Book Reviews

Geometrical Thinking

Space through the Ages. The Evolution of Geometrical Ideas from Pythagoras to Hilbert and Einstein. CORNELIUS LANCZOS. Academic Press, New York, 1970. x, 322 pp., illus. $11.50.

When Einstein, in 1928, was looking for a capable assistant, he happened to meet a 35-year old Hungarian mathematician who was working at the University of Frankfurt-am-Main. Impressed by the mathematician's knowledge and personality, Einstein invited him at once to work on certain problems in general relativity. At the first session Einstein asked him to solve a new type of field equation so that certain conditions would be satisfied. Proud of this assignment, Einstein's new collaborator got deeply absorbed in the problem and after three or four days, to his greatest satisfaction, found the required solution. At the appointed hour, happy about his success, he showed Einstein that the solution indeed met all the conditions required. "Quite remarkable," replied Einstein, adding after a short silence, "But don't you see, I gave you the wrong equation."

Einstein's collaborator of those days is the author of the present monograph on the evolution of geometrical thinking from the distant past to the present. Cornelius Lanczos, since 1954 professor at the Dublin Institute for Advanced Studies, is well known to scientists for his texts *The Variational Principles of Mechanics, Linear Differential Operations*, and *Applied Analysis*, as well as for his profound papers in professional journals, and to nonspecialists for his popular *Albert Einstein and the Cosmic World Order*.

Lanczos's latest publication lies between these two extremes; based on a course of about 40 lectures delivered to mathematicians, physicists, chemists, and engineers at North Carolina State University in 1968, it assumes some general familiarity with the elements of higher mathematics but no specialized knowledge of any of its branches. In fact, it is designed to present the history of geometrical thinking and its import for science not only to mathematicians but also to physicists, philosophers, logicians, and all those to whom man's conceptions of space are of interest.

The first three chapters of the book, about one third of the text, are devoted to a bird's-eye view of the historical development of geometrical ideas from the great civilizations of antiquity, through the Greek miracle, the times of Descartes and of Kant, the discovery of non-Euclidean geometries, up to the work of Minkowski and Einstein. The next three chapters offer a detailed reformulation of the monumental geometrical achievements of Gauss and Riemann and contain a lucid presentation of tensor algebra and tensor analysis. As the author rightly remarks, a student interested in relativity could, without preliminaries, come into possession of the mathematical background of Einstein's papers on the basis of these three chapters alone. In the next two chapters Einstein's theory of gravitation as an application of Riemannian geometry, and abstract spaces, such as the Hilbert space, so fundamental for the mathematical formalism of quantum mechanics, are discussed in an interesting and often original manner, interspersed with many apropos strokes of wit. The last chapter, the only one concerned with nonmetrical structures, deals with the foundations of projective geometry and, to bring it into the context of the preceding material, concludes with an exposition of Klein's ingenious establishment of a complete isomorphism between projective geometry of the Euclidean plane and the metrical geometry of the hyperbolic type (Bolyai-Lobachevski).

Obviously it was not the author's intention to offer an exhaustive history of geometrical thinking and its applications to other fields within and without mathematics. Surely conceptions such as Minkowski's "geometry of numbers" and its use in number theory, or "Minkowski's geometry"—not to be confused with the geometry of "Minkowski space"—to mention only two examples, are legitimately ignored, for they did little affect the mainstream of geometrical thinking. The material has been selected very well. Still, some readers may regret the almost complete exclusion of any topological subjects; for, although at present only of marginal importance (as for example in the geometrodynamics of Wheeler, Misner, *et al.*), topology may well offer fundamentally new avenues to profound insights in science. Other readers may miss a chapter in which an expert like Lanczos explains his view on epistemological issues connected with geometrical thinking, such as the controversy between the empiricist, conventionalist, and absolutist conceptions of congruence.

The most valuable feature of the book, however, lies not in its mere presentation of information but in its contribution to the furthering of what Felix Klein used to call "general mathematical culture." One of the means by which Lanczos achieves this laudable aim is his great skill in interpreting the formulas of geometry by revealing their association with other branches in science—a technique in the acquisition of which his experience of 1928 was probably of some avail. Also his optimism that, although Einstein's efforts in formulating a unified theory accounting for all physical effects on the basis of geometrical conceptions alone have not been successful, "his program will be tried again and again, until one day full victory will come" (p. 98) seems to stem likewise from his intimate collaboration with Albert Einstein.

Geometry has sometimes been called the art of reasoning correctly about figures which are poorly constructed. Cornelius Lanczos's book shows that geometry is more than that.

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Values and Experience

Class and Conformity. A Study in Values. MELVIN L. KOHN. Dorsey, Homewood, Ill., and Irwin-Dorsey, Georgetown, Ontario, 1969. xxiv, 320 pp. Cloth, $8.65; paper, $5.65. Dorsey Series in Anthropology and Sociology.

A recurring theme in sociological theory is that the social environment in which an individual finds himself will be responsible in large measure for the way he views the world around him and, as well, for a variety of subjective states. One of the most enduring aspects of social structure is the hier-