# Book Reviews

## Sincere and Other Strategies

**Theory of Voting.** ROBIN FARQUHARSON. Yale University Press, New Haven, Conn., 1969. xii + 84 pp., illus. $5.

Aside from the "consensus method," followed by the Society of Friends, procedures for arriving at collective decisions are usually institutionalized as explicit rules, embodied in constitutions and bylaws governing decision-making bodies, such as legislatures and committees. Besides specifying numbers of votes required to adopt a decision, the rules often designate the order in which issues are to be voted upon. The last mentioned rules are of importance where one of several alternatives is to be chosen and where a simple plurality is not sufficient to effect a decision. Given provisions for breaking ties, majorities can always be achieved if every vote is taken on just two alternatives at a time. Such procedures, analogous to paired comparisons in experimental psychology, are called *binary decisions*. For example, a motion having been made, an amendment may be offered. The body first votes on the amendment. If the amendment is adopted, the body then votes on the amended motion; otherwise on the original motion. In this way, one of three alternatives (adoption of the original motion, adoption of the amended motion, or rejection) is chosen by a sequence of Aye-No votes.

Given a set of voters and a set of alternatives, the situation is defined by assigning to each voter a preference order on the outcomes. The voter is now faced with one or both of the following questions:

1) Which of several voting procedures should he prefer?

2) Given a voting procedure, how should he vote each time the question is called?

That different procedures may lead to different outcomes is well known. Consider three voters with respective preference orders for outcomes A, B, and C: (ABC), (BCA), (CAB). If in a binary procedure the voters are first asked to choose between B and (A or C), and if each votes for the alternative that contains his top-ranking outcome (so-called *sincere* voting), then B will be eliminated. In the next vote, on A versus C, A will be defeated. Hence C will be adopted. On the other hand, if the first vote had been on C versus (A or B), C would have been eliminated, and A would have been adopted on the next vote.

Once a procedure has been decided upon, the question of how a voter should vote is far from trivial. It is by no means certain that it is always in a voter's interest to vote "sincerely." For example, if a voter dislikes an amended proposal even more than the original one, he may still act rationally in voting for the amendment in the hope that the amended proposal has less chance to pass than the original one.

In *Theory of Voting*, Farquharson gives a remarkably lucid exposition of such matters and in the process opens new avenues of exploration in the theory of $n$-person games. Game-theoretic concepts enter at the very start via the tree diagram of the decision process and via the notion of voting *strategy*. In a tree diagram, the branches at each node are the alternatives to be chosen among, always two in a binary procedure. Thus a node can be called an "issue" where a choice is offered between two not necessarily disjoint sets of possible final outcomes. The "surviving" set is then further subdivided, and the resulting issue is put to the next vote. The process ends in a single "surviving" outcome, the collective decision. This scheme is a direct analogue of the game in extensive form, where the nodes represent the moves of the game and the branches the choices available to the player who moves. The end points of the tree are the possible outcomes of the game.

A strategy has the same meaning as in game theory. It is a plan of action adopted by a voter, which specifies how he will vote on each successive issue in the procedure. An outcome is an equilibrium if no player could have effected a preferred outcome by using another strategy while the remaining players stuck to theirs. In short, given the strategies available to each voter, Farquharson's model is isomorphic to an $n$-person game in normal form. However, the special features of the voting process call for explorations of particular aspects of such games and so for a special terminology. The so-called "sincere" voting strategy has already been mentioned. It turns out that a *straightforward* strategy, one that dominates all others in the game-theoretic sense, is always sincere, but sincere strategies are not always straightforward. In addition, Farquharson introduces the concept of *sophisticated* voting strategy, indeed of several orders of sophistication, recursively defined in terms of the degree of a voter's awareness of the degree of sophistication of the other voters. The concepts are illustrated by examples that are both instructive and realistic. The following illustration may serve to convey the nature of the problems considered.

Jones, who is not a qualified librarian, has served for a year in a junior post; opinions about the quality of his performance are divided. After discussing what should be done, the library committee has agreed on the following procedure: The first motion to be put shall be (1) "That a senior post be created." If (1) is passed, then (2) "That the post shall be held only by a qualified librarian." If (2) is defeated, then the next question would be (3) "That Mr. Jones be appointed to the senior post." If (1) is defeated, then immediately following the committee would vote on the motion (4) "That Mr. Jones's contract is to be renewed in his present post." If (2) is adopted or (3) or (4) defeated, Mr. Jones would be dismissed and the vacant post would be advertised.

The procedure leads to one of five outcomes:

(A) Senior post, restricted (to a qualified librarian), advertised.

(B) Senior post, unrestricted, held by Jones.

(C) Senior post, unrestricted, advertised.

(D) Junior post, with Jones reappointed.

(E) Junior post, advertised.

There are 120 (5!) possible strong preference scales for the five outcomes. It turns out that straightforward voting strategies are available only to voters

who hold only 16 of these preference scales, namely (ABCDE), (ACBDE), (ABCED), (ACBED), (DEABC), (DE ACB), (EDABC), (EDACB), and the same eight in reverse order. Voters with these preference orders cannot do better than vote "sincerely." For others, "sophisticated" voting strategies are more rational.

Farquharson relates his theory to the "classical" *n*-person game theory as follows: While in the approach emphasized by Von Neumann and Morgenstern (*Theory of Games and Economic Behavior*) both coalitions and side payments are allowed, and in the approach emphasized by John Nash the noncooperative game, disallowing either coalitions or side payments, is taken as fundamental, in the present theory coalitions are taken into account but side payments are not possible; in fact, utilities of different players are not comparable because only an ordinal scale is assumed for preferences. Farquharson considers these assumptions to be more realistic. Furthermore, he writes, "It is no longer the case that the possibility of coalitions prevents any determinate solution. There can exist situations that are not dominated by any others" (p. 72).

The point about the ordinal preference scale is well taken inasmuch as practical applications of *n*-person game theory are severely restricted by the requirement of a cardinal utility scale. With regard to the cited remark, however, it might be pointed out that undominated situations occur also in the classical theory. For example, the core of an *n*-person game in characteristic function form, if it exists, consists of undominated imputations. Moreover, deterministic solutions are offered also by the Shapley value and in Harsanyi's bargaining model. This, however, is a minor point.

The great merit of the book is first of all in the rare combination of complete rigor and remarkable clarity, which makes its sophisticated ideas immediately comprehensible to *any* reader motivated to understand them. Secondly, the reader's interest is held at a high pitch by the graceful style and the felicitous examples, some taken from real life, ranging from deliberations in the Roman Senate to the five-year, $18-billion highway program offered by Tennessee's Democratic Senator Albert Gore (1955). Many readers interested in voting theory will be thrilled to read the cited letter of Pliny the Younger to Titus Aristo, containing a detailed and perceptive analysis of voting procedures in connection with a case before the Senate with which Pliny was concerned. This was 17 centuries before Marquis de Condorcet, the reputed father of mathematical voting theory, laid its foundations at the dawn of the Age of Democracy.

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## An Astronomer in Paris

**Science in France in the Revolutionary Era.** Described by Thomas Bugge, Danish Astronomer Royal and Member of the International Commission on the Metric System (1798–1799). With Extracts from Contemporary Works. MAURICE P. CROSLAND, Ed. Society for the History of Technology and M.I.T. Press, Cambridge, Mass., 1969. xvi + 240 pp. $10. Society for the History of Technology Monograph Series, No. 7.

Representatives from most of the states of Western Europe were invited to Paris in 1798 to examine the work done by the French on the metric system, to participate in its formal adoption, and, thereby, to endow the new standards with the greater authority derived from international support. Delays in this work provided the Danish representative, Thomas Bugge, with the time, and his own lively curiosity with the desire, to investigate virtually every aspect of the intellectual and artistic life of the French capital. Moreover, Bugge's official position—and the fact (unmentioned in Crosland's brief but useful biography) that, because he had been a correspondent of the former Academy of Sciences, his works and name were known to many Parisian scientists—gave him access to people and meetings closed to more ordinary visitors. And so he went everywhere—not only to every significant scientific and technological institution, but to almost every functioning educational establishment and to artistic centers as well. He was even allowed, despite the continuing war, to inspect such facilities as arms factories. With one major exception, to be noted below, his observations on all these things were marked by a kind of scientific objectivity. Thus, although he offered comments critical of the war, he never complained of the "looting" practices of the French armies; he was content simply to note the many works of art, scientific books and instruments, and other goods with which they were enriching France.

More than most such efforts, therefore, Bugge's account—translated into English from its original Danish in 1801 under the title *Travels in the French Republic*—is a valuable tool for the historian, providing data and dimensions lacking in "official" sources. Thus, where a contemporary document may inform us of the presentation of a specific lecture in the amphitheater of the Museum of Natural History, it will not tell us, as does Bugge, about that auditorium's poor acoustical qualities (a judgment with which, having heard speakers there in the summer of 1968, I must concur). Nor do the minutes of the meetings of the National Institute reveal the voice qualities and speaking abilities of the scientists reading papers there. These are the kinds of things offered to us in this selection of the most scientific sections of Bugge's account.

But Crosland has done more than simply make these scarce observations available again. He has both improved the earlier English version of Bugge's account itself and added some general contributions. One significant item under the former heading is that the 1801 edition, despite its suggestion to the contrary, was not a complete translation. Crosland not only tells us what it omitted, but provides some of that material—perhaps most importantly, some considerations on the metric system and a variety of comments by Bugge on governmental support of science and the works of several private societies and instrument makers which Crosland has placed together in a chapter entitled "Science—public and private," a logical grouping typical of his meaningful recategorizing throughout.

He has, further, made some changes —modernizations and clarifications—in wording, while eliminating some of the listlike offerings of the original. Recognizing the difficulties that most readers might have with the Republican calendar, he has provided Gregorian dates throughout, though happily avoiding the all too frequent practice of dropping the former after converting to the latter. As to general contributions, the most significant is his injection of additional materials, especially comparative evaluations taken from other travelers' accounts, a listing of which constitutes far and away the