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Megalithic Rings: Their Design Construction

It is proposed that prehistoric man used a unique method to scribe the simple megalithic designs.

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Sometime between 3500 and 1000 B.C. several thousand megalithic structures were erected in western Europe (*1*, p. 124). Some of those in the British Isles, such as Stonehenge, are thought to have served as astronomical observatories (*2*), while others, like the mound at New Grange, Ireland, seem to be burial places. Those that are ring-shaped fall into four categories of design: circles, flattened circles, ellipses, and eggs. Thom (*3*) in an investigation of the geometry of these designs has made a substantial contribution to the solution of these enigmatic shapes; he has given us a tractable geometrical analysis which is esthetic in its simplicity. Essentially, he has considered each ring perimeter as a set of arcs drawn from various centers within the design. Thom's geometrical analysis is given in Fig. 1; only the geometry of the ellipse and that of the type II egg are not his.

In this article I extend Thom's proposal and suggest the manner in which the designs were scribed. I confine my remarks to the simpler rings and do not discuss the compound rings, exemplified by the Avebury monument in Wiltshire, although there are features which suggest that even these fit the pattern of construction described here.

Thom's geometry suggests that ropes attached to anchor stakes placed at the arc centers were used to scribe the designs. Surprisingly, Thom has questioned the use of the rope and stake as a scribing tool (*3*, p. 32). He claims that the rings are too accurate to have been scribed by such a procedure. Presumably, he is referring to the propensity rope has for stretching. He suggests that the megalithic designers used two rods of standard length (a "megalithic yard") and measured the distances by carefully laying out the rods end to end much as one would use a yardstick. There are two sources of cumulative error in such a procedure. One is associated with the picking up and placing of the rods; the other is the problem of trying to keep the rods properly aligned. On the other hand, a little experience with a rope would quickly tell how much tautness is needed to keep the stretch to a minimum.

The assumption here is that two anchor stakes and two other stakes, "pivot stakes," were used in the construction of each simple ring, with the possible exception of the circle. Furthermore, it is assumed that the anchor stakes and the pivot stakes were always aligned at right angles (Fig. 1). On the surface, this relationship is of minor interest, but further consideration shows that, in constructing the various rings, the needed placing and movement of the anchor and pivot stakes may follow an evolutionary pattern.

Thom convincingly argues that the people who built these structures were obsessed with a concern for perfection—so much so that all their measures were laid out in integral units. The circular megalithic ring, with its perfect radial symmetry, must have especially appealed to them, particularly since its construction represents the utmost in simplicity. To a geometer, probably few things are more intuitively satisfying and esthetically appealing than an absolutely perfect circle drawn by rotating a radius around a point. Undoubtedly discovery of the irrational ratio between the diameter and the circumference was frustrating to the megalithic geometers. Quite possibly this discovery instigated the search for rings whose perimeters were such as to make this ratio integral. Perhaps it was at this point that the flattened circle was developed.

The Flattened Circles

Figure 1 shows the various simple rings, their geometry, and the proposed methods of construction. For the flattened circle of type A (Fig. 1, row 1), Thom suggests four centers (a_1, a_2, p_1, and p_2) from which four arcs are drawn. Suppose an anchor stake was placed at point a_1 and two pivot stakes were driven at points p_1 and p_2. With a rope of appropriate length tied to stake a_1 the designer could, in one sweep, inscribe all of the type A ring, except for the top arc, by moving the rope so that it swung around the pivot stakes when it came against them. The resulting figure is very nearly a perfect cardioid. The design could be completed by re-anchoring the rope at a_2 and marking the top arc.

Since the top arc smooths over the indention in the cardioid, one might guess that the builders dismissed this shape because it served as merely an auxiliary figure. There is evidence from other megalithic structures, however, that this was not the case, and indeed the cardioid may have been regarded

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as quite exceptional. The structures in which cardioid shapes seem to be important are the numerous passage tombs found throughout Britain and Ireland. Examples include the chambered tombs of the Severn Cotswold culture (*4*). One of these, the tomb at Parc le Breos Cwn at Glamorgan, is obviously cardioidal, albeit misshapen (Fig. 2).

Figure 3 is an outline of the exquisitely chambered passage tomb at New Grange, Ireland. This is considered to be one of the finest of the passage tombs; its date of construction has been placed at about the third millennium B.C. (*1*, p. 133). The end of the tomb is in line with the pivot points, and the entire chamber (exclusive of the passageway) just fits between the arcs below the cusp of the cardioid used to construct the design. These fits require the assumptions that the passage marks the vertical diameter and that the center of the finishing arc is located within the ring rather than on the perimeter (*5*).

How were the positions of the anchor and pivot stakes found? In all type A rings, points a_2, p_1, and p_2 lie on radii that divide the circle with center a_1 into three equal sectors. Thus the ring is a two-thirds-perfect circle with a flattened arc over the remaining third. Someone has remarked that perhaps all of these misshapen rings were attempts to make ring structures such that the ratio of circumference to diameter would be 3 (*2*, p. 150). That the radial lines divide the ring into thirds suggests that an equilateral triangle was constructed with p_1 and p_2 occupying two vertices and with the third vertex located at a point below a_1, the center of the triangle. The intersection of lines drawn from any two vertices to the midpoint of their bases would locate a_1. The construction of this triangle and its center would not be difficult, and it may be said with some confidence that the type A design construction was based on an equilateral triangle.

Neither the lower vertex of the triangle nor the cardioid cusp point in the type A ring are conspicuously evident in the final construction. Nor did they necessarily serve as pivot points during the construction as did the other triangle vertices. Their importance might have been enhanced if one served to mark the other. If the lower type A triangle vertex was located at the cusp point and the other two vertices, which remained pivot points, were placed in line with a_1, then a type B circle could be drawn in much the same manner as a type A circle. In a type B circle the length of rope needed for scribing the circumference was determined by measuring the distance from the anchor stake to the top vertex of the equilateral triangle by way of the outside of one of the pivot stakes (Fig. 1, row 2). This procedure would give a correct measure of the needed length of rope only if the pivot stakes were placed, as most of them were, one-third of the radial distance from the center. Only one of Thom's type B sites did not conform to this plan. Thus, as in the case of the type A design, one may say with some assurance that the equilateral triangle played a role in the construction of the type B rings.

The Ellipses or Oblate Circles

Thom and others describe a number of rings as "elliptical." These are nearly circular, but one axis seems to be slightly shorter than the other. However, if the anchor and pivot stakes were used, aligned at right angles for all designs, then the rings cannot be true ellipses; rather, they must be regarded as oblate circles.

Where were the stakes placed in constructing these sites? Note that the flattened circles are symmetrical about their lesser diameter, whereas the egg-shaped rings, dealt with below, are symmetrical about their greater diameter. The oblate circles, while not radially symmetrical, have a bilateral symmetry with respect to both their major and minor axes. This suggests

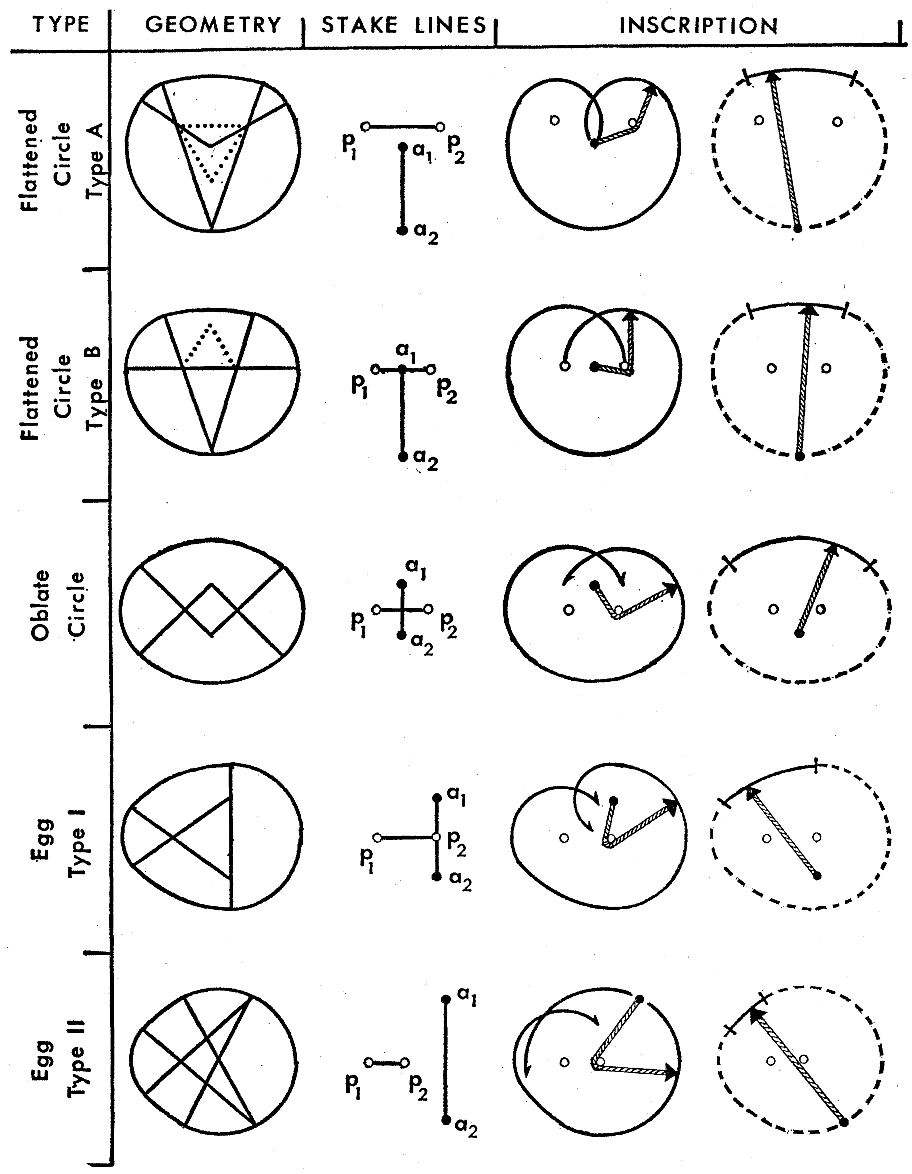

Fig. 1. The geometry, stake lines, and scribing method for the five classes of designs discussed. The solid circles are anchor stakes.

two things: first, the anchor and pivot lines reflect this double symmetry, and second, the oblate circle represents an evolutionary midpoint between the flattened circles and the eggs. If the anchor and pivot lines were made to intersect perpendicularly at their midpoints, then a rope tied to a_1 and then to a_2 could scribe the design in the same way that the flattened circle designs were constructed. Experimentation with a compass quickly demonstrates that an oblate circle can be constructed from these points—one which, to the eye, is indistinguishable from an ellipse.

A pattern to the shift of anchor and pivot lines begins to emerge. The anchor line seems to be drifting upward from its position in the type A flattened circle to its position in the type B ring, and finally in the oblate circle the two lines bisect each other. From here on, the only movement the anchor line can make to produce a new figure and yet remain at right angles to the pivot line is a lateral shift. This is precisely the step taken in the production of the egg designs.

The Egg-shaped Rings

The construction of a type I egg seems to have involved a shift in the anchor line just to the end of the pivot line (see Fig. 1, row 4). Before the design was scribed, a rope would be attached to a_1, then placed on the inside of p_2 at the right angle of the triangle formed by a_1, p_1, and p_2, and allowed to loop back to a_1. From this position all but one of the side arcs could be drawn in one sweep as before. The remaining arc could be drawn by re-anchoring the rope to a stake placed at a_2.

This shift of the anchor line carried with it a number of consequences. The most obvious one is the change in the orientation (symmetry) of the ring from its anchor line to its pivot line. Also, the pattern of the lines seems to mirror that of the type B flattened circle; in both, one line is at the end of the other. Despite these changes, the stake lines remain at right angles, and the arcs of the perimeter remain in the same positions relative to themselves and their centers (or, equivalently, the lie of the scribing rope relative to the four points is the same). This points to a certain topological equivalence between the simple rings discussed so far. Thom suggests that

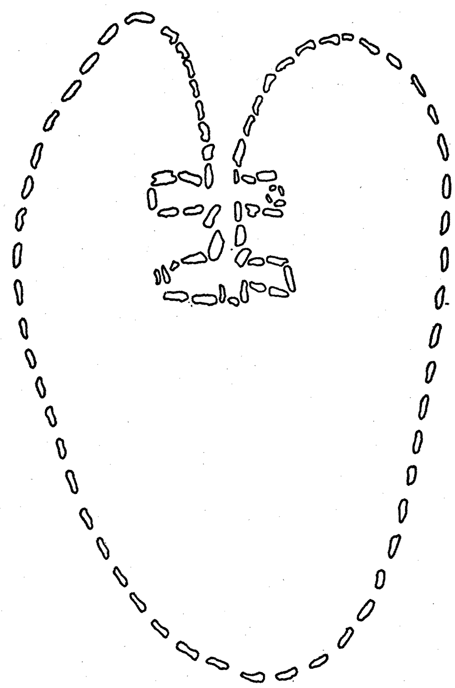

Fig. 2. Outline of the tomb at Parc le Breos Cwn. [After Daniel (*4*)]

the megalithic geometers knew rudimentary trigonometry. Perhaps they were nibbling at the edges of topology as well. They must have been impressed by the peculiar changes in the perimeters of these rings made by the straightforward manipulation of the stake lines—a quasi-topological observation at the very least.

A common feature of the type I egg is the appearance of perfect Pythagorean triangles formed by the stake points. Thom has discovered a number of these, particularly of the 3, 4, 5 variety. The perfect right triangle is conspicuously missing in the flattened circles. How did the Pythagorean triangle come into being here? In all

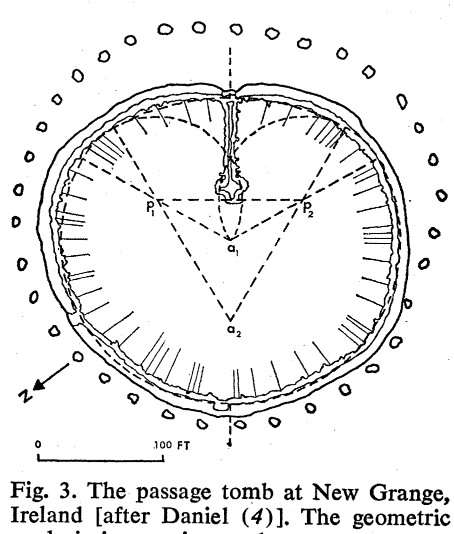

Fig. 3. The passage tomb at New Grange, Ireland [after Daniel (*4*)]. The geometric analysis is superimposed.

likelihood it did not, like Athena, spring fully matured from the brow of its creator. Probably there was a good deal of experimenting, perhaps starting with the equilateral triangles. If the builders were an inquisitive lot, as no doubt they were, they must have contemplated the distance between, say, the point at the cardioid cusp of the type B ring and the anchor point directly below. The side and altitude of an equilateral triangle and half of the base are in the ratio $1 : 3^{\frac{1}{2}} : 2$, which is close to $4 : 7 : 8$ in quarter units. This is nearly Pythagorean ($4^2 + 7^2 = 65 \approx 64$) but not so close that the difference would escape detection. If, as Thom suggests, these people were obsessed with integral measurement, the discovery of the nonintegral altitude of the equilateral triangle may have motivated them to seek a right triangle with integral sides.

The geometry suggested by Thom for the type II egg is considerably different from that given here. Our scribing method could not be reconciled with Thom's geometry, and this necessitated a search for another solution. Once the pattern of stake-line variation was established, the solution was quickly found. If the orientation of the stake lines of the type B circle had its counterpart in the type I egg, the stake lines of the type A circle should have its counterpart in the type II egg. That is, the anchor line of the type II egg should lie outside the pivot line, just as the pivot line lies outside the anchor line in the type A circle (see Fig. 1). This was indeed found to be the case.

There remained the problem of how far away from the pivot point the anchor line was placed. Since the distance between a_1 and the pivot line in the type A circle was determined by an equilateral triangle, it was thought that the same thing might be true for the type II egg. Unfortunately, any number of equilateral triangles can be constructed around the pivot stake p_2. As Fig. 1 suggests, the anchor stake might have been positioned at the intersection of the circumference of the larger circle and two of its trisecting radii. The principal differences in the perimeters of the type II egg produced by Thom's geometry and by the geometry suggested here is that Thom assumed that the top and bottom sides (as shown in Fig. 1) were straight, whereas here they are very shallow arcs. The use of shallow arcs rather than straight lines has precedent.

Thom suggested such arcs in considering Woodhenge (*3*, p. 74), and so did Borst in his analysis of the crypt of William the Englishman (*6*).

The designs of the four type II eggs listed by Thom were reconstructed on the basis of the geometry suggested here. Thom's measurements of the distance between the centers of the large and small near-semicircular ends and the radius of the large end arc were kept. With the geometry proposed here, the designs of two rings (Leacet Hill and The Hurlers in England and Wales, respectively) were virtually identical to the designs proposed by Thom. On the other hand, the designs of the Borrowston Rig and Maen Mawr monuments in Scotland and Wales could be constructed on the basis of Thom's measurements only if the locations of a_1 and a_2 were free to shift on the circumference of the larger arc. There are rationales for these diversities, however (see *7*).

The passage tomb at New Grange (Fig. 3) had an outside megalithic ring which was possibly a type II egg. A good construction of it can be produced if an equilateral triangle is used, as shown in Fig. 4. There are three places where the construction seems to miss its mark—the north, southeast, and southwest locations. This makes it uncertain that the type II egg was its model. However, this construction fits as well as, or better than, any of the others, including Thom's type II geometry.

If the passage tomb at New Grange is a type II ring, then an interesting question arises regarding the apparent evolution of the designs, for here the earliest (a type A flattened circle) and the latest are present in the same site. There are a number of ways in which this could have come about. The outside egg could have been constructed well after the inner flattened circle. It is difficult, however, to think why such a ring would be added to a site already occupied by a tumulus. If it should be found that the tumulus was built within an existing ring (whose presence perhaps indicated hallowed ground), this would be far more understandable. If the type A inner ring was developed (hence built) after the type II egg, implying a structural evolution in the reverse order of the one proposed here, then we must conclude that the Pythagorean triangle was discovered and then suddenly abandoned—a possibility which seems untenable. Such a conclusion also runs contrary to the earlier proposition that the Pythagorean triangle evolved from the type-B-ring equilateral triangle.

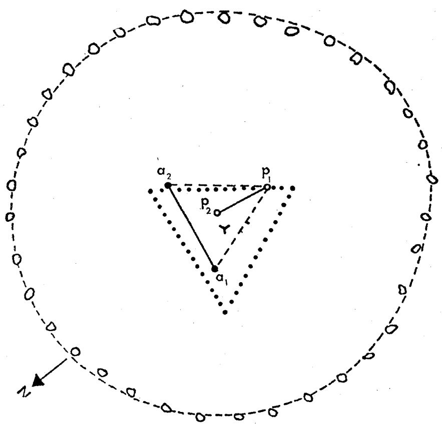

Fig. 4. The type II geometric analysis of the megalithic ring surrounding the New Grange passage tomb. The dotted triangle relates to the passage tomb mound. Note that the anchor stakes are not on the perimeter, hence that their location on the perimeter is apparently not a necessary feature of the type II egg.

Of course, the tumulus and the outer ring could have been constructed at the same time. In this case all ring types would have been known and used concomitantly. This could have happened if the different types evolved fairly rapidly, or if each ring design served a unique function. The specific function each design served remains a mystery.

Despite the difficulty in eliciting a rationale, there are reasons for believing that the outer ring was constructed after the tumulus. Although the center triangles of the ring and the tumulus are oriented in approximately the same manner, they do not coincide or share a common center (see Fig. 4). If the ring and the tumulus were built simultaneously, they undoubtedly would have the same center. If the ring had been built first, then its center could have been found fairly easily, and the design of the tumulus built around that center. Suppose the tumulus was constructed first, however. During the later construction of the ring it would have been necessary to ascertain the center of the tumulus from the top of the mound, if a common center was desired. This would have been difficult at best.

Final Comments

I have attempted to show how some of the simple megalithic rings were drawn, and this attempt, I believe, not only affects existing analyses of these sites but leads to new conjectures concerning the mathematical talents of the designers. Were the hypothesized scribings really used? Or are they another of a number of explanations that merely fit the field data? There are two indications that the evidence for the scribing method is more than circumstantial. The first is the existence of sites in which the cardioid appears to have been used. The second is the fact that the change in the pattern of the stakes from one ring to another is too orderly to be circumstantial.

Perhaps the best witness to the talents of these megalithic builders is the scribing method itself, for here is a procedure for geometric construction that is unique. There are only three known nonalgebraic or nongraphic geometric constructions: the Poncelet-Steiner circle method, the fixed compass or Mascheroni method, and the common method involving use of the flexible compass (*8*). No existing technique approaches that of the megalithic geometers, except for the well-known method of scribing an ellipse. The Poncelet-Steiner circle is the most restricted of the known constructions; it requires only a straightedge and a fixed circle. The Mascheroni method allows the use of any number of circles or arcs with a fixed radius, and the flexible compass further allows the use of arcs of varying radii. If all restrictions on radius length were removed so that the radius could be of any length at any time during the construction, then virtually any two-dimensional figure could be drawn. The proposed megalithic scribing method allows the length of the radius to change discretely and in one direction (toward shorter lengths) in the middle of a sweep. For this reason this technique might possibly constitute the next step in a hierarchy of construction methods. While this scribing method may not contribute profoundly to mathematical theory, it may at least have consequences of interest to recreational mathematics (*9*).

The megalithic geometers knew rudimentary trigonometry and may have had a grasp of simple topology. They had a standard length which, for all we know, may have been the precursor of the yard (*10*), and they had a unique method of geometric construction.

Is there something more? Perhaps much remains hidden in these remarkable sites.

References and Notes

1. S. P. Ó Ríórdáin and G. Daniel, *New Grange* (Praeger, New York, 1964).
2. G. Hawkins, *Stonehenge Decoded* (Doubleday, Garden City, N.Y., 1965).
3. A. Thom, *Megalithic Sites in Britain* (Oxford, London, 1967).
4. G. Daniel, *The Megalithic Builders of Western Europe* (Hutchenson, London, 1959), pp. 111–113.
5. Actually this tumulus represents what Thom calls a type D ring, one in which the two pivot stakes are placed at one-third the radius from a_1 rather than at the midpoint. As Fig. 3 shows, the pivots can be kept at the midpoint of the radius if a_2 is allowed to move inside the design.
6. L. B. Borst, *Science* **163**, 567 (1969).
7. The monument at Borrowston Rig requires anchor points that are placed farther apart on the circumference than the intersections made by the trisecting radii demand. In this design the circle of the smaller end arc passes through the center of the larger end arc, and the wider placement of the anchor stakes may have been made on this account. The site at Maen Mawr had anchor stakes that were closer together on the circumference. If the radial line from p_2 to either anchor is taken as a hypotenuse of a right triangle one side of which is half the anchor line, then the lengths of the sides of this triangle (in megalithic half-yards) are 14, 17, and 22, which is nearly Pythagorean ($14^2 + 17^2 = 485 \approx 484$).
8. M. Gardner, *Sci. Amer.* **221**, 239 (1969).
9. To mention one such consequence, with the megalithic method concentric rings, such as those found at Woodhenge, can be drawn without using any special mensuration technique. That is, a rope can be lengthened by an unspecified amount and a concentric design can be drawn at once. With a flexible compass, on the other hand, the large arc in a type II egg, for example, can be drawn without special measurement but the other arcs must be changed by x amount to remain equidistant from the perimeter of the original figure.
10. Thom presents a good argument for use of a standard length of 2.72 feet (one megalithic yard) in the construction of these structures, as well as of others in both the Old World and the New.

11. The writing of this article was supported in part by the Oklahoma State University Research Foundation.

Mechanism of Antibody Diversity: Germ Line Basis for Variability

Analysis of amino termini of 64 light chains indicates that much antibody variability is present in the germ line.

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The nature of the genetic control of antibody variability is one of the most fascinating and approachable problems in mammalian genetics. Vertebrate organisms appear to be capable of synthesizing thousands of different antibody sequences, each presumably encoded by a different antibody gene. How then do these genes arise? The somatic theory of antibody diversity postulates that antibody genes arise by hypermutation from a few germ line genes during somatic differentiation. In contrast, the germ line theory postulates that vertebrates have a separate germ line gene for each antibody polypetide chain the creature is capable of elaborating.

We discuss here certain patterns that have emerged from amino acid sequence analysis of antibody polypeptide chains. These patterns indicate that much of the sequence diversity is present in the germ line. We also discuss why the germ line theory seems to be the simplest explanation for antibody diversity.

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General Immunoglobulin Structure

All five recognized classes of antibodies (immunoglobulins) in mammals contain two distinct polypeptide chains, called light and heavy chains (*1*). For example, there are two identical light and two identical heavy chains per molecule in the major serum immunoglobulins, the immunoglobulin G class; but the light and heavy chains differ chemically in different antibodies. Since normal antibodies produced against the simplest of antigenic determinants are generally heterogeneous, advantage has been taken of the homogeneous immunoglobulins produced in large quantities by plasmacytomas in humans and in the highly inbred BALB/c mouse (*2*). Light chains are frequently excreted in the urine of individuals with certain plasmacytomas; these light chains have been called Bence Jones proteins. This article deals only with light chains because relatively little information on comparative sequences is available for heavy chains.

Light chains from most mammalian species including man are of two types, lambda and kappa, which are readily distinguished by serological and chemical criteria (*1, 3*). Light polypeptide chains have two parts: a common region (approximately residues 108 to 215), which is essentially invariant for a given light chain type and species and a variable region (approximately residues 1 to 107) which is different for each well-characterized protein (*4–6*). Presumably each variable region sequence directs the folding of a unique antibody light chain configuration.

Two patterns emerge when the amino acid sequences from many light chain variable regions are compared. (i) All kappa and lambda variable region sequences can be divided into subgroups on the basis of their similarity to one of eight prototype sequences (Fig. 1). (ii) Relatively minor deviation from the prototype sequences occurs among individual proteins of a given subgroup (Fig. 1) and is designated intrasubgroup variation. We first discuss the variable region subgroups and the reasons for concluding that each light chain subgroup must be encoded by at least one distinct germ line gene. We then discuss why we believe that the intrasubgroup variation may also be encoded by separate germ line genes.

Variable Region Subgroups of Kappa Chains

The nearly complete sequences of six variable regions of human kappa chains are known (Roy, Ag, Cum, Mil, Eu, and Ti), and partial sequences of more than 35 others have been determined. All variable regions of kappa chains can be assigned to one of three subgroups on the basis of their similarity to one of three sets of linked amino acid sequences (Figs. 1 and 2). These three sets of linked amino acids or three "prototype sequences" are derived by examining the proteins of