page 170, where there are eight typographical errors. Perhaps some of the other puzzling statements one finds in the book are also printer's errors.

W. DIXON WARD
Departments of Communication Disorders and Otolaryngology, University of Minnesota, Minneapolis

Limit and Convergence

The Development of the Foundations of Mathematical Analysis from Euler to Riemann. I. GRATTAN-GUINNESS. M.I.T. Press, Cambridge, Mass., 1970. xvi, 186 pp., illus. $10.

This compact volume is an austere critique, addressed to those with background in advanced calculus, of certain specific problems truly pertinent to foundational questions in analysis. It is as difficult to read as it is rewarding, for it offers few facile generalities, concentrating instead on the details of deep theorems. Topics treated include definitions (for functions of a real variable) of limit, continuity, the derivative and the integral, and the convergence of infinite series. The account opens abruptly with a controversy of the mid-18th century on the general solution of the "wave equation" for vibrating strings. D'Alembert insisted that a solution should be differentiable, Euler held that it need be merely continuous, and Daniel Bernoulli expressed it as an infinite series of sines. When in the early 19th century infinite series of trigonometric functions again arose in Fourier's work on heat diffusion, Cauchy questioned their validity. The crux of the matter was a theorem enunciated in Cauchy's *Cours d'analyse* of 1826:

> When the terms of a series are continuous functions of x in the vicinity of a particular value x_0 for which the series is convergent, the sum of the series is also a continuous function of x in the vicinity of x_0.

Cauchy had been one of the early analysts to give attention to conditions for the convergence of infinite series, and his *Cours* included the first batch of tests, several of which still bear his name. Nevertheless, not distinguishing adequately between series of constant terms and series of functions, he failed to achieve the concept of uniform convergence. Abel, who studied the behavior of a series of functions at the end points of its range of convergence, suspected that Cauchy's theorem admitted of exceptions; and in a paper of 1848 Philip Seidel, a student of Dirichlet, corrected the theorem through the introduction of what he called infinitely slow convergence, an idea hit upon independently by Stokes, who published it in 1849. More precise and comprehensive were the views expressed by Weierstrass in his lectures during the 1850's in which he specifically introduced uniform convergence through the delta-and-epsilon technique. Thus the "Age of Rigor" came to maturity in the "Weierstrassian analysis" which his students presented to the world.

The penetrating arguments by the author make a stark distinction between what he calls the "limit-achieving" concept of the 18th century and the "limit-avoidance" of the 19th. The latter view, akin to the later Weierstrassian "epsilontics," is attributed by Grattan-Guinness to Bolzano in an attempted arithmetization of analysis in 1817; and our author is perhaps overready to presume that Cauchy saw this paper and "learned from Bolzano how to reinterpret and reformulate the basic components of analysis in terms of limit-avoidance." He adds the ungenerous comment that, "needless to say, the name of Bolzano appears nowhere in the *Cours d'analyse*: Cauchy would have had more sense than to make Bolzano's work known to his rivals" (p. 78). Unlovely aspects of Cauchy's character have been noted by others, but disingenuousness ordinarily is not among them; and a contrary view holds that "of all the mathematicians of his period he is the most careful in quoting others" (see the article on Cauchy by Hans Freudenthal in the *Dictionary of Scientific Biography*, vol. 3, 1971, p. 134). When one considers cases of simultaneity of discovery during that period—non-Euclidean geometry, noncommutative algebras, complex integration and double periodicity, conservation of energy, and many others—the independence of Bolzano and Cauchy would appear to be unexceptional. One wonders also if the author may not have overstated the case for originality of "limit avoidance," for this is an arithmetization which is not far removed from the ancient geometrical integrations which the 17th century misguidedly called the "method of exhaustion," whereas in reality it might more accurately be described as "exhaustion avoidance."

The backbone of this book is the convergence of series, and a much-appreciated appendix (pp. 131–51) appropriately describes "The search for convergence tests." Somewhat less prominence is given to the steps by which successive refinements transformed Cauchy's concept of integration into the "Riemann integral" of the 1850's. On the basis of this and other developments described in this volume, the reader will agree easily with the author that by the time of Cauchy's death in 1857 the center of mathematical activity had shifted from Paris to the Göttingen-Berlin axis, with Riemann at the one end and Weierstrass at the other.

CARL B. BOYER
Department of Mathematics, Brooklyn College, Brooklyn, New York

Alchemy in Antiquity

The Origins of Alchemy in Graeco-Roman Egypt. JACK LINDSAY. Barnes and Noble, New York, 1970. xii, 452 pp., illus. $10.

This work, which is a continuation of the series of studies Lindsay has devoted to Greco-Roman Egypt, is certainly an important addition to the literature of the history of alchemy and will be welcomed by historians of science for its rich documentation, diversified bibliography, and references to many sources not touched upon by the earlier historians of alchemy. Because of his general knowledge of the history of the period, the author draws his material from many facets of Greco-Roman life and civilization, from metallurgy and cooking to mystical rites and philosophy.

The first four chapters of the book serve as a general introduction, in which the author discusses Platonic, Aristotelian, Neoplatonic, and Stoic natural philosophy and physics and the elements within them that in his view are responsible for the cosmological and philosophical background of alchemy. He stresses especially Aristotelian and Stoic ideas and draws much from the recent research in Stoic natural philosophy by Sambursky and others. In the author's view alchemy came into being from a mixture of Greek philosophical ideas and practical processes and techniques, among which he stresses cooking and brewing. He implicitly disregards the central importance in the genesis of alchemy of metallurgical rites and practices going back to Babylonia and ancient Egypt, which have been stressed by Eliade and