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* Present address: Department of Materials Science and Institute of Meteoritics, University of New Mexico, Albuquerque 87106.

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Where Was the Moon Formed?

Abstract. *Volatile substances have a low abundance in lunar surface rocks as compared to terrestrial rocks. If this depletion is explained in terms of a late accretion of volatile materials from a solar nebula with falling temperature, then the conclusion can be drawn that the moon accumulated not in earth orbit but as a separate planet, and that it was later captured by the earth.*

Anders and his colleagues (*1*) have proposed an interesting and far-reaching idea, namely, that the depletion of volatile substances (such as lead, bismuth, and thallium) in lunar rocks relative to terrestrial basalts might be due to the process of accretion rather than to a local heating and evaporation of these elements. One can postulate an inhomogeneous accretion process, which proceeds as the solar nebula is cooling so that the most volatile elements are condensed and accreted as a thin veneer on a nearly completed planet.

The first compounds to condense would be those of the refractory metals Ti, Mo, Nb, and Zr; then iron at about 1500°K, later nickel at about 1350°K, followed by various silicates. By the time a temperature of about 1000°K had been reached, all major elements and compounds would have condensed and the accumulation of planetary bodies could have proceeded toward completion. In the terminal stage of accretion, when the temperature had dropped below 600°K, volatile substances such as Pb, Bi, Tl, and In would have condensed (*2*).

Anders and his colleagues further suggested that the 10- to 100-fold depletion of volatile materials in lunar rocks can be explained in terms of the dynamics of the accretion process, and that the accretion rate of the moon would depend very much on the earth-moon distance (*1*).

From the point of view of theories of lunar origin, it is then important to know whether the accretion of the moon, including the final accretion of volatile substances, took place well outside of the earth's gravitational field or whether this accretion took place while the moon was in orbit around the earth. According to the first possibility, the moon would have to be captured subsequent to its formation. According to the second possibility, the moon formed from material in earth orbit and therefore never underwent capture.

We shall therefore calculate the ratio of the accretion rate for the earth to the accretion rate for the moon, ratio (earth : moon), as a function of the earth-moon distance, and compare this ratio with the experimentally observed ratio of 10 to 100.

Table 1. Ratio of specific accretion rates of earth and moon Z_E/Z_M as a function of earth-moon distance.

Geocentric velocity	Earth-moon distance d (earth radii)					
	1*	5	10	50	100	∞
$c << w_M$:	(0.83)	3.6	6.2	14.6	17.6	22
$c = w_E$:	(0.905)	1.56	1.71	1.86	1.88	1.9

* Calculated for a transparent earth. With a solid earth the distribution of accreting particles near the earth's surface is semi-isotropic, and the ratios at $d = 1$ should be multiplied by a correction factor $F = 2$. The correction becomes rapidly negligible with increasing distance (*4*), being < 10 percent at $d = 3$.

We assume an isotropic gaussian velocity distribution for the particles (containing volatile materials) relative to a circular orbit (at 1 astronomical unit) around the sun. The most probable speed in that reference frame is c, and the concentration of particles is N. The earth then accretes the volatile substances at a rate

$$A_E = 2\sqrt{\pi} R_E^2 \; N \; c \left(1 + \frac{w_E^2}{c^2}\right) \quad (1)$$

where w_E is the escape velocity from the earth's surface and R_E is the earth's radius. A moon in a circular orbit around the earth at a distance of d earth radii accretes this dust at a rate

$$A_M = 2\sqrt{\pi} \, R_M^2 \, N \, c \left(1 + \frac{w_M^2}{c^2} + \frac{7}{6}\frac{1}{d}\frac{w_E^2}{c^2}\right) \quad (2)$$

where R_M is the moon's radius and w_M is the escape velocity from the moon's surface.

These expressions for accretion rates are derived as follows (*3*). We assume that far from the earth the phase space density of the dust is given by

$$f_\infty(\mathbf{v}_\infty) = \frac{N}{(c\sqrt{\pi})^3} \exp\left(\frac{-v_\infty^2}{c^2}\right) \quad (3)$$

This equation describes an isotropic gaussian velocity distribution of geocentric speeds v_∞ and some most probable speed c, and a number concentration N. By Liouville's theorem, the phase space density at finite distances d from the earth's center is for a transparent earth exactly equal to that given in Eq. 3, since the gravitational field is conservative of energy (*4*); that is,

$$f_d(\mathbf{v}) = f_\infty(\mathbf{v}_\infty) \quad (4)$$

where

$$v^2 = v_\infty^2 + 2GM_E/d \quad (5)$$

and G is the gravitational constant and M_E is the mass of the earth. Let the selenocentric velocity vector be $\mathbf{u}$. The moon's dust accretion rate can then formally be written as

$$A_M = \iiint f_d(\mathbf{v}) u \; S_v \; d^3v \quad (6)$$

where S_v is the effective accretion cross section of the moon for a given (geocentric) velocity $\mathbf{v}$, and d^3v is an element of volume in velocity space. If r_b is the radius of the moon's sphere of gravitational influence, then from the laws of conservation of energy and angular momentum one easily finds

$$S_v = \pi R_M^2 \left[1 + \left(\frac{w_M^2}{u^2}\right)\left(1 - \frac{R_M}{r_b}\right)\right] \quad (7)$$

Now S_v depends only on u, and f_d depends only on v_∞; therefore, it is convenient to change the integration variables in Eq. 6 as follows. Let the moon's orbital velocity about the earth be $\mathbf{v}_0$; then

$$u^2 = v^2 + v_0^2 - 2\, v\, v_0 \cos \psi \qquad (8)$$

Furthermore,

$$d^3v = v^2\, dv \sin \psi\, d\psi\, d\phi \qquad (9)$$

where ψ is the angle between $\mathbf{v}$ and $\mathbf{v}_0$, and ϕ is the angle between the $\mathbf{v} - \mathbf{v}_0$ plane and the earth-moon line.

From Eq. 5 we find

$$v\, dv = v_\infty\, dv_\infty \qquad (10)$$

Furthermore,

$$\sin \psi\, (d\psi)_v = \frac{u\, du}{v\, v_0} \qquad (11)$$

We substitute these results into Eq. 6. Integration over ϕ gives a factor 2π. Upon rearranging, we have

$$A_M = \frac{2\pi}{v_0} \int_0^\infty f_\infty \cdot v_\infty\, dv_\infty \int_{b - v_0}^{b + v_0} S_v\, u^2\, du \qquad (12)$$

where

$$b \equiv \sqrt{v_\infty^2 + 2 v_0^2}$$

Since $R_M / r_b \ll 1$, we obtain

$$A_M \simeq 2\sqrt{\pi} R_M^2 N c \times \left[1 + \left(\frac{7}{3}\right)\left(\frac{v_0^2}{c^2}\right) + \left(\frac{w_M^2}{c^2}\right)\right] \qquad (13)$$

Using

$$w_E = v_0 \sqrt{2d} \qquad (14)$$

we derive Eq. 2 for the moon's accretion rate as a function of the earth-moon distance. As we let d approach ∞, we obtain the accretion rate for a "free" moon, which is analogous to Eq. 1.

We next proceed to calculate the ratio (earth : moon) for the specific accretion rates, referred to the unit surface area of the planetary body:

$$\frac{Z_E}{Z_M} \equiv \left(\frac{A_E}{A_M}\right)\left(\frac{R_M}{R_E}\right)^2 = \frac{[1 + (w_E^2/c^2)]}{[1 + (w_M^2/c^2) + (7/6d)(w_E^2/c^2)]} \qquad (15)$$

We can now discuss two cases, which depend on the value of most probable geocentric velocity c.

Case I: $c \ll w_M < w_E$; ($w_E = 11.2$ km/sec; $w_M = 2.38$ km/sec; $w_M/w_E = 0.213$):

$$\frac{Z_E}{Z_M} = \frac{w_E^2}{w_M^2 + (7/6d) w_E^2} = \frac{1}{0.045 + 1.16/d} \qquad (16)$$

Case II: $c \sim w_E$:

$$\frac{Z_E}{Z_M} = \frac{[1 + (w_E^2/c^2)]}{[1.05 + (1.16/d)(w_E^2/c^2)]} \qquad (17)$$

For $c = w_E$:

$$\frac{Z_E}{Z_M} = \frac{2}{1.05 + 1.16/d} \qquad (18)$$

Results for both cases are shown in Table 1.

It is useful also to examine the case $d \to \infty$, that is, the accretion ratio for "free" planets. From Eq. 15 we have

$$\frac{Z_E}{Z_M} = \frac{c^2 + w_E^2}{c^2 + w_M^2} \qquad (19)$$

The dependence on geocentric velocity c is shown in Table 2 for a moon outside of the earth's gravity.

Table 2. Ratio (earth : moon) of specific accretion Z_E/Z_M as a function of geocentric velocity c for the case of an independent moon.

Z_E/Z_M	c (km/sec)
22	0
18.8	1
11.5	2.32
4.9	5
1.9	11.2
1.1	29.8

Three points should be noted from Tables 1 and 2.

1) The absolute value of the ratio (earth : moon) for the accretion of volatile materials rises as high as 22, when referred to the surface areas. If, on the other hand, we consider that the volatile substances have been mixed with the whole planet, then the ratios in Table 1 should be multiplied by

$$(R_M \rho_M / R_E \rho_E) = 0.16$$

where ρ_M is the density of the moon and ρ_E is the density of the earth. This would make the maximum ratio about 3.6, well below the value of 10 to 100 seemingly required by the Apollo data.

2) If we assume no large-scale mixing following accretion, then the accretion model corresponding most closely to what is observed calls for the moon to be accumulated in a heliocentric orbit similar to that of the earth, with the "volatile particles" having also very nearly circular orbits. However, in this case (that is, for very small values of c), the accretion ratio depends very strongly on the orbit eccentricity e of both the earth and the moon; for example, the ratio is depressed below 22 if $e_M > e_E$, and is raised above 22 if $e_M < e_E$.

3) For free planets (that is, $d = \infty$) the accretion ratio rises from the value of 1 (if the planetocentric velocities are very large) to values of the order of the ratio of the square of the escape velocity, that is, w_E^2/w_M^2 or $\rho_E R_E^2/\rho_M R_M^2$. (The observed depletion of volatile substances in chondrites could be explained on the basis that during the accretion process chondrites were smaller than their competitors!)

At face value, these results are in agreement with the idea that the moon formed at a large distance from the earth, well outside of its gravitational field. After formation, both planets accreted a veneer of volatile materials which had orbits very similar to those of the moon and the earth, that is, nearly circular heliocentric orbits. This model comes closest to meeting the observed abundances of lead, thallium, and bismuth in the basaltic rocks of Apollo 11 and Apollo 12.

If this model is correct, then the moon was formed independently of the earth and later captured, presumably by a three-body interaction, and these events were followed by the dissipation of the excess energy through tidal friction in a close encounter (5).

S. F. SINGER

Office of the Secretary, U.S. Department of the Interior, Washington, D.C. 20240

L. W. BANDERMANN

Institute for Astronomy, University of Hawaii, Honolulu 96822

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