilonotics," is attributed by Grattan-Guinness to Bolzano in an attempted arithmetization of analysis in 1817; and our author is perhaps over-ready to presume that Cauchy saw this paper and "learned from Bolzano how to reinterpret and reformulate the basic components of analysis in terms of limit-avoidance." He adds the ungenerous comment that, "needless to say, the name of Bolzano appears nowhere in the *Cours d'analyse*: Cauchy would have had more sense than to make Bolzano's work known to his rivals" (p. 78). Unlovely aspects of Cauchy's character have been noted by others, but disingenuousness ordinarily is not among them; and a contrary view holds that "of all the mathematicians of his period he is the most careful in quoting others" (see the article on Cauchy by Hans Freudenthal in the *Dictionary of Scientific Biography*, vol. 3, 1971, p. 134). When one considers cases of simultaneity of discovery during that period—non-Euclidean geometry, non-commutative algebras, complex integration and double periodicity, conservation of energy, and many others—the independence of Bolzano and Cauchy would appear to be unexceptional. One wonders also if the author may not have overstated the case for originality of "limit avoidance," for this is an arithmetization which is not far removed from the ancient geometrical integrations which the 17th century misguidedly called the "method of exhaustion," whereas in reality it might more accurately be described as "exhaustion avoidance."

The backbone of this book is the convergence of series, and a much-

appreciated appendix (pp. 131–51) appropriately describes "The search for convergence tests." Somewhat less prominence is given to the steps by which successive refinements transformed Cauchy's concept of integration into the "Riemann integral" of the 1850's. On the basis of this and other developments described in this volume, the reader will agree easily with the author that by the time of Cauchy's death in 1857 the center of mathematical activity had shifted from Paris to the Göttingen-Berlin axis, with Riemann at the one end and Weierstrass at the other.

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Alchemy in Antiquity

The Origins of Alchemy in Graeco-Roman Egypt. JACK LINDSAY. Barnes and Noble, New York, 1970. xii, 452 pp., illus. \$10.

This work, which is a continuation of the series of studies Lindsay has devoted to Greco-Roman Egypt, is certainly an important addition to the literature of the history of alchemy and will be welcomed by historians of science for its rich documentation, diversified bibliography, and references to many sources not touched upon by the earlier historians of alchemy. Because of his general knowledge of the history of the period, the author draws his material from many facets of Greco-Roman life and civilization, from metallurgy and cooking to mystical rites and philosophy.

The first four chapters of the book serve as a general introduction, in which the author discusses Platonic, Aristotelian, Neoplatonic, and Stoic natural philosophy and physics and the elements within them that in his view are responsible for the cosmological and philosophical background of alchemy. He stresses especially Aristotelian and Stoic ideas and draws much from the recent research in Stoic natural philosophy by Sambursky and others. In the author's view alchemy came into being from a mixture of Greek philosophical ideas and practical processes and techniques, among which he stresses cooking and brewing. He implicitly disregards the central importance in the genesis of alchemy of metallurgical rites and practices going back to Babylonia and ancient Egypt, which have been stressed by Eliade and

many other earlier scholars of the subject.

In chapter 4, which concludes the background for the main subject of the book, a thorough discussion is given of the origin of the term "alchemy" in which all the different theories of Greek (*chymia*, *chēmeia*, and *chymos*), Egyptian (*kmt*), and even Chinese (*chin-i*, as proposed recently by Mahdihassan) origin are considered. The author concludes that *chēmeia-chymia* is related to both *chymos* and *chyma* and that "at the early stages there was no very clear differentiation between juice-extraction and smelting or alloying" (p. 89).

Chapters 5 through 17 are devoted to a systematic "history" of alchemy in antiquity, beginning with two chapters on Bolos of Mendes, whom the author considers to be the founder of systematic alchemy, and ending with the important figure Zosimos, with whom theory and practice became indissolubly wed, and the last of the alchemists of antiquity like Stephanos. The author stresses the presence of the two contending schools of Jewish and Egyptian alchemy, the importance of Iranian influences as seen in Ostanos and Mithraic elements, and the crucial role played by the craft guilds throughout this period of the genesis and development of alchemy. In many of the chapters new material has been brought together for the first time in English, and some of the alchemical figures like Bolos and Maria the Jewess are seen more clearly here than in any of the standard histories of alchemy.

Despite its rich documentation and historical analysis, this work falls short in explaining the symbolic significance of alchemy. Of the four basic levels of meaning in alchemy, the cosmological, the psychological, the medical, and the physical, the author limits himself mostly to the last and considers the modern conception of nature the only valid background against which the teachings of alchemy can be judged. It is strange that after the appearance of works by T. Burckhardt, W. Pagel, and M. Eliade the author should not consider seriously the symbolic significance of alchemy and the world view which makes of it something completely other than a protochemistry. Lindsay does recognize the purely quantitative nature of modern chemistry and writes, "Modern chemistry was not just alchemy without the nonsense; it was alchemy tamed, reduced wholly to a quantita-

tive level, and thus giving up its ghost" (p. 387). Moreover, in his conclusion he regrets the fact that the quantitative Galilean science did not take into consideration the intuitions of alchemy. But he refuses to judge alchemy in terms of its own universe with its multiple levels of meaning rather than in terms of the "two-dimensional" world of post-Galilean science.

A surprisingly large number of misprints does not detract from the value of the book with its rich documentation of the early history of alchemy and its bringing together of material not easily available in the most commonly used works on the subject.

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Practical Botany

Genetic Resources in Plants. Their Exploration and Conservation. A conference, Rome, September 1967. O. H. FRANKEL, E. BENNETT, R. D. BROCK, A. H. BUNTING, J. R. HARLAN, and E. SCHREINER, Eds. Published for the International Biological Programme by Davis, Philadelphia, 1970. xxii, 554 pp. + plates. \$17.50. IBP Handbook No. 11.

This volume is dedicated to N. I. Vavilov, the father of modern crop plant exploration. An appendix by D. Brezhnev of the Vavilov Institute of Plant Industry, Leningrad, contains the encouraging news that the research on cultivated plants which was broken off at the time of Vavilov's banishment and death has now been resumed on a broad scale with the help of modern genetic techniques.

The book itself contains 44 chapters, each of them written by a different author, except for three by Sir Otto Frankel and two by his associate editor, Erna Bennett of the U.N. Food and Agriculture Organization (which sponsored the symposium). Most of the authors are well-known authorities in the genetics of cultivated plants and forest trees or in the field of plant exploration and ecology. They represent 13 nations.

Most of the chapters are brief digests of the principal facts about their topics. The longest are those of Frankel on "Genetic conservation in perspective," of H. G. Baker on "Taxonomy and biological species concepts in cultivated plants," of E. Bennett on "Tactics of plant exploration," of I. A.

Watson on "The utilization of wild species in the breeding of cultivated crops resistant to plant pathogens," and of J. F. Harrington on "Seed and pollen storage for conservation of plant gene resources." These titles give a good idea of the scope of the book. In addition to general articles, there are brief treatments of gene pool explorations in wheat, maize, potatoes, peas, sweet potato, cotton, temperate zone tree fruits, and various groups of forest trees.

Some of the theoretical concepts emphasized are the following: The hypothesis of Vavilov, that the original center of cultivation of a crop plant species can be recognized because it contains the greatest amount of variability in modern times, is criticized by both Harlan and Zohary, who point out that various factors, particularly introgressions between crop species and their wild relatives, accompanied by a low intensity of varietal selection, can build up large secondary centers of variability. Baker makes a strong plea for biosystematic treatments of cultivated plants and their wild relatives, in contrast to treatments that are based solely on gross morphology. He emphasizes the fact that characters of flowers, fruits, and seeds, which in wild species are usually conservative because of strong selective pressures that maintain character combinations favoring seed dispersal, can be drastically altered in a short time by artificial selection for characteristics that are desirable to man and that reduce natural seed dispersal, a characteristic that is undesirable for the cultivator. Consequently, taxonomic systems that are based upon these characteristics can give an exaggerated impression of the differences both between cultivated species and their wild relatives and between different varieties of a cultivated species. J. G. Hawkes points out difficulties in the way of rigid applications of the species concept, and recommends a pragmatic approach that will be most useful to the breeder dealing with each particular group.

Several factors affecting the adaptation of crop plants to their environment are listed by J. P. Cooper, and W. Hartley points out that artificial selection has extended the range of most crop species far beyond that of their wild relatives.

On the practical side, Bennett gives careful instructions on how to plan an expedition for exploration for crop