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Arguments concerning Relativity and Cosmology

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A recent article by de Vaucouleurs (*1*) induces me to give a brief account of my view regarding the cosmological problem (*2*), which in some respects is similar to his but in other respects differs from it. In the first place my view is based on a critical review of the arguments that led Einstein to his cosmology, from which the whole branch of relativistic cosmologies descended, and, second, of their replacement by a program following the general trend of physics ever since Galilei. Most beautiful examples of this trend are the contributions to quantum theory and to relativity theory by Einstein himself; yet, in both cases he came to a philosophical point of view strangely deviating from this trend.

With respect to general relativity, its essential foundation, later called *the principle of equivalence,* was already clearly indicated and applied to the behavior of light in a gravitational field in Einstein's paper of 1911 (*3*), based on imaginary experiments in a laboratory which is either freely falling in a gravitational field or is correspondingly accelerated in a region where gravitation is absent. His conclusions regarding the behavior of light were later essentially verified, and he was led to the hypothesis that no deep-going difference exists between "genuine" gravitation and inertial force.

In fact, this kind of equivalence became the main foundation of his gravitational theory, although Einstein himself came to regard its foundation to be *general relativity,* that is, the extension of special relativity—as a symmetry of nature—to arbitrary nonuniform motion. This view he found strongly supported by Mach's philosophical criticism of the Newtonian concept of *absolute space,* as needed for the true *physical* definition of motion. To this Mach opposed a *relativistic* explanation of the inertial forces as being due to nonuniform motion with respect to the bulk of matter in the universe, considering these forces as a kind of gravitation analogous to the magnetic field from moving electrical charges. In practice there was not much difference between Mach's view and Newton's; according to both the reference frame for the validity of the ordinary laws of motion coincides very approximately with that defined by the sky of the fixed stars. But the idea of Mach, that this coincidence pointed to the *cause* of the inertial forces, was undoubtedly the main incitement for Einstein's cosmological attempt.

However, as will be shown by means of the equivalence principle, Mach was right insofar as Newton's *absolute* space is not absolute but, still, Newton's concept is nearer to physical reality than Mach's idea about inertial forces. In fact, it may be said that the sky of fixed stars has no more to do with inertial forces than the roof with its lamps in the weightless room of a satellite has on the separation of cream from milk in a centrifuge rotating there. Therefore, it was hardly fair of Mach to speak in this connection of Newton's metaphysical inclination toward the "absolute." Rather it seems that he himself inclined toward the opposite exaggeration: going too fast from coincidence to cause, when the cause is sufficiently antimetaphysical—here material bodies.

Role of the Equivalence Principle

In his fundamental paper of 1916 (*4*) Einstein generalized his original considerations (*3*) in two respects, for variable and for arbitrarily strong gravitational fields. Thus, in the first place, he made the very reasonable assumption that, because the removal of the gravitational field is more and more accurate, the smaller the region to which the freely falling frame of reference belongs, the more rigorously would the conclusions formed (*3*) hold for a nonhomogeneous and time-dependent gravitational field. Hence, going to the limit, bodies in such a *local inertial frame* would move according to the law of inertia, and electromagnetic fields would satisfy the Maxwell equations. The second claim would simply mean that the laws in such a frame would satisfy the special relativity principle. This implies that at a given space-time point there are a multitude of frames connected by Lorentz transformations, one of them being at rest at the chosen space-time point. The latter circumstance is very important. It means that the principle of special relativity is neither upset, nor is it generalized, by Einstein's gravitational theory; only the conditions for its validity are thereby recognized.

The main problem for Einstein was now to obtain a general mathematical definition of the gravitational field and the general form of the laws known in the gravitation-free case of special relativity. Both these problems he solved by an ingenious use of the Minkowskian invariant characterizing the space-time metric of special relativity. As is well known, this invariance generalizes the Pythagorean theorem of the invariant expression of space distances as ex-

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pressed in terms of coordinate distances.

What Einstein established was, indeed, closely analogous to the geometry of curved surfaces founded by Gauss and based on Euclidean geometry in the infinitesimal, *thereby providing a definition of lengths and angles in the surface*. Thus at any nonsingular point of the surface, an infinitesimal region may be identified with the corresponding region of the tangent plane, and the distance ds from that point to an adjacent point—the line element—may be expressed in terms of the differentials of two Cartesian coordinates (x and y) in that plane according to the Pythagorean theorem

$$ds^2 = dx^2 + dy^2 \qquad (1)$$

Expressing the same infinitesimal interval in the coordinates used for the mapping of a finite region of the surface, ξ and η, this theorem takes the form

$$ds^2 = \alpha d\xi^2 + 2\,\beta d\xi d\eta + \gamma d\eta^2 \qquad (2)$$

where α, β, γ are functions of ξ and η. If we use for ξ and η a set of orthogonal coordinates ($\beta = 0$), as is always possible—such as latitudes and longitudes on a sphere—and letting the x and y axes in the tangent plane coincide with their directions at the chosen point, then

$$\begin{aligned} dx &= (\alpha)^{\frac{1}{2}}\, d\xi, \\ dy &= (\gamma)^{\frac{1}{2}}\, d\eta \end{aligned} \qquad (3)$$

give the definition of lengths and directions in the surface by means of the mapping coordinates. If this procedure is applied to the surface of a sphere, it follows that the *straightest lines* there are the great circles.

Now Riemann, in his generalization of two-dimensional Gaussian geometry to "spaces" of any number of dimensions, based the metric on a line-element ds with a generalized Pythagorean theorem

$$ds^2 = \sum_{i,k} g_{ik}\, dx^i dx^k \qquad (4)$$

And he showed that at any nonsingular point—the g_{ik}, being functions of the coordinates x^i, which map a finite region—a local coordinate frame may be found in which the first coordinate derivatives of the g_{ik} vanish at the point, their directions at the point coinciding with those of the mapping coordinates. These so-called *geodesic* or Riemann frames are clearly a direct generalization of the tangent plane coordinates in surface geometry, implying the validity of the "Euclidean geometry," that is, a geometry of constant g_{ik} in a region sufficiently small for the neglect of the higher derivatives of the g_{ik}. Here on earth this corresponds to regarding a lake, which is small with respect to the surface of the earth, as being plane.

In this connection it is interesting to consider for a moment the reasons given in Greek antiquity for assuming the earth to be approximately spherical. They were to some extent empirical but largely speculative. The latter reasons had to do with the cosmology reigning through the Middle Ages in the form given by Aristotle, a main point of which was the idea that natural motion of heavy matter was directed toward the center of the universe, from which would follow its accumulation around this point, thus forming the spherical earth. To this came the experience of the curvature of the sea as shown by the gradual appearance of approaching ships, beginning with the tops of the masts. This, however, could only prove that the comparatively small region of the earth then known formed a part of an approximately spherical surface, which might very well be that of a hill floating on a plane ocean. The same result could have been obtained by accurate measurements of the sums of the angles in large triangles formed by straightest lines, as later suggested by Gauss. But long before a circumnavigation of the earth had been performed, a strong empirical argument for its sphericity was known, namely the circular form of its shadow on the moon during lunar eclipses, regardless of the place on the earth from which the eclipse was seen. But without the third dimension of height no such proof was to be had, the essential reason being the infinitesimal definition of the metric of curved surfaces. In cosmology no such extra dimension exists, and, since we are confined to a limited region of the universe, there is no way of obtaining empirical reasons for its topology—if the equivalence principle holds.

Now, all this corresponds in every detail to Einstein's establishment of general relativity: in the first place to its mathematical formalism, but also—as he explicitly stated (*3*)—with respect to the definition of space and time by means of measurements in an inertial frame. Thus the space-time metric is given by the line element in an inertial frame, here taking the Minkowskian form

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2\, dt^2 \qquad (5)$$

belonging to the immediate surrounding of a point situated in a space-time region mapped by four general coordinates, in which it takes the Riemannian form (Eq. 4). Taking the chosen space-time point as the origin of the Cartesian coordinates and the zero-point of time and using orthogonal coordinates x^1,x^2,x^3,x^4 coinciding as to "direction" with x,y,z,t at the point, we have in analogy with Eq. 3

$$\begin{aligned} dx &= (g_{11})^{\frac{1}{2}}\, dx^1 \\ dy &= (g_{22})^{\frac{1}{2}}\, dx^2 \\ dz &= (g_{33})^{\frac{1}{2}}\, dx^3 \\ dt &= (|g_{44}|)^{\frac{1}{2}}\, dx^4/c \end{aligned} \qquad (6)$$

which shows how space and time, the meaning of which is given by the local inertial frames, is determined when the g_{ik} are given functions of the four coordinates.

The circumstance that the first coordinate derivatives of the g_{ik} vanish at the chosen point of a geodesic frame shows that they must be related to the gravitational potential, their derivatives representing the gravitational field. Now, this is in full agreement with the equations of motion of a body in a gravitational field obtained by translation of the mathematical expression of the law of inertia from an inertial frame into general coordinates. In fact, in the former frame the "orbit" in space-time is a "straight" line, which in the general case is a "straightest" line, just as an element of a great circle on the sphere corresponds to a straight line on the tangent plane. Moreover, for weak gravitational fields these equations go over into the corresponding ordinary equations, with g_{44}/c^2 taking the place of the gravitational potential.

From the above account of the equivalence principle—essentially Einstein's own—it follows that the laws of motion of particles, neutral as well as charged ones, and the Maxwell equations for the electromagnetic field valid in the presence of a gravitational field—to mention only those laws of nature which are still the best known, being treated in detail already in Einstein's 1916 paper—are completely determined by the corresponding laws valid in an inertial frame. How could Einstein then believe that inertia is due to the bulk of matter in the universe? In his first cosmological paper of 1917 (*5*) he hardly mentions this hypothesis. But already in his book *The Meaning of Relativity* (*6*) of 1921 he tries to motivate it by the result of Thirring (*7*):

that the gravitational field inside a hollow sphere, rotating with respect to an asymptotically inertial frame of reference, resembles the Coriolis and centrifugal forces due to rotation. Generalizing this result Einstein thus concluded that the inertial mass of a body is increased by the gravitational effect of other bodies, summarizing in the following way: "We must see in them a strong support for Mach's ideas as to the relativity of all inertial actions. If we think these ideas consistently through to the end we must expect the *whole* inertia, that is the *whole* $g_{\mu\nu}$-field, to be determined by the matter of the universe, and not mainly by the boundary conditions at infinity." With the whole $g_{\mu\nu}$-field he means here that including the Minkowskian metric. However, the term in Einstein's formulas, which looks like an addition to the inertial mass, is removed locally by a coordinate transformation, in full accordance with the equivalence principle. It is just a somewhat uncommon contribution to the gravitational field.

The last sentence of the quotation bears upon Einstein's meaning that boundary conditions go against his idea of general relativity because the concept *inertial frame* is not covariant. However, independently of the structure of the universe at large, boundary conditions are necessary for the definition of those solutions of his equations for the gravitational field which determine the field governing the planetary motions. Moreover, according to the equivalence principle the properties of an isolated system are defined by means of them. Thus, among other claims to be satisfied for this purpose—the system being anything from a single particle to a stellar object—is the removal of all gravitational fields not belonging to the system itself. Hence the definition of its total energy and momentum, that is, its mass, depends on an asymptotically inertial frame. In fact, this follows from the conservation theorems derived by Einstein himself. Let us, as an example, consider the determination of the mass of the solar system, thereby making good the statement that the sky of the fixed stars has merely a casual relation to the inertial forces.

Thus, the center of gravity of the solar system is freely falling in the gravitational field from the other stars in our galaxy; since this field is not only very weak but also practically homogeneous over the range of the system and, of course, practically unchanging during the time of our observations, we see that a frame of reference fixed to its center of gravity is as nearly inertial as could be wished. Hence, measurements of the gravitational field by instruments at rest in this frame would give the value of its mass. Since this frame of reference is not visibly changing with respect to the sky of fixed stars, it follows that these act just as lamps indicating the frame. Moreover, this frame coincides by definition with Newton's absolute space and is thus in agreement with the equivalence principle, in contrast to Mach's idea. Since Newton was mainly concerned with the motion of the planets, it is not astonishing that he did not pay attention to the fact that the laws of motion take the same form in any freely falling frame of reference as in his absolute space and, if he had, he could hardly have made any use of it. The way Einstein did this is certainly worth our highest admiration, not less so because also he shared the general human condition of making mistakes when attempting something new.

As the above considerations show, general relativity is—unlike special relativity—not, as Einstein believed, a symmetry of nature. On the other hand, its mathematical formalism is completely covariant with respect to arbitrary transformations of the four coordinates, as follows naturally from the role of such transformations with respect to changes of gravitational fields. Because the object of interest in surface geometry is only partly similar to that in relativity theory, it is important to be on guard against overstressing the analogy. Thus, the fact that some nonlinear transformations correspond to alterations of the gravitational field—a most real physical quantity—has hardly any geometrical analogy. Also changes of coordinates in the description of a given surface plays a rather trivial although often a practical role.

An important objection, which may have led Einstein to undervalue the role of the equivalence principle, ought to be mentioned here. Although the g_{ik} and their derivatives which give mathematical expression to gravitation are a consequence of this principle, Einstein's equations for the gravitational field itself are only partly based on it, simply because in an inertial frame there is no gravitational field to translate. However, like all classical laws, general relativity has to be subjected to the claims of quantum theory. And there are reasons to believe that a corresponding reformulation of the equivalence principle may meet this objection, because in quantum theory there is not the same division between the presence or the absence of a quantity as in classical physics. This question, however, is beyond the scope of this article.

Cosmological and Ordinary Solutions

Einstein's original attempt toward a cosmology was mainly based on physical arguments—what he believed to be fundamental physical principles and conclusions from astronomical observations—which, like everything in physics, were open to criticism by other physical arguments. Thus, his assumption of an average constant density was due to the belief, then shared by astronomers, that the relative velocities of stars are small compared with that of light and of random distribution, like those of molecules in a gas. From this he was led to his static closed universe. Later on, after Hubble's discovery of the regular expansion demonstrated by the red shifts of the distant galaxies and its interpretation by means of the solutions of Einstein's field equations discovered by Friedmann (and later independently by Lemaitre), there came a tendency toward an axiomatic foundation of cosmology defended by pointing to the singleness of the universe in contrast to all other objects of physical investigation. The strangest application of this kind of axiomatics was certainly the use of the so-called *cosmological postulate* (that the average situation of the universe is the same everywhere and at all times) for the invention of new laws of physics in order to make the change demonstrated by the Hubble expansion only apparent, namely the creation of new matter at a rate so chosen that the vanishing galaxies are always replaced by new ones.

However, a part of the universe sufficiently large to contain a fair representation of the possibilities given by the laws of nature would fulfill the claim of this "postulate," and the obvious conclusion would be that our expanding metagalaxy is too small to represent the universe, there being other metagalaxies in other phases of evolution, some expanding like our own and some contracting. This reasoning will not exclude, of course, the possibility of a much larger closed universe, than assumed by Einstein, one containing a great number of subsystems of the kind of our metagalaxy. A recent attempt of this kind has been made by Horák (8), who returned to Einstein's original closed universe solution but postulated a much smaller density and correspondingly larger size. His belief that this would save Mach's idea about inertia is open, however, to the above criticism. Since the local metric is independent of the topology at large—also a bounded system of the Einstein type or Friedmann type has a spatially constant positive or negative curvature—it follows, in analogy to what was said above about the earth being spherical, that no observations we can make could prove or disprove whether the universe corresponds to a solution of this kind. Perhaps a deep-going generalization of current quantum field theory may lead to the result that its general ground state is not a vacuum but has a finite though extremely low average density. Still, this is hardly probable, and, for my part, I think it wiser at present not to invent cosmologies.

Although there is hardly any difference between a cosmological Friedmann solution and a bounded one so long as observations are not approaching the border and the extrapolations backward in time do not approach the state of the so-called fireball, the difference becomes significant when these conditions are not satisfied; in fact, it is enormous at the early stage assumed for the fireball. This is due to the Schwarzschild limit where the condition securing the continuity at the boundary of the system breaks down. Thus, in a spherical model cut out of a cosmological solution and, hence, having the Schwarzschild-Droste line-element outside of the sphere, namely

$$ds^2 = \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r}\right)} + r^2 d\Omega^2 - c^2\left(1 - \frac{2GM}{c^2 r}\right) dt^2 \quad (7)$$

where M is the total mass of the sphere with the boundary at $r = R$, G being the ordinary gravitational constant, c the velocity of light, and $d\Omega$ the line-element of the unit sphere, the Schwarzschild condition is given by R being greater than $2GM/c^2$. Since

$$M = \frac{4}{3}\pi\mu R^3 \quad (8)$$

where μ is the average density of the sphere, this gives the condition

$$M^2\mu \leq \frac{3c^6}{32\pi G} = 1.46 \times 10^{83} \text{ g}^3 \text{ cm}^{-3} \quad (9)$$

This shows, indeed, a striking difference between some conclusions drawn from cosmological solutions and what is possible in a bounded metagalaxy. Thus, if the mass of the metagalaxy is approximately 10^{54} g, which is probably of the right order of magnitude, Eq. 9 gives an upper limit for the density which is slightly greater than 10^{-25} g/cm^3, while that in the fireball state is estimated to about 10^4 g/cm^3. We shall show below that this is not contradictory to the existence of the observed isotropic radiation of approximately 3°K, although the temperature will all the time be extremely low compared with that of the fireball.

A Model of the Metagalaxy

This model is certainly but a rough approximation to the real structure of the metagalaxy at its present state, but it has the advantage of mathematical simplicity like the corresponding cosmological model, from which it only differs by having a boundary. But I hope that the explicit formulas and the numerical example given below may be of help in testing it, which may then lead to a more realistic model. Although it seems not to be decided whether the Friedmann solution, from which it is cut out, is expanding toward infinity or not, we shall limit ourselves to the former case corresponding to the rather reasonable estimates of the density and the Hubble parameter to be used in the numerical example. We write the line-element as follows

$$ds^2 = a(\tau)^2 d\sigma^2 - d\tau^2 = a(\tau)^2\left(\frac{d\eta^2}{1+\eta^2} + \eta^2(d\theta^2 + \sin^2\theta d\phi^2)\right) - d\tau^2 \quad (10)$$

Here $a(\tau)$, a function of the time variable τ alone, determines the expansion, while η, θ, and ϕ determine the place of any stellar object inside the metagalaxy independently of time. Let now T be the reciprocal of the Hubble parameter and μ the matter density as a function of time. Then by the Friedmann solution the following relations are satisfied

$$\frac{1}{cT} = \frac{1}{a}\frac{da}{d\tau},$$
$$\frac{8\pi\mu}{3} G\mu T^2 = \frac{a_0}{a_0 + a},$$
$$a_0 = \frac{8\pi}{3}\frac{G\mu a^3}{c^2} \quad (11)$$

where a_0 is a constant of motion because μa^3 is so. Thereby the criterion for expansion toward infinity is that the left side of the middle equation (like the right side) be smaller than unity. When this is so, then the values of a at the present time and a_0 will follow if T and μ are now known.

Let us assume that the boundary of the system is at η_B. Then its total mass M and its total rest-mass M_0 (that is, the sum of the masses of the subsystems) are given by

$$M = \frac{4\pi}{3}(\mu a^3 \eta_B^3),$$
$$M_0 = 2\pi\mu a^3\{\eta_B(1+\eta_B^2)^{\frac{1}{2}} - \ln[(1+\eta_B^2)^{\frac{1}{2}} + \eta_B]\} \quad (12)$$

Some idea of the η_B-value may be gained from the red-shift observations. Let a be the length $a(\tau)$ at the present time, and $a(0)$ its value at the time when the observed light was emitted. Then, denoting the geodesic distance in the (η, θ, ϕ) space between source and observer by σ, we have

$$\frac{\nu_0}{\nu} = \frac{a}{a(0)}$$

$$\ln\left(\frac{\left(1+\frac{a}{a_0}\right)^{\frac{1}{2}} + \left(\frac{a}{a_0}\right)^{\frac{1}{2}}}{\left(1+\frac{a(0)}{a_0}\right)^{\frac{1}{2}} + \left(\frac{a(0)}{a_0}\right)^{\frac{1}{2}}}\right)^2 = \sigma \quad (13)$$

where ν is the observed frequency and ν_0 that of the source. Placing our galaxy at $\eta = \eta_0$ and $\theta = \theta_0 = \pi$ of the θ-axis, while the source is at an arbitrary point (η, θ, ϕ), then (independent of ϕ) the distance appearing in Eq. 13 is given by

$$\sigma = \ln\frac{(1+\eta^2)^{\frac{1}{2}} + (\eta^2 - p^2)^{\frac{1}{2}}}{(1+\eta_0^2)^{\frac{1}{2}} + (\eta_0^2 - p^2)^{\frac{1}{2}}}$$
$$tg\,(\theta - \theta_0) = \frac{AB_0 - A_0B}{AB + A_0B_0}$$
$$A = (\eta^2 - p^2)^{\frac{1}{2}} - p(1+\eta^2)^{\frac{1}{2}}$$
$$B = (\eta^2 - p^2)^{\frac{1}{2}} + p(1+\eta^2)^{\frac{1}{2}}$$
$$A_0 = (\eta_0^2 - p^2)^{\frac{1}{2}} - p(1+\eta_0^2)^{\frac{1}{2}}$$
$$B_0 = (\eta_0^2 - p^2)^{\frac{1}{2}} + p(1+\eta_0^2)^{\frac{1}{2}} \quad (14)$$

Here p is an integration constant of the geodesic

$$\eta^2(d\theta/d\sigma)$$

which has to be eliminated in order to express σ as a function of η, η_0, and θ.

As an example we chose the following values for T and μ

$$T = 4 \times 10^{17} \text{ seconds}, \quad \mu = 10^{-30} \text{g cm}^{-3} \quad (15)$$

which seem reasonable according to present estimates, the former value corresponding to 25 km/sec per light-year. Thus we get very nearly

$$a_0 = 1.25 \times 10^{27} \text{ cm}, a = 12.5 \times 10^{27} \text{ cm} \quad (16)$$

Although the boundary is hardly sharp, an estimate of η_B may be obtained by means of observations of the highest "cosmological" red shifts at different angles of observation. Thus, as far as the model goes, the highest red shifts would come from a direction which passes through the center of the model, the lesser red shifts being placed more or less regularly according to the angle made by the direction of observation with this line. A statistical treatment of the red-shift observations with this in view would, if a sufficient number of objectives are available, either disprove the model or give an approximate value of η_B and our own value η_0 with respect to the center.

Conjectures Regarding the Evolution of the Metagalaxy

In contrast to the cosmological solutions, which are just assumed, the attempt to consider the metagalaxy as an example of a type of regular stellar objects—different from, but on the line of those known in great numbers, such as stars and galaxies—poses the problem of its evolution from an initial state of reasonable probability to its present state of an enormous accumulation of galaxies and galaxy clusters expanding in a way approximated by Hubble's empirical law.

The natural assumption as to its beginning would seem to be a big cloud of stray particles of the simplest and stablest kind—protons and electrons and, perhaps, hydrogen atoms—very thin, but somewhat denser than its surroundings, slowly contracting because of the gravitational attraction of its parts. A first suggestion as to its development occurred to me when reading the chapter in Bondi's book on cosmology (*9*) on the Eddington relations. According to them the mass of what Eddington regarded as the universe—a closed Einstein space—and its linear dimensions would be of the order of magnitude $N^2 m_p$ and Nd, respectively, where m_p is the proton mass, d the so-called radius of the electron and N the ratio of the electrostatic and the gravitational attraction of a proton and an electron. Hence

$$M \sim N^2 m_p, R \sim Nd \quad (17)$$

with

$$N = \frac{e^2}{G m_p m_e} = 2.27 \times 10^{39}$$
$$d = \frac{e^2}{m_e c^2} = 2.82 \times 10^{-13} \text{ cm} \quad (18)$$

e being the elementary quantum of electricity and m_e the mass of the electron. These relations obtained some further support by the fact that the Hubble parameter $1/T$ is of the order of magnitude c/Nd. But to consider them as indicating a "deep" connection between the laws governing the Macrocosmos and those governing the Microcosmos, as Eddington and his followers did, seemed to me a most doubtful deviation from the main trend of physics. However, although the cosmological quantities are still rather uncertain, to regard these relations as mere chance coincidences would seem an exaggeration to the opposite side. Thus, I thought that it would be worthwhile to look for a natural origin of them, which might tell us something about the main forces at work in the development of the original contracting cloud to the present expanding state of the metagalaxy. This view is strengthened by the fact that the length a_0, which is constant during the expansion, is of the order of magnitude Nd according to the above estimates (in fact almost exactly $2Nd$, which is probably a chance coincidence in view of the uncertainty of the values of T and μ used).

Thus, on the assumption that electromagnetic radiation formed during the contraction of the cloud—due to the presence of electric particles and magnetic fields—played a main role in turning contraction into expansion, due to Thomson scattering by the electrons, the opacity κ of the cloud would be an important quantity. We define it as the ratio between the "radius" $R = a\eta_B$ of the cloud and the mean free path of a photon according to the Thomson scattering formula

$$1/(8/3)\pi d^2 n_e$$

where n_e is the average number of electrons per unit volume in the cloud. Hence

$$\kappa = \frac{8\pi}{3} d^2 n_e a \eta_B \quad (19)$$

In my first attempt I assumed the opacity to be of the order of magnitude unity at an early state of the metagalaxy, when its mass was approaching the highest value permitted by the Schwarzschild condition. With n_p, the number of protons per unit volume, equal to n_e—as would be demanded by electrical neutrality if only ordinary matter were present—this gave rise to the following relations

$$\frac{8\pi}{3} d^2 \frac{\mu}{m_p} R = \frac{2d^2}{R^2} \frac{M}{m_p} \sim 1,$$
$$\frac{2GM}{c^2 R} \sim 1 \quad (20)$$

from which follows

$$M \sim N^2 m_p, \; R \sim Nd \quad (21)$$

in accordance with Eddington's relations.

This result led me to the introduction of Nd and $N^2 m_p$ as units of length and mass, respectively, in the equations describing the evolution of the cloud. In fact, if the main forces governing this evolution are gravitation and the mentioned action of radiation, those equations take a most simple form, giving a more general meaning to the Eddington relations than the estimates (Eq. 21).

The assumption of matter and antimatter in equal amounts came as an afterthought, although the possibility of stars of antimatter, not to be distinguished from ordinary stars by observing their spectra, was evident to me as to most physicists from the early 1930's, when the discovery of the positive electron seemed to restore the symmetry of positive and negative electricity. However, the problem here was to find a reasonable separation process available at the low, average density and temperature of the metagalaxy according to the proposed model. For this purpose, after my general arguments were sufficiently clear to give me confidence of being on the right track, I turned to Alfvén, who thereupon became most enthusiastic of the possibility that annihilation constitutes a hitherto neglected cosmic force. This then led to a number of promising attempts both regarding the origin of the galaxies as well as to a further development of the mentioned attempt to describe the evolution of the metagalaxy (*10*). Among the latter I particularly mention work by Laurent and Söderholm (*11*), where the mentioned relativistic equations are not only generalized by the inclusion of annihilation and a better treatment of the behavior of the radiation at the surface of the cloud but also by subjecting them to a numerical treatment generalizing an earlier nonrelativistic treatment by Bonnevier (*12*). Here we shall not enter further on these problems, the aim of this article being to present the more convincing general arguments for replacing relativistic cosmology by a bounded model of the metagalaxy.

Returning to the relations of Eq. 21, it should be mentioned that, from the above estimates, we get the following values of the mass M, of the "radius" R_s and the opacity κ_s at the Schwarzschild limit, which are constants of the motion, whether this limit is approached or not, namely

$$M = N^2 m_p \eta_B{}^3$$
$$R_s = 2Nd\eta_B{}^3$$
$$\kappa_s = \frac{1}{2\eta_B{}^3} \times \frac{n_e}{n_p} \quad (22)$$

One further point may be added to these conjectures. If the above estimates are approximately correct, and if the same holds for the assumption $\eta_B = 1$ (reasonable if the quasar red shifts are "cosmological"—which seems still doubtful) we have a rather strong general argument from Eq. 12 in favor of annihilation having played a role during the evolution of the metagalaxy. Thus, on the assumption of an original cloud of very small density, we shall expect that its mass M' was practically equal to its rest mass. During the evolution an amount ΔM of mass is lost by radiation including neutrinos, and, mainly due to annihilation, an amount ΔM_0 of rest-mass is lost. Thus we shall have

$$M' = M + \Delta M = M_0 + \Delta M_0$$

or

$$\frac{\Delta M_0}{M} = \frac{\Delta M}{M} + \frac{M - M_0}{M} \quad (23)$$

Now, it follows from Eq. 12 that M, the present mass, is greater than M_0—owing to the loss of rest mass and gain of kinetic energy, both probably because of annihilation. With $\eta_B = 1$ we get thus $(M - M_0)/M = 0.2$, implying that ΔM_0 is more than 20 percent of the present mass.

So-Called Fireball Radiation

The discovery of an isotropic electromagnetic radiation corresponding approximately to a temperature of 3°K, predicted many years ago by Gamow as the remainder of the so-called fireball radiation (belonging to an early stage of a cosmological solution), is still considered as the *pièce de résistance* of relativistic cosmology. It is therefore interesting that the existence of such a radiation is by no means a priori excluded in the present model. In fact, its being what has been left over from the acceleration process mentioned in the former section seems even probable. Thus, if we first disregard the effect of the open boundary on what happens in this respect in our neighborhood—which seems to be far away from the boundary, the same Friedmann solution as that used by Gamow is also possible in the bounded model. This may be defined by the following equations

$$a_0 = \frac{8\pi G\mu}{3c^2} a^3$$
$$\frac{8\pi G u}{3c^4} a^4 = k a_0{}^2$$
$$\frac{8\pi G}{3} \mu T^2 = \frac{a_0}{a_0 + a\left(1 + k\frac{a_0{}^2}{a^2}\right)} \quad (24)$$

Here k is a dimensionless constant, which, with the above assumptions about μ and T and if we assume that the energy density u corresponds to that at 3°K, is equal to 8.4×10^{-3}.

Now, this solution is exactly isotropic, there being no energy current *in the expanding frame*, a state which could certainly not be maintained with the open boundary, although the gravitation would, to a certain extent, stabilize it. Somewhat schematically we shall assume that the end of the acceleration process [at $a = a(0)$] leaves the radiation in approximate temperature equilibrium with the electrons, thereby following the expansion of the cloud, the energy current density S being equal to zero. Since $k(a_0{}^2/a^2)$ is very small in the range in question, we may now neglect it when studying the further development of the radiation. We shall then be somewhere between two extremes: that the opacity is still being big enough to maintain the equilibrium of the radiation with the electrons—in this case the current will stay zero—or that the opacity is negligible, the radiation expanding in the gravitational field given by the metric (Eq. 24) belonging to $k = 0$. The latter case, still based on the assumption of spherical symmetry, is governed by the energy-momentum equations and takes here the form

$$a\frac{\partial}{\partial\tau}(ua^4) + \frac{1}{c}\frac{(1+\eta^2)^{\frac{1}{2}}}{\eta^2} \cdot \frac{\partial}{\partial\eta}\left[\frac{\eta^2 S a^4}{(1+\eta^2)^{\frac{1}{2}}}\right] = 0$$
$$a\frac{\partial}{\partial\tau}\left(\frac{Sa^4}{1+\eta^2}\right) + \frac{c}{3}\frac{\partial}{\partial\eta}(ua^4) = 0 \quad (25)$$

These equations, which in a geodesic frame go over into the corresponding classical equations, have the following rigorous solutions

$$u = \frac{u_0 a_0{}^4}{a^4}[1 - \lambda(1+\eta^2)^{\frac{1}{2}}]$$
$$S = \frac{c}{3} u_0 \frac{a_0{}^4}{a^4} \cdot a\lambda \frac{d\lambda}{d\tau}(1+\eta^2)^{\frac{1}{2}}\eta \quad (26)$$

where λ is a function of τ alone, satisfying the equation

$$a\frac{d}{d\tau}\left(a\frac{d\lambda}{d\tau}\right) = \lambda \quad (27)$$

Since a is a function of τ, we may introduce a variable ξ defined by

$$d\xi = \frac{d\tau}{a} = \frac{da}{[a(a_0 + a)]^{\frac{1}{2}}} \quad (28)$$

the latter expression following from the relations defining the metric.

Putting $\xi = \xi_0$ and $\lambda = \lambda_0$ for $a = a(0)$, the state supposed to have no current, we have

$$a = a_0 \cdot \sinh^2\frac{\xi}{2}$$
$$\lambda = \lambda_0 \cosh(\xi - \xi_0) \quad (29)$$

This solution should be completed by a study of the behavior of the radiation beyond the boundary, where the gravitational field is given by the Schwarzschild-Droste solution, a problem we shall not enter on here. It is probable, however, that λ_0 is small and also that $a(0)$ has a value greater than a_0, making $\xi - \xi_0$ (ξ corresponding to the present value of a) comparatively small. Also the red-shift observations make it probable that the value of η_0 (our η) is rather small compared to η_B. Writing S in the form

$$S = \frac{c}{3} u \times \frac{\lambda_0 \sinh(\xi - \xi_0)}{1 - \lambda_0(1+\eta^2)^{\frac{1}{2}}} \times (1+\eta^2)^{\frac{1}{2}}\eta \quad (30)$$

it follows that the present value of S is probably small compared with $(c/3)u$, which corresponds to the isotropic part of the observed radiation.

As to the four constants appearing in the above formula, u_0, λ_0, ξ_0, and η_0, only one relation between them is known from observations of the radiation. The eventual discovery of a slight deviation from its isotropy may lead to one more relation between the constants and also to the knowledge of the direction from our place to the center of the metagalaxy, the value of η_0 being possibly determined by means of red-shift observations as mentioned above.

Summary

In the first place I have reviewed the true foundation of Einstein's theory of general relativity, the so-called *principle of equivalence*, according to which there is no essential difference between "genuine" gravitation and inertial forces, well known from accelerated vehicles. By means of a comparison with Gaussian geometry of curved surfaces—the background of Riemannian geometry, the tool used by Einstein for the mathematical formulation of his theory—it is made clear that this principle is incompatible with the idea proposed by Mach and accepted by Einstein as an incitement to his attempt to describe the main situation in the universe as an analogy in three dimensions to the closed surface of a sphere. In the later attempts toward a mathematical description of the universe, where Einstein's cosmology was adapted to the discovery by Hubble that its observed part is expanding, the so-called cosmological postulate has been used as a kind of axiomatic background which, when analyzed, makes it probable that this expansion is shared by a very big, but still bounded system. This implies that our expanding metagalaxy is probably just one of a type of stellar objects in different phases of evolution, some expanding and some contracting. Some attempts toward the description of this evolution are sketched in the article with the hope that further investigation, theoretical and observational, may lead to an interesting advance in this part of astrophysics.

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Interpersonal and Economic Resources

Their structure and differential properties offer new insight into problems of modern society.

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Man doth not live by bread only.
DEUTERONOMY 8 : 3

Human needs are seldom satisfied in solitude; because people depend on one another for the material and psychological resources necessary to their well-being, they associate to exchange these resources through interpersonal behavior. In the study of these exchanges there has been a traditional division of tasks. Economists have long been concerned with the exchange of money with goods, and, more recently, with labor and with information, while psychologists and sociologists (*1*) have investigated transactions that involve more subtle resources, such as attraction, devotion and affect, esteem, respect, and status. This professional specialization does not, however, obviate the fact that the same behavior is often influenced by both economic and noneconomic factors: one may, for example, prefer a less paid but prestigious job to another where salary is higher but status is lower; and a small shop may attract customers by giving them the individual attention they miss at the less expensive but more impersonal department store. In view of this interplay of economic and noneconomic resources in the conduct of human affairs, it appears unrealistic to expect that social problems will be solved by material means alone. ". . . There are no 'economic' problems; there are simply problems and they are complex," observes Myrdal (*2*) in discussing international development. Closer to home one can see model housing projects built a few years ago turning into model slums, possibly because their dwellers were provided with houses, but not with self-pride and a sense of community.

Attempts to bridge the dichotomy between economic and noneconomic resources came mainly from sociologists and social psychologists (*3*) who sought to interpret every interpersonal behavior as an exchange, characterized by profit and loss. Extension of the economic model to noneconomic resources, however, produced difficulties for the social exchange theory. The fact, for instance, that resources like information and love can be given to others without reducing the amount possessed by the giver has been considered contradictory to the very notion of exchange (*4*) since this effect does not occur in transactions of money and goods. Likewise it makes little sense to consider economic transactions of a person with himself; one can, on

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