Reports

Apollo 12 Seismic Signal: Indication of a Deep Layer of Powder

Abstract. *The seismic signal caused by the Apollo 12 lunar module is interpreted in terms of propagation between source and receiver through a layer of powder in which sound velocity increases with depth. This increase, which is due to compaction, extends over several kilometers and leads to a concentration of seismic waves toward the surface. Computer simulations with the use of ray acoustics and on the assumption of a randomly undulating lunar surface approximate well the observed signal. Seismic amplitudes are greatly enhanced in such a medium compared to solid rock, so that the observed signal requires less power to be transmitted than previously estimated.*

The seismograph carried by the Apollo 12 mission gave results (*1*) that appear to be of great importance for the understanding of the lunar surface. The seismic signals, recorded at an epicentral distance of about 76 km from the lunar module (LM) impact, were totally different from any that have ever been seen on the earth. The same kind of signal, but with lower intensity, was recorded on many other occasions, while no other type of signal that appeared was recognized for certain as being of lunar origin. Meteorite impacts may well have been responsible for all signals of this type [referred to as type "L" by Latham *et al.* (*1*)].

We shall discuss here only the signal due to the spacecraft impact, not only because this was the largest and clearest signal, but also because the associated time and distance were known independently. The knowledge that other signals of a similar type are common indicates that the explanation has to be of such a nature that it can be generally applied, rather than that it is specific to a particular configuration or set of events connected with the spacecraft impact.

The signal for the spacecraft impact, as observed on the instrument recording vertical displacements, is shown in Fig. 1. The spectral content is between 1 and 6 hz. For comparison Fig. 2 (*2*) shows the type of wave train that would be observed in similar circumstances on the earth. While of course there is a good deal of variation, in all cases the terrestrial signal would have risen to a maximum very much sooner, in the interval when the lunar signal is still exceedingly small and the whole phenomenon would have been over before the lunar signal had even reached its maximum.

There are a number of other differences that have to be considered also. On the earth a large fraction of the wave energy in such a case would be due to surface waves in a solid material capable of sustaining shear forces. Such waves usually show the phenomenon of dispersion, so that different frequency components arrive at different times. No clear evidence of dispersion appears to have been seen in the lunar signals. Second, a transverse wave possesses polarization which, in a three-axis instrument as was employed on the moon, would result in general in a correlation between the outputs of the separate channels. For a pressure wave the three outputs may of course be quite independent and show no correlation. In all lunar "L type" signals, including the spacecraft impact signal, no correlation is seen. Multiply reflected surface waves could cause some of this evidence to be lost, but nevertheless, in the complete absence of dispersion or polarization for most of the received signal power, it seems probable that the signals result mainly from pressure waves.

The first conclusion that seems to emerge is that on the moon there is not a sheet of solid rock providing the usual type of seismic channel between the impact and the receiving site. Had there been such a sheet of rock, even with an overlay of a few meters of powder, the terrestrial-type signals would have been seen. Since these would have occurred early, before the observed signal had risen much, they would not have been obscured. A few meters of soft ground overlying rock might lower the amplitudes received by decreasing the coupling, but since such dimensions would be very short fractions of the wavelength, no great change in the nature of the signal could be expected. The conclusion is therefore that a continuous sheet rock at a shallow depth is absent on the moon in the region of observation.

The next hypothesis that might be discussed is that there is a sheet of rock but that it is not continuous. On the moon, in the absence of water percolating down from the surface, cracks in the rock may fail to become densely filled and cemented, down to a depth of as much as 50 km. Acoustically such cracks may represent boundaries of very much greater compliance and, therefore, provide a substantial reflection for both P and S waves (pressure and shear waves). Could the observed signal have been produced by sound waves that were so multiply reflected as a consequence of such cracks that the result is a diffusion-like propagation only? This suggestion is made by Latham *et al.*, who point out that the smoothed-out signal would fit a diffusion phenomenon from a sudden and localized source. They also stress, however, some of the difficulties associated with this interpretation. These concern the very high degree of reflection necessary between boundaries of the rocks and the very low attenuation that would be required at the same time. The last signal, for example, seen after 55 minutes, had it traveled as a wave of mean velocity 4 km sec^{-1}, would have covered a cumulative distance of 13,200 km before

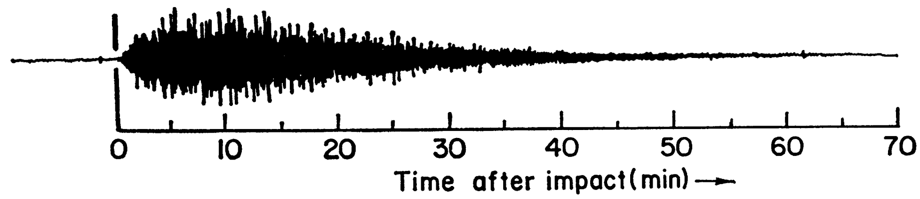

Fig. 1. Signal received by the long period vertical component lunar seismometer, due to the LM impact at a distance of 75.9 km [after Latham *et al.* (*1*)]. The signal takes 4 to 8 minutes to rise to maximum amplitude and is still detectable nearly 1 hour later.

detection, and the fact that it is still seen would imply a remarkably low physical attenuation. On the other hand, just the circumstances of a heterogeneous medium with internal boundaries of a large change in compression and shear modulus is usually a situation of very high attenuation. A further objection to such a model is that it does not account for the absence of evidence for S waves.

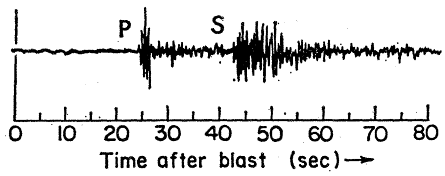

Fig. 2. Vertical component of a signal recorded at the Harvard Seismograph Station, as a result of a dynamite blast at 152.5 km [after Leet (*2*)]. The north-south and east-west components were very similar. Although the path is twice that for the Apollo 12 LM impact, the main portion of this terrestrial signal, consisting of distinct P and S waves (pressure and shear waves), has registered itself within the first minute, and no significant signal amplitude is observed thereafter.

We have investigated the circumstances of acoustic wave propagation in deep deposits of powder in which the velocity of sound increases with depth as a result of compaction. Uncompacted rock powders can have propagation speeds for P waves as low as 100 m sec^{-1}. At full compaction, at whatever depth this may occur, the P wave speed may be $\sim$6 km sec^{-1}. In such a situation there would be a most unusual propagation channel in which the velocity of sound changes by a large factor between the top and the layer of full compression. The acoustic properties of a powder that are relevant to this discussion have not yet been adequately observed. However, some theoretical considerations, partially corroborated by experimental evidence, should be mentioned.

The attenuation of both P and S waves in a powder is extremely dependent on amplitude. For large amplitudes irreversible changes take place at the contact points between grains, and energy is dissipated there. Even without any sliding at contact points, the deformation of the solid is greatly enhanced compared with any deformation that would occur for a wave of similar energy traveling through a compact solid, the ratio of enhancement being of the order of that of the cross-sectional area of the grain to the effective area of the contact point. We can at present only guess how large this factor may be, but it is probably more than 10,000. The amplitude at which the comparatively nondissipative propagation would set in is therefore smaller by some such factor than would be the case for a compact solid.

Almost all laboratory investigations of powders have been concerned with amplitudes far too large to come into the nondissipative regime and are thus not representative of the very low amplitudes with which we are concerned over most of the transmission path in the lunar case. Once the amplitudes are so low that at contact points on an atomic scale no displacements are produced and the deformation of the material is within its most accurately elastic range, then there is no reason to expect any higher attenuation than would be the case in the compact solid (*3*).

A deep layer of dust on the moon may thus provide a very good acoustic transmission channel. For such a medium, the sensitivity of the usual type of seismometer is greatly enhanced over that on or close to solid rock. The reason for this is that the instrument is not matched to the wave, but merely records the local value of the displacement amplitude. The relation between the power P crossing unit area and the displacement amplitude a is given by

$$P = \tfrac{1}{2}\, a^2 \omega^2 \rho c$$

where ω is the frequency, ρ is the density of the medium, and c is the velocity of sound. If the amplitude is observed, the power in the wave that is inferred is thus proportional to ρc. For the lunar surface material the density may be one-half, and the velocity of sound would be 1/40 that of the compact rock, so that 80 times less power will produce a given displacement amplitude. This anomalously high sensitivity of the instrument is an important factor in accounting for the apparent great strength of the received signal and is relevant to any discussion of the conversion efficiency of impact to seismic energy and of attenuation along the path.

In order to gain some insight into the acoustic properties of such a medium, we have performed a computer simulation with the use of simple acoustic ray theory. We consider a medium having a certain low sound velocity at its upper surface which increases rapidly with depth. We make a number of simplifying assumptions for the calculation which will no doubt cause it to be inaccurate but nevertheless leave it showing the main features of the phenomenon.

First, we deal only with ray theory rather than with the complete wave equation. This will be inaccurate where the change in the velocity of sound is large over a distance of a wavelength, and of course the conversion on the surface between P and S waves will be omitted. We believe in fact that only P waves are of primary concern here. This is because near the source a yet smaller amplitude of S waves has to be reached before linear propagation characteristics commence, and because at surface reflections the conversion from P to S waves is not favored in uncompacted powder of exceedingly low shear modulus.

A second simplification for the calculation is that we deal with the two-dimensional model only, in which the rays never deviate from the vertical plane containing the impact point and the seismometer. For the case we consider, in which sound speed increases monotonically with depth, all rays are refracted to make trajectories concave upward. For the simple case of a linear velocity profile, all the rays are in fact circular arcs. We assume that the velocity c varies with depth z according to

$$c = c_0\,(1 + Az)$$

where c_0 and A are constants. Then, as shown by Brekhovskikh (*4*) for the analogous case of underwater propagation, a ray reflected from the surface at an angle χ_i below the horizontal will return to the surface after a time

$$t_i = \frac{1}{Ac_0} \ln \frac{1 + \sin \chi_i}{1 - \sin \chi_i}$$

having traversed a horizontal distance given by

$$r_i = \frac{2}{A} \tan \chi_i$$

If the surface of the moon were perfectly flat, then a ray leaving the origin initially at an angle χ_0 would be reflected into the angle χ_0 at each subsequent step, and after n cycles would have moved downrange a distance nr_0 in a time nt_0.

We know, however, that the lunar surface in the vicinity of the seismic experiment is not flat but is composed of gentle undulations characteristic of mare regions. If the undulations are random, then at the ith reflection, a ray will be sent away at an angle

$$\chi_i = \chi_{i-1} - 2\alpha_i$$

where α_i is the angle of random slope encountered (positive if tilted up away from the origin). After n reflections, the ray will have traveled from the origin a net distance

$$R_n = \frac{2}{A} \mid \sum_{i=0}^{n} \tan \chi_i \mid$$

Since the α_i are given random signs and magnitudes (up to some cutoff α_{max}), $\tan \chi_i$ will occasionally change sign, that is, the ray will be reversed in direction. Thus a ray encountering random slopes will have traveled a considerably smaller net distance from the origin in a given time than would be the case for a perfectly flat surface. This introduction of a modified random walk into the seismic energy propagation is in fact responsible for the long duration of the observed signal in our model (Fig. 3).

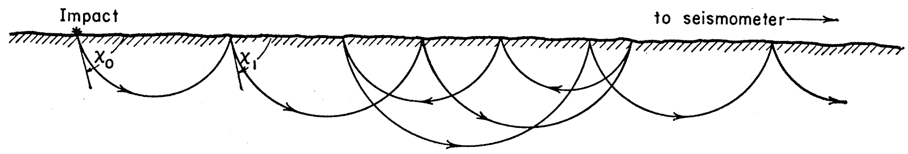

Fig. 3. Progress of a typical seismic ray away from its impact source on the lunar surface. The cycles are refracted concave upward in a medium in which acoustic velocity increases with depth. The randomly undulating surface reflects the ray into a different angle of descent at the start of each cycle, occasionally reversing its direction.

The two-dimensional approach used here is equivalent to assuming that the distribution of surface slopes is radially symmetric about the origin. That is, the slopes are random with distance but are independent of azimuth. The error introduced by this is one that will become more severe the greater the distance from the impact site. The surface irregularities in actual fact will deviate the rays into other azimuthal directions and thus the character of the propagation will approximate more that of two-dimensional diffusion. Energy will be spreading out more slowly than we have calculated, and the error will be greatest at the latest times in the event.

We assume that the total seismic energy E delivered instantaneously at the origin begins to propagate isotropically over 2π steradians. The fraction that goes into the range of angles χ_0 to $\chi_0 + d\chi_0$, that is, into the solid angle $2\pi \cos \chi_0 d\chi_0$, is then $E \cos \chi_0 d\chi_0$. This energy returns first to the surface in an annulus of area $2\pi R_0 dR_0$ and is reflected down again at all azimuths by some new angle χ_1. After n reflections the energy reaches the surface spread into an annulus of area $2\pi R_n dR_n$. Thus the energy per unit surface area of a ray that departed the origin at angle χ_0 becomes, after n reflections,

$$\mathcal{E}_n = \frac{E \cos \chi_0 \, d\chi_0}{2\pi \, R_n \, dR_n}$$

$$= \frac{EA}{4\pi} \cdot \frac{\cos \chi_0}{R_n \sum_{i=0}^{n} \sec^2 \chi_i}$$

This last step involved differentiating R_n and using the fact that $d\chi_i = d\chi_{i-1} = d\chi_0$. It should be noted that the expression for $\mathcal{E}_n$ accounts only for the energy decline caused by geometrical spreading of a ray and ignores physical attenuation.

In our computer simulation of the seismic record, a ray with some given initial angle χ_0 is followed through its consecutive random reflections. For each of $n = 0, 1, 2, \ldots, N$ cycles, the associated quantities R_n, $\mathcal{E}_n$, and T_n are calculated, where

$$T_n = \sum_{i=0}^{n} t_i$$

ranges up to a cutoff T_N set by the duration of the observed lunar seismic record. The process is repeated using a uniform distribution of starting χ_0 until a sufficiently large data set of (R_n, $\mathcal{E}_n$, T_n) is generated. Each member of the set represents a seismic pulse of energy per surface area $\mathcal{E}_n$ falling at time T_n some distance R_n from the impact. We select out all those pulses with R_n falling within a 10-km range gate centered about the epicentral distance of 75.9 km, and plot the associated $\mathcal{E}_n^{1/2}$ against T_n to obtain a simulated seismic amplitude record.

The most important factor in the simulation is the random selection of a surface angle α_i at each reflection. For simplicity the α_i are chosen at random from within an unweighted range $\pm \alpha_{max}$. This cutoff for maximum surface slope is here chosen to be $\pm 10°$, which is consistent with data from long wavelength lunar radar studies (5).

A simulated record is characterized by the choice of linear velocity profile, which in turn is defined by the surface velocity c_0 and the profile gradient $c_0 A$. With the use of reasonable values for the constants c_0 and A, the resulting simulated seismic record has the general character of the observed signal, that is, a relatively rapid initial rise and an extended decline in intensity. The entire record is scaled in time by the choice of c_0, and the time after impact of the peak signal strength may be adjusted to the observed 7 minutes by choosing $c_0 = 150$ m sec^{-1}. This is somewhat larger than the 108 m sec^{-1} surface velocity measured during a thruster test at the Apollo 12 site (1). However, one might expect the "surface" velocity sampled over a 75.9-km path to differ from that measured over ~20 m.

We have now only to select the constant A. It appears that the time of peak signal strength is not particularly sensitive to A, but a reasonable approximation to the observed rates of signal rise and decline is obtained by setting $A = 9$ km^{-1}. This means that the acoustic velocity increases by a factor of 10 in the first 1 km of depth, in this case to 1.5 km sec^{-1}. If it is kept in mind that a linear velocity gradient is almost certainly a crude oversimplification for the upper region of the moon, then this value is in reasonable agreement with laboratory measurements on samples of lunar fines (6).

A representative computer simulation of the Apollo 12 impact signal, with $\alpha_{max} = 10°$, $c_0 = 150$ m sec^{-1}, and $A = 9$ km^{-1}, is shown in Fig. 4. The simulation shows a higher peak at maximum but levels out more in time than does the envelope of observation. The latter effect was expected because the actual signal has undergone physical attenuation, which was not included in the simulation. Thus, the excess rate of decline in the observed record, amounting to perhaps 0.4 db min^{-1} over that of Fig. 4, would indicate for an amplitude loss factor $e^{-\omega t/2Q}$ that $Q \approx 2000$ for the medium, where we have used a signal frequency of 1 hz. This is only meant to be illustrative of a method for finding the quality factor Q, and the above-mentioned value for Q should not be taken too seriously. As was pointed out, the simulation contains oversimplified assumptions, variations in which lead to equally suitable though altered fits to the observational curve.

The simulated signal, though not an exact fit, does reproduce all the general characteristics of the observed signal and, in particular, the extended "tail." The importance of the surface undulations in this regard is emphasized by the fact that a simulation with the use of the same parameters as applied in Fig. 4 but with a flat surface, that is, with all the $\alpha_i = 0$, appears radically different. With zero slope everywhere, the resulting signal rises rapidly to a maximum at $t =$ 8.4 minutes (the travel time of the surface rays) where it cuts off abruptly. Similar behavior is in fact observed for the analogous situation in ocean acoustics (*4*).

The use of a model for the random surface slopes more sophisticated than a uniformly weighted distribution up to a 10° cutoff would undoubtedly improve the simulated fit to the observed signal. So would the introduction of a nonlinear velocity profile, specified by some arbitrary set of adjustable parameters. But this could be done in a number of ways, and there is at present insufficient data to prefer one description over another. For this reason we used a linear profile, the simplest case producing the principal observed signal characteristics. The actual profile of course flattens out when full compression is approached at some depth, but this is probably unimportant because the main part of the seismic energy is confined by refraction within the top few kilometers. For example, the velocity profile used to produce Fig. 4 sets a velocity of 6 km $\sec^{-1}$ at a depth of 4.3 km. The behavior below this depth should have little effect upon the shape of the observed signal except for the precise timing of the first signal.

A discussion of the actual physical character of the medium would have to deal with a number of other complexities. Compaction of the medium, and therefore the velocity of sound, would depend primarily not on the mere static weight of overburden but on the peak pressures from which the medium had suffered beneath craters of impact origin. The general increase of velocity with depth would probably be due chiefly to the statistics of the cratering process, which causes the material at a shallow depth to be plowed over by the smaller impacts and therefore to be unconsolidated and have a low velocity, while at the same time contributing pressure pulses below that cause compaction and a high sound velocity. While this process must produce in general the type of vertical dependence of velocity that we have discussed, it will in detail cause a great deal of additional refraction in random directions. Whether this introduces the element of randomness more than the surface irregularities is not known. It may be possible to discover this by comparison of the transmission properties over paths with smooth and rough surfaces.

Internal refraction will also tend to deviate the rays more out of the plane containing the impact and the seismometer into the transverse directions. Scattering from the small angles by which the surface generally deviates from level has a remarkably large effect on the ray paths in the vertical plane, where the angle of launch from the surface is very critical in determining the distance of the point of return to the surface. In the transverse direction there is no such criticality, and small surface angles will therefore only slowly build up the transverse component of rays. Refraction due to internal inhomogeneities may be much more important in this respect.

The timing of the signal peak as a function of distance between source and receiver will depend on the degree to which the rays have been deflected into transverse directions. Without transverse

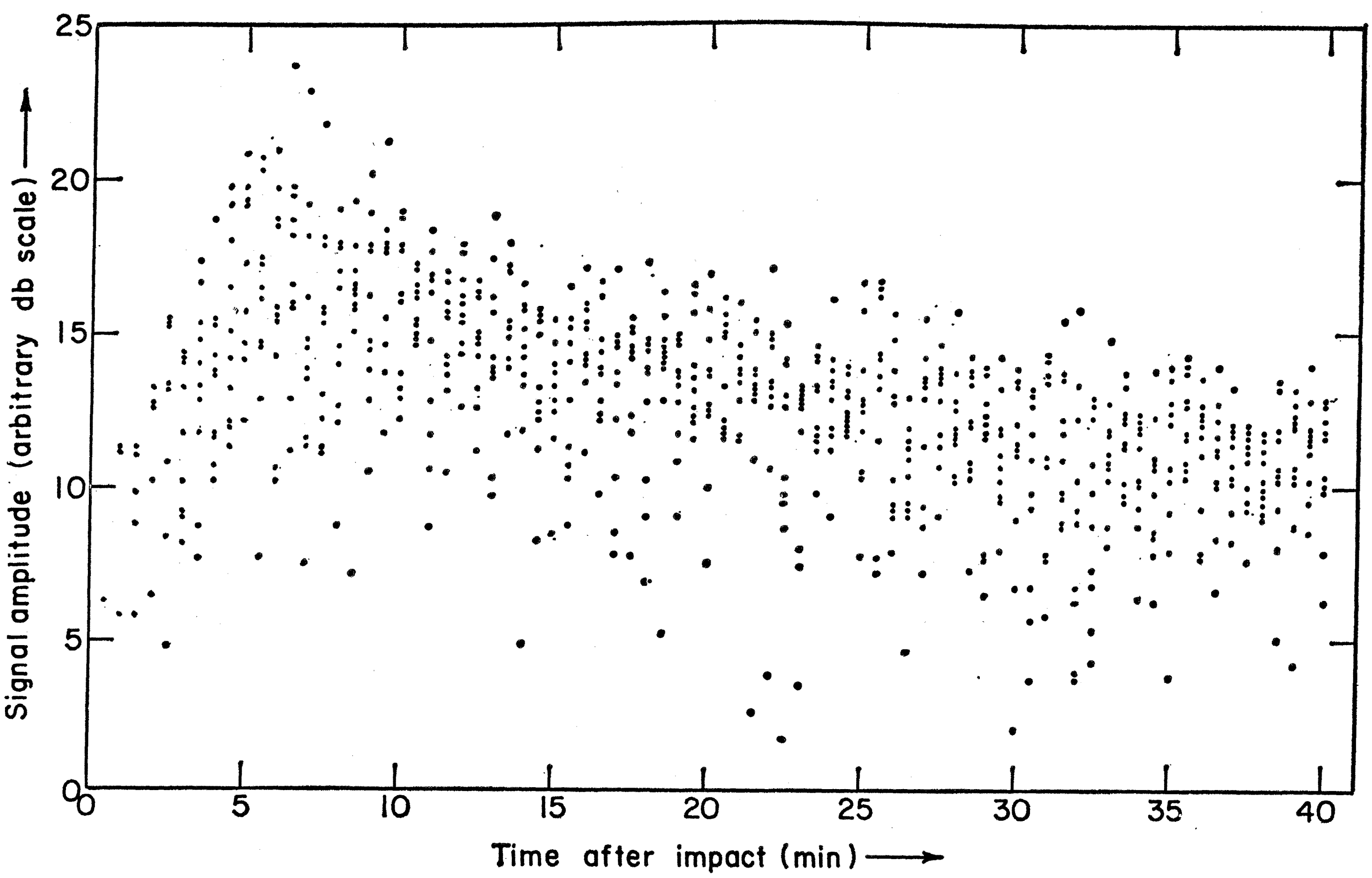

Fig. 4. Computer simulation of the lunar seismic record, showing a maximum intensity at about 7 minutes after impact followed by a gradual decline. The simulated signal falls off more slowly in time than the actual signal does, as expected, because it does not include the effects of physical attenuation. The simulation shown here uses surface slope angles ranging up to ±10°, a surface velocity of $c_o = 150$ m $\sec^{-1}$, and a velocity of 6 km $\sec^{-1}$ attained at a depth of 4.3 km.

scattering, the time delay to the peak would be directly proportional to the epicentral distance, while in the presence of transverse scattering the delay will approach proportionality to the square of the distance that diffusion would give. The increased seismic paths proposed for future experiments should demonstrate this effect, and, if the present discussion represents the essentials of the phenomenon, one would expect the peak amplitude to be reached much sooner than if the process were one of two-dimensional diffusion.

Appendix

The result of the impact of the Apollo 13 S-IVB casing has just been reported (*7*). The signal, measured at a distance of 142 km from the impact, appears to be of the same general nature as the LM signal of Apollo 12. However, there is reported to be a clearly recognizable onset of signal at 32 seconds after impact, and this increases the confidence that the first arrival signal suspected at 23 seconds for the Apollo 12 case is also genuine. These values can be used to specify more closely some of the properties of the medium.

The main shape of the signal received is not very critically dependent on the value of the velocity gradient and is almost independent of the circumstances below the depth where a velocity of more than 40 times the surface velocity is reached, since very few rays and therefore very little of the energy reaches that depth. Nevertheless, the onset time for the first arrival of the fastest wave is dependent on these quantities. If we assume that solid rock or fully compacted material exists at a certain depth, and if we assume for it a speed of approximately 6 km sec^{-1}, increasing only slowly with depth thereafter, one can derive the initial signal arrival time as a function of this depth. Using a surface velocity of 150 m sec^{-1} and a linear velocity gradient from there down to the layer of full compaction, we derive the following relations between that depth h and the first arrival time T for the Apollo 12 LM and for the Apollo 13 S-IVB impact events.

h (km)	Apollo 12 T (sec)	Apollo 13 T (sec)
3	16.1	27.1
6	19.6	30.6
9	23.1	34.1
12	26.5	37.5

The times quoted for the first arrival signals are approximately 23 seconds for Apollo 12 and 32 seconds for Apollo 13, and this would agree in both cases with a depth for full compaction of between 6 and 9 km. This depth would be less for a lower velocity in the fully compacted material.

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7. Apollo 13 Seismic Experiment Press Conference, 15 April 1970.
8. We thank H. Bondi for valuable discussions concerning wave propagation in inhomogeneous media. We also wish to thank L. Whitehill for suggestions relating to the calculations. Work on lunar studies at Cornell is supported by NASA grant NGL-33-010-005. S.S. is supported by a NASA traineeship.

23 April 1970; revised 22 June 1970

Phase Change Instability in the Mantle

Abstract. *In the presence of a temperature gradient, phase changes of the type believed to exist in the upper mantle, in which the less dense phase lies above the dense phase, may be unstable. Approximate calculations show such phase change instabilities are possible for both the 400-kilometer olivine-spinel phase transition and also for partial melting at shallower depths. The resulting flow patterns may provide a driving mechanism for the new global tectonics.*

Thermal convection within the earth's mantle has been proposed to explain continental drift (*1*). The Rayleigh number for the mantle, based on the value of viscosity inferred from postglacial uplift of Scandinavia (*2*), is several orders of magnitude larger than the critical Rayleigh number for the onset of convection (*3, 4*). Also, estimates of surface velocity and heat flux from constant property theories of thermal convection are in good agreement with observations (*5*).

Seismological (*6*) and geochemical (*7, 8*) evidence indicates that one or more phase changes occur in the upper mantle at a depth of about 400 km. The most important is likely to be the olivine-spinel phase transition. It is of interest to consider the influence of such a phase change on mantle convection. The volume and the entropy changes have the same sign for this phase transition (*7, 8*), so that heat is evolved when going from the olivine (light) to the spinel (heavy) phase and is absorbed when going from the heavy to the light. In this case the Clapeyron curve has a positive slope. Seismic evidence shows that the dense phase lies beneath the light phase; under isothermal conditions such a phase change is stable. If the fluid moves upward, the dense phase transforms to the light one and heat is absorbed. Thus the fluid is cooled and there is a downward stabilizing body force. On this basis it has been argued that the mantle phase change would act as a barrier to thermal convection (*4*). However, others (*9*) have argued qualitatively that large-scale convection could penetrate the phase change interface. A quantitative analysis of the influence of phase transformations on the stability of a fluid has recently been made (*10*). In this report we apply this theory to phase changes in the mantle.

We consider a simple model in which a two-phase fluid is confined between horizontal planes separated by the distance $2d$. The phases are assumed to be in thermodynamic equilibrium, so that the location of the phase boundary is determined by the intersection of the Clapeyron curve with the pressure-temperature curve for the fluid. In order for the light phase to lie above the heavy phase, it is necessary that the slope of the pressure-temperature curve exceed the slope (assumed positive) of the Clapeyron curve. The univariant phase boundary is initially midway between the planes. Both phases are assumed to have the same values of absolute viscosity μ, thermal conductivity k, and specific heat at constant pressure c_p (μ, k, and c_p are constants). A constant negative temperature gradient of absolute magnitude β, and a constant pressure gradient $-\rho g$ (g is the gravitational acceleration and ρ is the density) are present in this static state. The change in density $\Delta\rho$ at the phase boundary is an essential feature of the model. However, elsewhere each phase can be assumed to be an incompressible fluid of density ρ (for the olivine-spinel phase change the fractional density change $\Delta\rho/\rho$ is only about 0.1); the thermal expansion of the fluid is not considered. Since the coefficients