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The Case for a Hierarchical Cosmology

Recent observations indicate that hierarchical clustering is a basic factor in cosmology.

G. de Vaucouleurs

In questions of science the authority of a thousand is not worth the humble reasoning of a single individual.
—GALILEO GALILEI

True knowledge can only be acquired piecemeal, by the patient interrogation of nature.
—SIR EDMUND WHITTAKER

Once upon a time philosophers and cosmographers insisted that the motions of the planets must be circular and uniform. An irrelevant aesthetic concept of "perfection" and a more valid mathematical need for simplicity were at the root of this long-held error. Nowadays, theoretical cosmologists insist that the large-scale distribution of galaxies must be homogeneous and isotropic, and most astronomers believe that the expansion of the universe is linear and isotropic and that it proceeds at a uniform rate measured by the Hubble "constant" H (*1, 2*).

Some Historical Perspective

Modern theoretical cosmology was born 50 years ago with General Relativity; its subject matter was determined a few years later with Hubble's final proof of the old concept that nebulae are "island universes." Its first observational test was Hubble's discovery just 40 years ago of the universal red shift in the spectra of distant galaxies. Its second potential test by galaxy counts, which has been available for more than 30 years, has so far miserably failed for a variety of technical reasons, although the principle remains valid and further observational progress may reestablish its value. The third and presently most exciting test came with the discovery just a few years ago of the so-called 3°K background radiation (this discovery is so new, in fact, that its relevancy to cosmology still awaits the test of time). Other data, of course, may have a bearing on the problem—if, in fact, the universe is "one and indivisible," almost every fundamental law or principle of physics, chemistry, and astronomy must have some cosmological connotation. Familiar and perhaps overworked examples are Mach's principle, the abundance of the elements, and the (extra atmospheric) brightness of the night sky (*3, 4*). Nevertheless, the few facts and figures which in the past 40 years have been given prominence as particularly relevant to cosmology are still too little understood and often too poorly established or too recently discovered to form a solid basis for a "final" solution. Also we may well still lack some fundamental knowledge of physical laws on the very large (cosmic) scale or on the very small (particle) scale, or both, to even hope for a realistic solution at the present time. Is it not possible, indeed probable, that our present cosmological ideas on the structure and evolution of the universe as a whole (whatever that may mean) will appear hopelessly premature and primitive to astronomers of the 21st century? Less than 50 years after the birth of what we are pleased to call "modern cosmology," when so few empirical facts are passably well established, when so many different oversimplified models of the universe are still competing for attention, is it, may we ask, really credible to claim, or even reasonable to hope, that we are presently close to a definitive solution of the cosmological problem?

Those who are so optimistic as to answer affirmatively have in effect already made a choice, primarily for philosophical, aesthetic, or other extraneous reasons, from among the vast array of possible homogeneous isotropic universes of general relativity; thus solving the cosmological problem reduces to the almost trivial matter of fitting a few empirical constants, which, some suggest, may take only a matter of a few years.

I cannot subscribe to this view, first, because promoters of other cosmologies will not meekly "abjure, renounce, and detest" their errors. Quite recently, for instance, opponents of the steady-state theory confidently announced the demise of this concept which was said to be inconsistent with some counts of radio sources and the existence of the 3°K background radiation. But the latest pronouncements of at least one defender of this particular theory show that he still maintains his original views (*5*).

A second reason is that even within the framework of the orthodox "primeval atom" or "big bang" theory we have witnessed in the past 40 years frequent and drastic changes in the fundamental "constants." For example, estimates of the Hubble constant de-

The author is professor of astronomy, The University of Texas, Austin, 78712.

creased from $H = 560$ kilometers per second per megaparsec in 1931 to present values in the range $50 < H < 110$ kilometers per second per megaparsec (1 megaparsec $= 3.25 \cdot 10^6$ light-years $= 3.18 \cdot 10^{24}$ centimeters). The situation is worse still for values of the so-called deceleration parameter which have fluctuated wildly from year to year, without any clear trend. And so it seems unbelievable that we are now in A.D. 1969 on the threshold of reaching the promised land of the true and only cosmology.

Let us look, for example, at a graph (Fig. 1) of successive estimates of "the age of the world" made during the past three centuries, from Bishop Ussher's specific, if unfortunate, 17th century assertions to the Helmoltz-Kelvin gravitational contraction ages in the mid-19th century, to the ages based on early 20th century radioactive dating, to the expansion ages of the mid-20th century, and to present estimates based on the evolution of stars in globular clusters. During this entire three-century span, estimates of the age of the universe have increased exponentially at the surprisingly uniform logarithmic rate of 1.9 per century (the doubling time of 16 years just about matching the growth rate of astronomical progress in general). It is true that the curve may well suddenly stop rising and level off beyond A.D. 1969. If so, we live at a truly remarkable time, the time when the age of the universe is finally fixed, this despite the fact that the expression "age of the universe" cannot be defined without a prior definition of "universe" and of a universal time scale, or without a scheme of universal evolution, and, indeed, a solution of the cosmological problem. The least that can be said is that the historical record is a warning against excessive optimism.

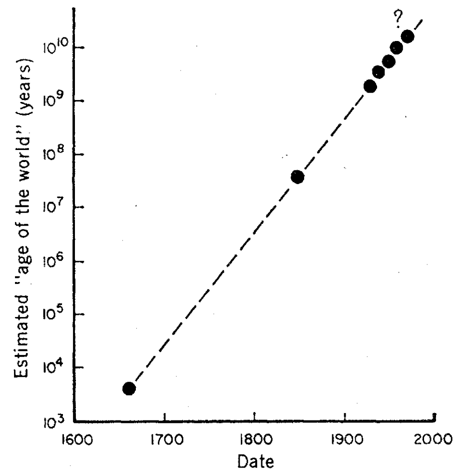


Fig. 1. Estimates of the "age of the world" have grown exponentially during the past three centuries. What is the probability that a limit has finally been reached?

Questionable Assumptions of Orthodox Cosmology

With few exceptions modern theories of cosmology have come to be variations on the homogeneous, isotropic models of general relativity. Other theories are usually referred to as "unorthodox," probably as a warning to students against heresy. When inhomogeneities are considered (if at all), they are treated as unimportant fluctuations amenable to first-order variational treatment. Mathematical complexity is certainly an understandable justification, and economy or simplicity of hypotheses is a valid principle of scientific methodology; but submission of all assumptions to the test of empirical evidence is an even more compelling law of science. Facts of observation cannot be ignored indefinitely or dismissed as unimportant. A stubborn discrepancy of 8 arc minutes between Tycho's observations of the longitude of Mars and the most elaborate pyramiding of circular epicycles led in Kepler's exuberant but correct view "to a complete revolution of astronomy." The history of science, of course, is full of examples of stubborn, "ugly" little facts that destroy "beautiful" theories, but most scientists have learned not only to live with this reality but also to hunt zealously for such discrepancies, which usually lead to further progress, to improved theories, and sometimes to completely new ideas.

Unfortunately, a study of the history of modern cosmology (*2*) reveals disturbing parallelisms between modern cosmology and medieval scholasticism; often the borderline between sophistication and sophistry, between numeration and numerology, seems very precarious indeed. Above all I am concerned by an apparent loss of contact with empirical evidence and observational facts, and, worse, by a deliberate refusal on the part of some theorists to accept such results when they appear to be in conflict with some of the present oversimplified and therefore intellectually appealing theories of the universe (*6*). It is not merely that, as Otto Struve once remarked, "In a sense the observer knows too many facts to be satisfied with any theory"; it is due to a more basic distrust of doctrines that frequently seem to be more concerned with the fictitious properties of ideal (and therefore nonexistent) universes than with the actual world revealed by observations.

If this sounds too harsh a judgment, let us consider some of the questions that are routinely raised and even "answered" in our modern cosmological symposia and congresses: what is the mean density ρ of the universe? Granting that we agree on a definition of the universe (countable? observable? horizon limited?), seldom, if ever, do we hear raised the following *a priori* questions: What precisely do we mean by the average density? What is the evidence to support the notion that a mean density can be defined? In short, how do we *know* that the universe is homogeneous and isotropic? In fact, since ρ is so evidently *not* a constant independent of space coordinates in our neighborhood, how large a volume of space do we need to consider before the average density in this volume may be accepted as a valid estimate of ρ? And what proof do we have that the same value of ρ would obtain in another equal, disjoint volume of space or in a still larger volume? Or again, for another central question of modern cosmology: What is the value of the Hubble constant H? Granting again that we agree on the interpretation of galactic red shifts as classical Doppler shifts (for which there *is* good evidence), seldom is there a discussion of the *a priori* question: What is the concrete evidence to support the assumption that the expansion parameter is a universal constant? Why must it be a constant independent of place and direction? In short, how do we *know* that expansion is linear and isotropic? And since ρ is not in fact a constant in our neighborhood, how can H be a constant? Or is it possible that it is a stochastic variable that fluctuates with ρ and, if so, again, how large a distance must we consider before a stable statistical average value emerges for H, irrespective of direction in space? Or again for a third example of a standard topic: What precisely is the age of the universe? This question requires the adoption of a very specific class of cosmological models before it can be given any sense. For an irreverent comment on this question, see Fig. 1.

These and similar questions cannot

be answered by aesthetic prejudices or considerations of mathematical simplicity; correct answers can only be discovered by a searching, critical study of the empirical evidence. Clearly, simplifying assumptions and first-order (or even zero-order) approximations are legitimate tools of the theoretical trade; their value is not in question here, and occasionally nature will cooperate. Not infrequently, the simplest assumptions will give a fair—even a good—approximation of observations. Newton's law is a shining example.

But if nature refuses to cooperate, or for a time remains silent, there is a serious danger that the constant repetition of what is in truth merely a set of *a priori* assumptions (however rational, plausible, or otherwise commendable) will in time become accepted dogma that the unwary may uncritically accept as established fact or as an unescapable logical requirement. There is also the danger inherent in all established dogmas that the surfacing of contrary opinion and evidence will be resisted in every way.

Clustering and Superclustering over 10 to 100 Million Light-Years

Let us now turn to specific facts and figures that warn against premature confidence in current "orthodox" models, in particular, against their fundamental assumptions of homogeneity, isotropy, and the existence of a definite mean density (*7*). First, let us recall some of the drastic changes in the observational evidence on the large-scale distribution of matter in the universe since the early surveys of faint galaxies of the 1930's. From sampling surveys with small-field reflectors (primarily the Mount Wilson 60-inch and 100-inch reflectors) (*8*), a picture emerged, about 1935, of a so-called "general field" of more or less "randomly" distributed galaxies, broken only in rare places by an occasional large globular cluster of galaxies, a few megaparsecs across, of which the nearest and best known example is in Coma. These great clusters, being easy to recognize at large distance, were (and still are) used as convenient markers for a study of the velocity-distance relation, the proof of an expanding universe. Except for effects attributed to local absorption in our galaxy, the number density N of galaxies at the magnitude limit of the Mount Wilson counts (about $m = 19.4$ on the current scale) seemed to be roughly independent of direction over at least one hemisphere (there are serious experimental difficulties in maintaining a constant limiting magnitude in the celestial southern hemisphere from an observatory in the Northern Hemisphere). Such results certainly encouraged theoretical cosmologists to adopt homogeneous, isotropic models as realistic approximations of the physical universe. Thus galaxies played the role of molecules in a gas, and from galaxy counts, estimates of distances, and average masses derived from the rotation of a few nearby galaxies, it was in principle a simple matter to derive the mean density of the corresponding volume of space, and this, evidently, was the required value of ρ.

Astronomers soon realized, however, that the concept of a randomly distributed general field of galaxies was the result of poor statistics: first, random sampling with a field of view much smaller than the angular scale of density fluctuations will tend to mask large-scale clustering; second, the Gaussian distribution of the logarithm of N observed by Hubble really means that N is subject to contagion, that is, clustering, as several authors quickly pointed out (*9*). At about the same time, shortly before World War II, surveys with wide-field astrographs at Harvard Observatory and especially with the 18-inch Schmidt camera at Mount Palomar, proved that clusters and groups are the rule rather than the exception (*10, 11*); apparently most, if not all, galaxies are members of some group or cluster, with typical populations of, perhaps, 10 to 100 in the first three magnitudes. The exhaustive galaxy counts with the Lick 20-inch astrograph (*12*) over two-thirds of the sky to $m = 19.0$, and the searching surveys of galaxy clusters with the 48-inch Schmidt camera at Mount Palomar (*13, 14*) to $m = 20.5$ provided in the 1950's overwhelming evidence of the universal prevalence of galaxy clustering on a wide range of linear scales from a few megaparsecs up to at least 50 megaparsecs; this scale of galaxy clustering corresponds to the so-called "superclusters."

Evidence has accumulated in favor of the hypothesis, first advanced by Zwicky (*11, 14*) in 1938, that clusters or more precisely "cluster cells" are "space fillers" that occupy all space available as "suds in a volume of suds" (*15*). Indeed, in a recent systematic, exhaustive survey of the 55 nearest groups of galaxies (those within 16 megaparsecs), there were very few galaxies that could not be assigned to a definite group or cluster (*16*). Isolated, intercluster galaxies (or, if you will, clusters of $N = 1$ member in statistical terminology) are apparently very rare, a fact that has obviously important, if still hidden, physical as well as cosmological implications. Diameters of typical groups are generally between 1 and 3 megaparsecs (*16*); clusters have diameters between 2 and 5 megaparsecs; and there is now good evidence for clustering on a much larger scale of, say, 30 to 60 megaparsecs, that is, for "superclusters."

Some controversy has arisen concerning the concept and reality of "superclusters," that is, of condensations of galaxies on a scale much larger than conventional groups or clusters which typically do not exceed a few megaparsecs in diameter. On the one hand, Zwicky and his collaborators (*17, 18*) have repeatedly asserted that clusters of (globular) clusters of galaxies do not exist and their evidence is not denied. On the other hand, Abell—working from his catalog of clusters based on plates from the same Mount Palomar 48-inch camera—has given definite statistical evidence that at least some of these large clusters have a (nonrandom) clumpy distribution on a typical clustering scale of 50 megaparsecs; he has offered specific examples of such associations or loose groups of clusters, all having about the same red shift (*19, 20*). Statistical analyses by Neyman, Scott, and Shane (*21*) of the counts made at the Lick Observatory have shown that galaxy distribution models based on the assumption of single clustering (that is, of a random distribution of independent cluster centers) do not account in detail for the parameters of the observed galaxy distribution and that the hypothesis of multiple clustering, that is, clusters of clusters, is, therefore, probably necessary. Recently, Karachentsev (*22*) has analyzed the distribution of "very distant" and "extremely distant" clusters at high northern galactic latitudes in Zwicky's catalog; these clusters are at distances of several hundred megaparsecs. Again, as in the Lick Observatory counts, positive correlation, indicative of clustering of cluster centers, exists over areas several degrees in diameter, corresponding to a linear diameter of some 40 megaparsecs for the average supercluster; some five to ten major clusters are included in each supercluster (and, of course, many more small groups).

The mean volume of space ("supercluster cell") occupied by a supercluster has a diameter on the order of 60 megaparsecs. Finally, I have discussed on several occasions since 1953 the growing evidence for a Local Supercluster (*23–25*), encompassing the majority of the nearby galaxies and groups with a center approximately in or near the Virgo cluster. The influence of this supercluster on galaxy counts can be detected at least down to $m = 16$ in the northern galactic hemisphere (*26*). Our Galaxy is in an outlying location, in our Local Group, near the southern edge of the system. Here, too, the major diameter is on the order of 30 to 60 megaparsecs, depending on the distance scale adopted. Furthermore, studies of radial velocities have indicated that in our neighborhood, say, within 100 million light-years, the velocity field is neither isotropic nor linear; this is apparently the result of differential expansion and rotation of the flattened supersystem (*25, 27–29*). A total mass of the order of 10^{15} solar masses was derived from the rotation of the supercluster (*27, 28*). The total solar motion due to galactic and supergalactic rotation (*25*) causes a slight asymmetry of the order of 0.1 percent in the intensity of the 3°K background radiation; this asymmetry may have been detected recently by refined observations (*30*).

Most astronomers who have studied the problem closely have been convinced by this evidence and now accept the reality of superclustering on a scale of the order of 50 megaparsecs. Some of the controversy is more a matter of nomenclature than of fact; Zwicky apparently designates condensations as large as this as "clusters" and admits that such clusters exhibit much "subclustering" (*13, 18*); Kiang (*31*) and Kiang and Saslaw (*32*) have concluded from a statistical analysis of Abell's catalog (*19*) that clustering exists on all possible scales from small groups to the largest superclusters, and they contend that in this respect the concept of "cluster" or "supercluster" is not significant. I presume that a statistical study of human agglomerations would disclose a continuous spectrum of city sizes from isolated farms, hamlets, and townships, to major towns, capital cities, and perhaps megalopolises; I would *not*, however, conclude from this argument that, because a distinction between, say, Johnson City, Texas, and Washington, D.C., has no clear-cut statistical basis, it is therefore not physically significant. This argument remains valid even if clusters occasionally overlap (as cities do too), a fact that can be taken into account in the statistical theory of "interlocking" clusters.

There is a further danger to a purely statistical or "fluctuations" approach to the theory of clustering or superclustering; it neglects the fundamental effects of collective gravitation and its possible counterbalancing of the general expansion. In other words, the physics of the formation, evolution, and possible dissolution of clusters and superclusters in the framework of the expanding background or frame of reference must be considered, not merely the instantaneous aspect of a fluctuating field of massless particles. Unfortunately, concepts and theories of cluster formation and dynamic evolution are still in a very primitive state (*33*). In a sense, a self-consistent theory of clustering can be developed only within the framework of a specific cosmological model; such a model is necessary to fix initial conditions and the surrounding cluster field.

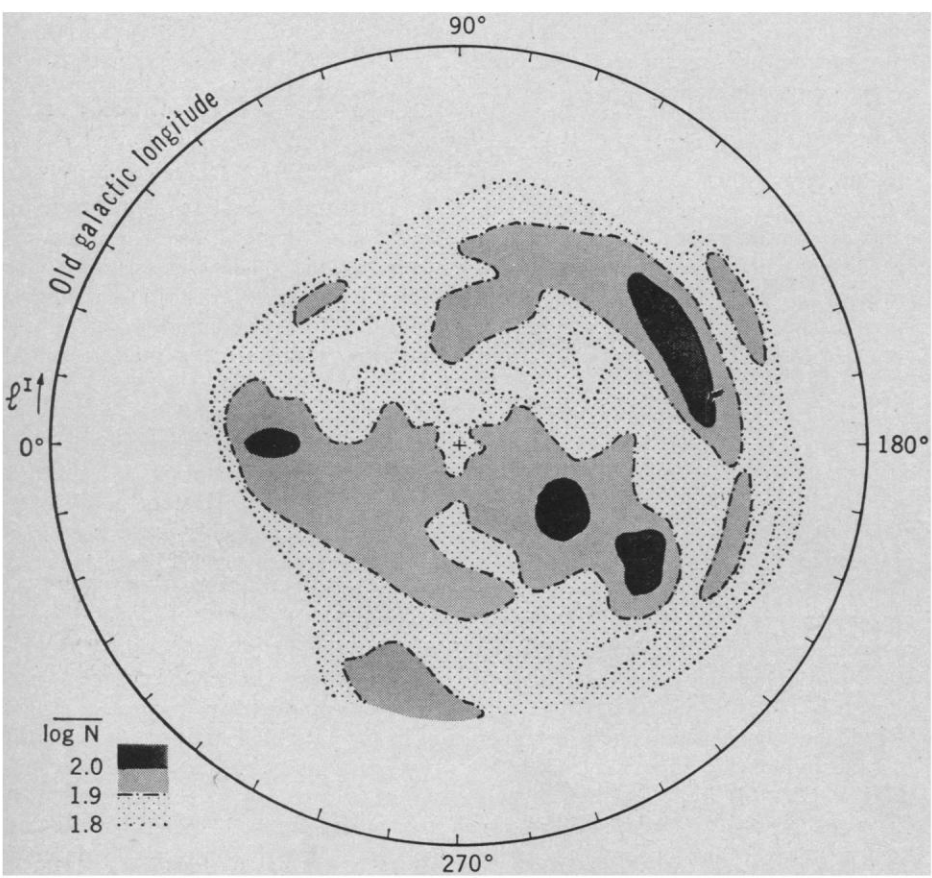


Fig. 2. Smoothed isopleths of the Mount Wilson counts of faint galaxies in the northern galactic hemisphere show large areas of above-average density separated by lanes of lower density. Three or four possible supersuperclusters, or third-order clusters, on a scale of 200 to 300 megaparsecs, are in evidence. North galactic pole is at center of map, equator at circumference.

Higher-Order Clustering over 100 to 1000 Million Light-Years

The question naturally arises whether still larger organizations, that is, higher-order clustering, exist on a scale much greater than the typical supercluster. At present there is no definitive quantitative evidence, principally because no effort has been made to detect fluctuations on this enormous scale, but also in part because one runs out of data. I believe, nevertheless, that there is some indication of nonrandom density fluctuations on a scale of the order of 60° of arc in the smoothed isopleths of Hubble's galaxy counts (*8*) with the 100-inch Mount Wilson reflector (Fig. 2) reaching the faint limiting magnitude ($m \simeq 19.4$). At the estimated limit of the survey, about 300 megaparsecs, this size corresponds to a diameter of about 300 megaparsecs, which is large enough to accommodate many typical superclusters. Similarly, in the galaxy counts to $m \simeq 18.6$ in the southern galactic polar cap, made long ago at Harvard Observatory, Shapely (*34*) noted the presence of a strong density gradient along a 90° arc as one moves away in the northwest direction from a giant galaxy cloud in Fornax. Here, too, the linear scale is in the hundreds of megaparsecs (*35*).

If, then, the deepest, most encompassing surveys of faint galaxies do not begin to approach the required statistical uniformity, if clustering forces still operate strongly on a scale of hundreds of megaparsecs, what is the evidence for large-scale homogeneity and isotropy? How far do we need to go before, at last, we begin to encompass a volume of space big enough to be a fair sample of the universe, with its presumed characteristic mean density ρ? We have been talking about this "mean" density for so long that we almost believe it exists, and many authors have attempted to estimate it from counts of galaxies to $m = 18$ (or even $m = 13$!) in blissful ignorance of the overwhelming, pervasive influence of this hierarchical clustering on ever larger scales—demonstrated by the present, concrete evidence of existing galaxy counts. We can only conclude that, if indeed there is a definite mean density (of normal galaxies) in the universe, it can be estimated only for the whole counted region, unless a still larger volume is required to constitute a genuine "fair sample." There is no proof that even the whole of the presently counted region is a fair sample, because to test this hypothesis would require that one count a roughly equal volume, entirely outside the present one, to check whether the two disjoint volumes lead to approximately equal densities. But if we could do that, that is, reach out to at least twice the present range, we could just as well (in principle) count a volume of space almost an order of magnitude greater than the presently counted volume. Then, however, we would probably run into serious difficulties of interpretation because these deeper counts, to an estimated limiting magnitude $m \simeq 21$ or fainter, involve such questions as the effects of red shifts on apparent magnitudes (the so-called K-correction) (*7*), possible effects of intergalactic absorption over very long paths, and problems of galaxy evolution over the correspondingly long time intervals ($> 2 \cdot 10^9$ years).

To get a feel for this problem of the mean density ρ of the universe, we can replace the hypothetical test out to, say, twice the present distance by actual tests over the range of smaller distances for which data do now exist. This range for galaxy counts is of the order of 300 megaparsecs, thus defining a volume of space of the order of 10^8 cubic megaparsecs, (10^{82} cubic centimeters), not a negligible test domain.

Definition of a Mean Space Density

For clarity let us restate the definition of ρ and the object of the test: If space is homogeneous and isotropic, the average density ρ of the universe is defined as the mean density of a "big enough" volume of space such that the same mean density obtains for any arbitrary increase in the radius of the sample region or for any other, disjoint region of volume at least equal to that of the original test region.

Since matter is evidently clustered on a small scale, this definition implies that, except perhaps for statistical fluctuations, the average density is that of a volume of space large enough to contain at least several clusters of the largest order of clustering (say, on current ideas, simple cluster centers) and that there are no larger clusters of a higher order of clustering, that is, the cluster centers of the highest order of clustering actually realized have a statistically uniform (Poisson) spatial distribution.

In the 1930's astronomers stated, and cosmologists believed, that, except perhaps for a few clusters, galaxies were randomly distributed throughout space; in the 1950's the same property was assigned to cluster centers; now the hope is that, if superclusters are here to stay (and apparently they are), at least they represent the last scale of clustering we need to worry about, and that *their* centers may be denizens of an isotropic homogeneous expanding universe [see, however, (*35*)]. Ignoring for a moment the evidence of superclustering on the scale of 200 to 300 megaparsecs foreshadowed by the galaxy counts, we can at least check whether there is empirical evidence for a leveling off of the average space density of galaxies as sampling regions of increasing radii are considered out to the range of the Lick Observatory survey, the deepest with a well-established limiting magnitude.

In order to place the problem in proper perspective, we will consider the average density of astronomical bodies from the highest value, that of neutron stars, to the lowest, that of the whole counted region. We may always assume that the observer happens to be located at the center of each volume of space in which we estimate the mean density, just as we are, perforce, in the center of the observable region. The problem, then, is that of the relation between the radius R and the mean density ρ of various domains of space. Further, to express all results in conventional physical units, say, centimeters and grams per cubic centimeter, we will need to adopt masses derived either from rotation or from the virial theorem. The well-known discrepancy of one and one-half orders of magnitude between rotational masses of individual galaxies and statistical masses of galaxies in pairs, groups, and clusters is not at issue here. The old debate on the respective merits or flaws of various methods of estimating galaxy masses is not closed (*36–38*), although it is quiescent at the moment. Whether the "missing mass" is located in an extensive corona of low-mass stars with large mass-luminosity ratios around each galaxy or is thinly spread out in the form of ionized gas or other optically invisible forms of matter scattered in intergalactic space between cluster galaxies is of little import here; in all cases, a given mean mass can be statistically attached to each counted galaxy and the actual mass must be bracketed by the lower and upper limits set by the rotational and statistical methods.

Density-Radius Relation and Carpenter's Density Restriction

Now we believe that, to be optically observable, no stationary material sphere can have a radius R less than the Schwarzschild limit

$$R_M = 2GM/c^2$$

corresponding to its mass M (G is the gravitation constant and c is the velocity of light). In a plot of the correlation between mean density ρ and characteristic radius R of cosmical systems of various sizes (Fig. 3), the line

$$\rho_M = 3c^2/8\pi G R_M^2 \qquad (1)$$

or, in c.g.s. units,

$$\log \rho_M \simeq 27.2 - 2 \log R_M \qquad (1a)$$

defines an extreme upper limit or envelope. The ratio $\phi = \rho/\rho_M$ of the actual density to the limiting value for a system of observed radius R may be called the Schwarzschild filling factor. For most common astronomical bodies (stars) or systems (galaxies), the filling factor is very small, on the order of 10^{-4} to 10^{-6}.

A second, lower natural limit to the density of a nonrotating system of free particles is that which is fixed by the virial theorem condition for statistical equilibrium between the total kinetic energy T and the gravitational potential energy Ω,

$$2T + \Omega = 0$$

If ρ^* is the equilibrium density, this condition may be written in the form

$$\rho^* \simeq 3\sigma_v^2/4\pi R^2 G \qquad (2)$$

where σ_v is the velocity dispersion, or

$$\log \rho^* \simeq 6.5 + 2 \log \sigma_v - 2 \log R \qquad (2a)$$

If ρ is less than ρ^*, the system is unstable and will evaporate in a relatively short time; if ρ is greater than ρ^*, the system is dynamically stable and will tend to shrink toward the equilibrium condition (Eq. 2). Real systems with nonzero values of net angular momentum depart from this relation, but the departure is relatively minor. In large systems of stars and galaxies, σ_v is often in the range of 100 to 1000 kilometers per second, so that

$$\log \rho^* \simeq (21.5 \pm 1) - \log R \qquad (2b)$$

which corresponds for the filling factor to

$$\log \phi = \log (\rho^*/\rho_M) \simeq -5.7 \pm 1$$

There is naturally no definite *lower* limit to the density of matter in a given volume of space, if σ_v is negligible and forces other than gravitation are important; for example, $\phi \simeq 10^{-8}$ for the planets and $\phi \simeq 10^{-12}$ for the solar system as a whole.

We can now compare observational data on stars and stellar systems with these theoretical limits. The no-longer so hypothetical neutron stars may come close to the Schwarzschild limit as shown in Fig. 3, where the dashed segment in the upper left corner illustrates a range of theoretical models for which $-2.5 < \log \phi < -0.6$. The next group of very dense stars, the white dwarfs, is represented (Fig. 3 and Table 1) by some well-observed white dwarfs for which $-5.0 < \log \phi < -2.7$. The sequence of ordinary stars is illustrated by the sun and a few representative points for main sequence stars from M8 dwarfs to O5-B0 supergiants and on down to M2 supergiants. Giants with distended atmospheres fall three to six orders of magnitude lower. Infrared stars would extend the density-radius relation downward by several orders of magnitude (dashed line) as illustrated by a hypothetical star of 100 solar masses at the stage where it just begins to radiate ($R \simeq 1000$ astronomical units). Here the filling factor is in the range $10^{-7} < \phi < 10^{-5}$ for stars and $10^{-9} < \phi < 10^{-7}$ for protostars. It is rather remarkable that on the grand view of Fig. 3, where details of stellar models matter little, all families of stars follow closely the same density-radius relation, although each is governed by very different basic physical laws (perfect gases as opposed to degenerate and nuclear matter) and do not form a continuous evolutionary sequence (for example, no stable objects populate the gap between white dwarfs and neutron stars).

An order-of-magnitude relation is thus defined for stellar bodies

$$\log \rho \simeq -2.7 (\log R - 11.0) \qquad (3)$$

which applies at least in the range $-14 < \log \rho < +14$, or $6 < \log R < 16$, and in this range the filling factor ϕ decreases from about 10^{-1} to about 10^{-9} (Fig. 3).

An even more intriguing and as yet unexplained situation develops as we move from stars to star clusters, galaxies, clusters of galaxies, and eventually to the whole countable extragalactic space depicted in the lower half of Fig. 3. As one moves downward in the figure, the symbols refer to (i) compact dwarf elliptical galaxies, (ii) normal giant elliptical galaxies, (iii) normal giant spiral galaxies, (iv) small, compact groups of galaxies, (v) larger groups and clouds of galaxies, (vi) small clusters, of the Virgo or Fornax I type, (vii) large clusters of the Coma type, (viii) the Local Supercluster, (ix) the nearby region ($R < 30$ megaparsecs) in which detailed studies, for example, of red shifts and luminosity function, are possible, and (x) the largest volume of space ($R < 250$ megaparsecs), for which

Table 1. Mass-radius-density data.

Class of objects	Examples	log M (g)	log R (cm)	log ρ (g cm^{-3})	log ϕ	Ref.
Neutron stars		33.16	5.93	14.75	−0.6?	(*53*)
		32.54	7.44	9.60	−2.5	(*53*)
White dwarfs	L930-80	33.45	8.3:	7.93	−2.7	(*54*)
	αCMaB	33.30	8.77	6.37	−3.2	(*54*)
	vM2	32.90	9.05	4.13	−5.0	(*54*)
Main sequence stars	dM8	32.2	9.95	1.76	−5.6	(*55*)
	Sun	33.30	10.84	0.15	−5.5	(*55*)
	A0	33.85	11.25	−0.55	−4.7	(*55*)
	O5	34.9	12.1:	−2.0	−5.0:	(*55*)
Supergiants stars	F0	34.4	12.65	−4.2	−6.1	(*55*)
	K0	34.4	13.15	−5.7	−6.6	(*55*)
	M2	34.7	13.75	−7.2	−6.9	(*55*)
Protostars	IR	35.3?	16.2?	−13.9?	−8.7?	(*55*)
Compact dwarf elliptical galaxies	M32, core	41.0	19.5?	−18.1	−6.3	(*56, 57*)
	M32, effective	42.5	20.65	−20.0	−5.9	(*58*)
	N4486-B	43.4	20.5	−18.75	−5.0	(*56, 59*)
Spiral galaxies	LMC	43.2	21.75	−22.65	−6.3	(*60*)
	M33	43.5	21.8	−22.5	−6.1	(*61*)
	M31	44.6	22.3	−22.9	−5.5	(*62*)
Giant elliptical galaxies	N3379	44.3	22.0	−22.35	−5.6	(*38, 63*)
	N4486	45.5	22.4	−22.3	−4.7	(*56, 64*)
Compact groups of galaxies	Stephan	45.5	22.6:	−23.1:	−4.7	(*38, 65*)
Small groups of spirals	Sculptor	46.2	24.1	−26.7	−5.7	(*38, 66*)
Dense groups of ellipticals	Virgo E, core Fornax I	46.5	23.7	−25.2	−5.0	(*38, 67*)
Small clouds of galaxies	Virgo S Ursa major	47.0	24.3	−26.5	−5.1	(*38, 67*)
Small clusters of galaxies	Virgo E	47.2	24.3	−26.3	−4.9	(*38, 67*)
Large clusters of ellipticals	Coma	48.3	24.6	−26.1	−4.9	(*36, 38*)
Superclusters	Local	48.7:	25.5:	−28.4:	−4.7	(*24*)
HMS sample to $m \simeq 12.5$			26.0:	−29.6	−4.6	(*68*)
Lick Observatory counts to $m \simeq 19.0$			26.8	−30.5	−4.1	(*12*, p. 55)

reliable galaxy counts are available. Numerical details and references are given in Table 1.

This lower half of Fig. 3 is a revised, updated version of a graph first published 10 years ago (*24*, *36*), and which was itself inspired by Carpenter's discovery of a "density restriction" governing the maximum population of groups and clusters of galaxies (*39*). If we consider all clusters with a characteristic radius $R \pm dR$, there is apparently an upper limit to the space density of galaxies ν_R or to the mass density ρ_R that can exist within the corresponding volume; this limit defines a linear envelope in the scatter diagram of $\log \rho$ as a function of $\log R$ above which no system is observed. This envelope is well below the Schwarzschild limit ($10^{-6} < \phi < 10^{-5}$), and it is clearly not caused by observational selection, since a system having the same diameter as, say, the Coma cluster, but with 10 to 100 times its galaxy population would have been among the first to be discovered. In order to define the envelope as shown in Fig. 3, we need therefore to consider only a sample of the densest systems corresponding to a given range of typical radii.

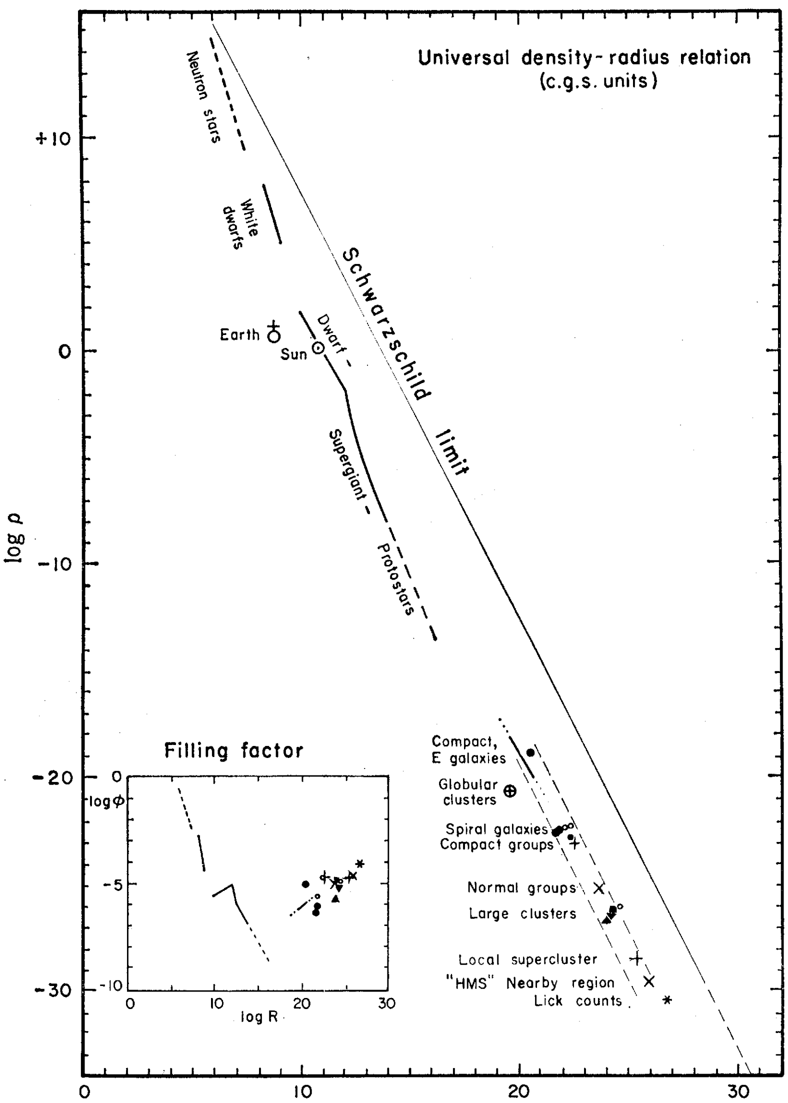
It is remarkable that these points also define a linear relation, but it is not a direct extension of the line for the stellar bodies. For systems of stars and galaxies the filling factor ϕ apparently increases with R from about 10^{-6} to 10^{-4} when R increases from 10^{18} to 10^{27} centimeters. A linear fit gives the relation

$$\log \rho = -21.7 - 1.7 (\log R - 21.7) \quad (4)$$

The slope lies within the range (−1.5 to −1.9) of earlier preliminary estimates (*24*, *36*, *39*). Comparing Eqs. 3 and 4, we see that the slope

$$\partial (\log \rho) / \partial (\log R) = S$$

of the density-radius relation is significantly different for stellar bodies for which $S = -2.7$ and for star and galaxy systems for which $S = -1.7$. Nevertheless, most of the data points are



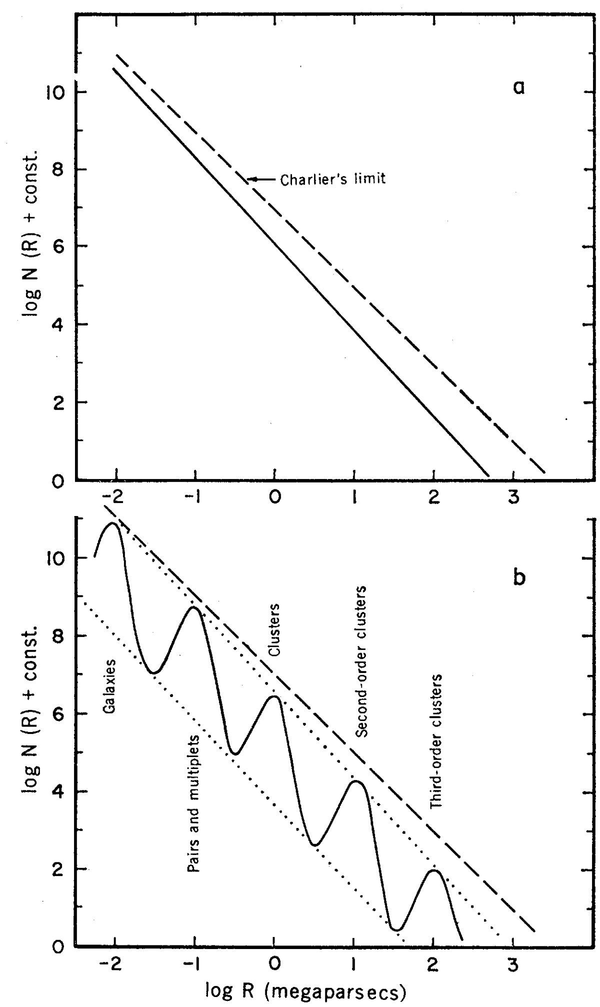


Fig. 3 (left). Universal density-radius relation gives the maximum average density of matter (in grams per cubic centimeter) in spherical volumes of radius R (in centimeters) from neutron stars (dashed line at top) to the largest domain in which galaxies have been counted (asterisk at bottom). The Schwarzschild limit (thin line) and filling factor ϕ (inset) are shown. The range of densities by the virial theorem for stellar and galaxy clusters is shown (thin dashes). Fig. 4 (right). Idealized frequency functions of star and galaxy clusters illustrate two possible pictures of a hierarchical universe. (a) Clustering of galaxies occurs on all possible scales with no preferred sizes; the number density of clumps per unit volume decreases smoothly as their radius R increases; (b) clustering of galaxies occurs on all scales but with greater numbers of clumps with characteristic radii near 10 kiloparsecs (galaxies), 100 kiloparsecs (pairs and multiplets), 1 megaparsec (groups and clusters), 10 megaparsecs (superclusters), 100 megaparsecs (third-order clusters), and so forth. In both (a) and (b) the average slope of $\log N$ as a function of $\log R$ must be steeper than − 2, the Charlier limit for convergence.

within a strip parallel to the Schwarzschild relation (Eq. 1a) for which $S = -2$; this strip corresponds to a mean filling factor $\phi \simeq 10^{-4}$ to 10^{-6}. There are few reliable data for individual objects in the range $13 < \log R < 18$. This may result from observational selection if most such objects are dark; for example, infrared stars, protostars, and the small dark clouds of interstellar matter known as globules have diameters in this range. Luminous objects in this range, which would appear as highly compact globular clusters with diameters of a fraction of a parsec, should be easily detected if they were present in the galactic neighborhood of the sun, but no such object is known. Quasars are other possible candidates that might fill this gap; but we know too little of their masses and radii to place them in Fig. 3 with any degree of confidence at present.

Cosmological Implications and Charlier's Hierarchical Models

The density-radius relation for stellar and galactic systems has interesting cosmological implications. There is no indication out to the largest observed value of R defined by the Lick Observatory counts to $m = 19.0$ that a limiting constant value of ρ is reached which could be *the* average density ρ_0 of the universe (or at least of optically visible condensed matter). If a constant nonzero value of ρ_0 exists, it is not reached within the range of distances sampled by galaxy counts. In the range $18 < \log R < 27$ the larger the volume of space sampled, the lower the mean density of countable galaxies; that is, the volume of space in which galaxies have been counted is not the "fair sample" of space postulated by the isotropic homogeneous models of cosmology, and the value of ρ inserted in such models on the basis of existing galaxy counts may well be irrelevant. *A fortiori* values of the space density (and also of the expansion parameter H) derived from studies of nearby galaxies within the Local Supercluster are even less likely to fulfill the theoretical assumptions.

There is a little-discussed class of cosmological models in which the average density ρ can converge to an arbitrarily small value as the radius of the test volume increases indefinitely in a manner which could be consistent with Eq. 3.

The concept of a hierarchy of systems, that is, galaxies, clusters of galaxies, superclusters, or clusters of the second order, and so on, was first introduced by Charlier (*40*) in 1908 and refined in 1922 as a possible classical solution to Olbers' paradox (41). Charlier showed how with this hierarchical concept a Euclidean-Newtonian universe could be built up which would avoid the Olbers' divergence as long as the radii R and population N of systems of order i and $i+1$ satisfy the inequality

$$R_{i+1}/R_i > \sqrt{N_{i+1}} \qquad (5)$$

where N_{i+1} is the number of systems of order i in the system $i+1$. It is easy to demonstrate that, if this condition is met, the infinite series which gives the total light flux at any given point is convergent, that is, the light flux is finite and the contribution of the distant systems is arbitrarily small. The discovery of the universal red shift, which decreases the energy of photons from distant sources, and the successful development of finite world models neatly solved the Olbers' paradox and removed the necessity of invoking the Charlier solution. Nevertheless, the concept of a hierarchical structure was not disproved by the convenient emergence of different types of solutions to the Olbers' paradox. Nor is there any need to adhere to an oversimplified geometric description of a hierarchical structure, which is perhaps a convenient model to demonstrate the theorem but is probably not an essential one. In reality, the scale of density fluctuations may very well form a continuous spectrum; that is, clustering could probably occur on all possible scales—for example, as in a theory of hydrodynamic turbulence on a cosmic scale, in which we might compare the clusters of various orders to a hierarchy of eddies—and still satisfy the rather weak condition expressed by Charlier's inequalities, as long as enough relative "void" is left between the eddies. Here two cases must be distinguished, depending on the shape of the frequency function of cluster radii $N(R)$. In the first case $N(R)$ is a monotonically decreasing function of R (Fig. 4a), as implied by Kiang (*31*) and Kiang and Saslaw (*32*). In the second case $N(R)$ has a series of relative maxima at some preferred values of R (Fig. 4b), say, $R_{-2} \simeq 0.01$ megaparsec (galaxies), $R_{-1} \simeq 0.1$ megaparsec (pairs and multiplets), $R_0 \simeq 1$ megaparsec (groups and clusters), $R_1 \simeq 10$ megaparsecs (superclusters or second-order clusters), $R_2 \simeq 100$ megaparsecs (third-order clusters), and so forth, as suggested by the data presented here. The empirical evidence and the statistical methods of analysis are not yet good enough to clearly distinguish between these two cases (*42*, *43*). It is clear that there is a need for an extension of Charlier's work to quasi-continuous models of density fluctuations that would replace the original, oversimplified discrete hierarchical model. It is equally clear that this fluctuating density field should be considered within the framework of the world models of general relativity.

It would seem, in principle, that nothing prohibits the introduction of the hierarchical concept of an indefinitely clustered density distribution in the more promising relativistic or steady-state models. In practice, of course, major mathematical difficulties may be encountered. A first-order treatment of small fluctuations (say, ~ 10 percent) has been developed successfully (*44*), but it is not sufficient for a discussion of the actual situation in the real universe where "fluctuations" are of the order of 1000 percent of the local average density and, as indicated by Fig. 4 and Eq. 4, the average density decreases by nearly two orders of magnitude when the radii of the systems considered differ by one order of magnitude.

In a sense a hierarchical model is homogeneous because, except possibly for statistical fluctuations, two equally large disjoint volumes of radius R_n, each encompassing at most one supersystem of order n, can have the same average density $\rho(R_n)$. This type of homogeneity satisfies the restricted cosmological principle (no privileged position in space) but, because of Charlier's inequality, it does not imply that ρ should be a constant independent of n. All observers, wherever located (but within the hierarchy), will find that the average density decreases as the range R of their counts increases. In a Newtonian-Euclidean universe devoid of intergalactic matter, R could presumably increase indefinitely, and, if so, $\rho(R)$ would asymptotically approach zero as in Charlier's original concept. In a relativistic world model, all the basic concepts of the homogeneous models would presumably still remain, including the possibility of closed or open universes. In the case of a closed static universe, the radius of curvature (R)

would be related to the density ρ of the largest-scale supersystem defined by $R_M \simeq (R)$, and $\rho(R) = \rho_M$ is the Schwarzschild radius defined by Eq. 1. As a purely arithmetic curiosity, if the value of ρ given by Eq. 4 is substituted into Eq. 1, then $\rho = \rho_M$ for $\log R \simeq 40$, where $\log \rho \simeq -52.8$(!) Obviously this result cannot be taken seriously because of the enormous extrapolation and observational uncertainties in the basic data.

Nevertheless, the point remains that if $\log \rho = -30 \pm 1$ throughout (independent of R for all observers), as is currently assumed in most orthodox theories, we must postulate that intergalactic space is filled by a uniform and relatively homogeneous invisible gaseous medium comprising 90 to 99 percent of the smoothed-out mean density. If so, we are forced to conclude that the mass condensed in galaxies, clusters, and superclusters makes an almost trivial contribution to ρ and that it is possible, indeed probable, that the values of ρ currently derived from galaxy counts are of little significance with respect to the problem of the mean density of homogeneous models.

Even if we grant, as a working hypothesis, that galaxies may not be the major contributors to the mass density, we may at least use them as tracers, which show where matter is condensed; it would be very strange indeed if condensations of visible and invisible matter were mutually exclusive. If so, it seems difficult to believe that, whereas visible matter is conspicuously clumpy and clustered on all scales, the invisible intergalactic gas is uniform and homogeneous. This is perhaps conceivable for radiation, but not for matter, whether it be diffused or condensed. Certainly interstellar matter is extremely clumpy and irregularly distributed; so are the high-latitude, high-velocity neutral hydrogen clouds in the galactic corona; why then should intergalactic gas be smoothly spread out throughout the universe?

If indeed there is something to the latest version of the steady-state hypothesis (*5*), where matter is injected (by some as yet unknown law of physics) into the visible universe at the centers of galaxies (*45*), then, evidently, the distribution of invisible matter must be closely related to and just as clumpy as the distribution of galaxies, and a hierarchical structure *à la* Charlier must be included in any realistic cosmological model.

Cosmology in a Clumpy Universe

If, therefore, we grant that clumpiness in the distribution of matter in the universe is a basic property of fundamental importance for cosmology and not merely a local nuisance that can be ignored in the grand smoothed-out view, we must pay much more attention than we have thus far to the possible consequences of this situation.

Although, clearly, much detailed work will have to be done before we can assess the consequences of this hierarchical concept for different cosmological models, we may perhaps consider the existing homogeneous, isotropic models, as analogous to the osculating parabolic elements of a cometary orbit. The curvature, expansion rate, and deceleration of a given homogeneous model may approximate conditions over some distance range (volume) within which a local average density ρ_R obtains, but it cannot be extrapolated to the whole universe past and present, any more than the parabolic orbit can be extrapolated to infinity in space and time. Within the volume considered, density fluctuations will wrinkle the geodesics and alter the expansion parameters; there is already clear evidence that the Hubble expansion rate is reduced by gravitation within the Local Supercluster (*27*, *29*), perhaps almost cancelled within the Local Group (*46*), and, of course, completely overwhelmed by it within individual galaxies.

This leads one to view the Hubble parameter as a stochastic variable, subject in the hierarchical scheme to effects of local density fluctuations on all scales. A simple analogy is a light ray weaving to and fro as it traverses successive domains of different sizes, densities, and inhomogeneous gravitation fields. The path of the light ray will experience more deflection, but over a shorter length, as it crosses small domains of relatively high density (for example, a cluster), or less deflection, but over larger paths, as it propagates through larger domains of lower density (for example, a supercluster). Detailed calculations for specific models of the hierarchical structure will be needed to evaluate the net effect of this mechanism over increasing path lengths (*47*, *48*).

Such calculations may require a kind of statistical approach to relativity in which the model parameters will depend in some complicated manner on characteristic scale lengths, all satisfying Charlier's inequality. On any scale an osculating homogeneous model may be defined, but it should not be extrapolated to much larger (or smaller) scales. In this sense if the current homogeneous-isotropic models seem to converge or point to a definite solution of the cosmological problem, it may well be merely a reflection of the limited range of present data on red shifts, galaxy counts, and source counts. And so, perhaps, once again we are mistaking the horizon for the end of the world.

Summary

We have to choose between two sets of possibilities. First set: (i) estimates of the age of the world which have grown exponentially for the past 3 centuries (Fig. 1) will suddenly reach a final constant level within a few years of A.D. 1969—obviously possible, but a little surprising; (ii) the obvious nonrandom clustering which dominates the galaxy distribution on all scales out to the limit of the deepest survey (Fig. 2) will suddenly vanish and be followed by the emergence of statistical uniformity as soon as we consider volumes equal to or greater than the totality of the presently counted volume—again possible, but, on present evidence, a not too likely event; (iii) as ever larger volumes of space are considered, the correlation between the maximum density of matter and radius, demonstrated over a range of more than 20 orders of magnitude in radius and 45 orders of magnitude in density, suddenly stops operating beyond the last observed point (Fig. 3) to level off at the presumed value of the mean density postulated by current homogeneous models —again not completely impossible, but certainly a highly artificial hypothesis on the basis of the evidence at hand (what other law of physics or astronomy has been checked over a greater range?).

Now in order to accept the present orthodox cosmologies we must assume that all three suppositions above will be verified. The combined probability of all three being simultaneously resolved in the affirmative appears to be very small indeed, since each requires a sudden break just beyond the last datum, of which there is no hint in the observed range.

The second set of possibilities involves merely accepting the empirical evidence as it stands: (i) the concept of "age of the world" has a long historical record of rapid change and, anyway, lacks definiteness, except in specialized world models which are far from established; (ii) clustering of galaxies, and presumably of all forms of matter, is the dominant characteristic of the structure of the universe on all observable scales with no indication of an approach to uniformity; (iii) the average density of matter decreases steadily as ever larger volumes of space are considered out to the limit of the counted region, and there is no observational basis for the assumption that this trend does not continue out to much greater distances and lower densities.

It would seem that the time has come to give serious consideration to hierarchical models either within the framework of otherwise conventional relativistic cosmology, if that is possible, or within the context of the steady-state theory which is eminently compatible with the kinds of large-scale properties discussed here (*4*).

A beginning has already been made in this direction; thus Hoyle and Narlikar (*47*) have considered the effects of a creation rate which varies with preexisting local density in a kind of hierarchical steady-state model, and Rees and Sciama (*48*) have discussed possible effects of large-scale inhomogeneities in evolutionary relativistic models (*49*). Wertz (*50*) made a preliminary survey of possible variations on the hierarchical scheme, starting with the simplest homogeneous isotropic hierarchical (or "polka dot") cosmological models and branching out to include hierarchical-homogeneous as well as inhomogeneous anisotropic models, both finite and infinite, and even hierarchical schemes with homogeneous background and inhomogeneous models with varying elementary (that is, galaxian) masses. It is evident that a systematic morphological analysis of all possible hierarchical models will reveal the great richness of the concept compared with the already large range of choices offered by homogeneous models. The task of developing self-consistent theoretical models within the framework of this vast array of hierarchical systems and of rigorously testing each of them against the observed properties of the universe appears rather formidable. It seems safe to conclude that a unique solution of the cosmological problem may still elude us for quite some time!

References and Notes

1. See, for example, W. H McCrea, *Science* **160**, 1295 (1968); H. Y. Chiu, *Sci. J.* **4**, 33 (1968).
2. Among the better introductions to modern cosmology the following are especially recommended: P. Couderc, *The Expansion of the Universe* (Faber & Faber, London, 1952); G. C. McVittie, *Fact and Theory in Cosmology* (Macmillan, New York, 1961); H. Bondi, *Cosmology* (Cambridge Univ. Press, New York, 1960). For philosophical perspectives on the history of modern cosmology I recommend the following: J. Singh, *Great Ideas and Theories of Modern Cosmology* (Dover, New York, 1961); J. D. North, *The Measure of the Universe* (Clarendon Press, Oxford, 1965). The last reference is exceptionally thorough and comprehensive as well as more technical.
3. H. Bondi, *Rep. Progr. Phys.* **22**, 97 (1959).
4. W. Davidson and J. V. Narlikar, *ibid.* **29**, 541 (1966).
5. F. Hoyle, *Quart. J. Roy. Astron. Soc.* **10**, 10 (1969).
6. For example, in a recent review article on "The cosmology of our universe" (*1*), we read that "In cosmological theory it is convenient to regard galaxies as the basic elements of the Universe" (even though Zwicky showed more than 30 years ago that clusters, not galaxies, are the basic blocks); more amazingly, as a summary of the observational evidence, we read: "The distribution of galaxies is generally uniform up to a distance of 2×10^9 light-years, with a variation of a few percent. In general, the distribution is isotropic and homogeneous. . . . The matter density due to galaxies . . . is 3×10^{-31} gram per cubic centimeter." This is little more than sheer fiction as I will show in this paper.
7. Readers unfamiliar with some astronomical terms or concepts may find useful the following explanations: Galaxies are counted on photographic plates by visual inspection through low-power magnifiers; anything fuzzy (unless it is obviously a galactic nebulosity) is counted as a galaxy down to the faintest detectable smudge that can be just differentiated from the sharper star images; the limiting brightness at the detection threshold evaluated in the traditional stellar magnitude scale (which increases logarithmically with decreasing intensity) is the magnitude limit m of the counts. The results, corrected for various instrumental factors, are expressed as the areal number density of galaxies on the celestial sphere, that is, the number of galaxies per square degree $N(m)$ brighter than the limiting magnitude m of the survey. The first systematic survey by Hubble at Mount Wilson Observatory in the early 1930's was limited to small telescopic fields, each covering less than the area of the moon, at 5° and 10° intervals regularly spaced in galactic latitude and longitude. The most recent and thorough survey by Shane and his collaborators at Lick Observatory completely covered the sky north of declination —23° (including about two-thirds of the area of the sphere) to a slightly brighter limit of magnitude. At the large distances of the faintest galaxies counted in these surveys, the apparent luminosity is reduced not only by distance itself but also by geometric and spectral effects of the expansion of the universe which must be corrected (the so-called K-correction) before the results can be used in cosmology. The existence of clustering among galaxies far in excess of the accidental clumping that arises by chance in a random (Poisson) distribution of independent particles is quantitatively demonstrated by a comparison of the frequency distribution of $N(m)$ determined in many small different areas with that expected for a Poisson distribution. The average size of the clumps is derived by correlation techniques which make possible the evaluation of the degree of coherence or incoherence of counts in separate nearby areas of variable separation S as a function of S. For example, positive correlation exists in the Lick Observatory counts over areas up to 8° in diameter, corresponding to an average clump size of 30 to 40 megaparsecs at the estimated limiting distance of the survey (this size is typical of the second-order clustering).
8. E. P. Hubble, *Astrophys. J.* **79**, 8 (1934).
9. B. J. Bok, *Bull. Harvard Coll. Observ. No. 895* (1934); P. Bourgeois and J. Cox, *C. R. Hebd. Seances Acad. Sci. Paris* **204**, 1622 (1937); *Ciel Terre* **54**, 287 (1938); A. G. Mowbray, *Publ. Astron. Soc. Pacific* **50**, 275 (1938).
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11. F. Zwicky, *Publ. Astron. Soc. Pacific* **50**, 218 (1938).
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35. In the conclusion of his study of superclustering of cluster centers in the Zwicky catalog, Karachentsev (*22*) is led "to question the correctness of the initial premise of a random distribution of the supercluster centers" and concludes that "some associative property is conspicuous in their distribution," in agreement with the evidence of Fig. 2 and other data discussed in this article.
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41. The Olbers' paradox (*Bode's Jahrbuch*, Berlin, 1826), p. 110, concerns the question "Why is the sky dark at night?" In an infinite, transparent Euclidean-Newtonian universe uniformly populated with stars, any line of sight would

eventually encounter the surface of a star and everywhere the sky should be about as bright as the sun. The problem was first discussed in the 18th century by de Chéseaux, of Lausanne, and the related question of infinite gravitational potential was further analyzed by Seeliger in 1895 (*Astron. Nachr. No. 3273*). One possible solution which survived as late as 1918 in Shapley's views of the galactic system and 1922 in Kapteytn's heliocentric stellar universe was the hypothesis that the world of stars is not infinite but rather a single island universe in an empty cosmos. Charlier's concept was a neat geometric solution (in nonrelativistic terms) to the problem of "how an infinite world may be built up." A hierarchical structure of the cosmos was first envisioned by the Alsatian, J. Lambert, in his somewhat fanciful "Kosmologischen Briefen" (Augsburg, 1761). An excellent historical summary (in French) on the Olbers' paradox and its sequels was published in 1966 by R. Chameaux [*Bull. Soc. Astron. Toulouse* **57**, No. 485 (1966)]. Another very interesting discussion of the paradox and its cosmological implications was presented in June 1965 by A. G. Wilson in an unpublished lecture to the Los Angeles Astronomical Society (Rand Corporation reprint P-3256) [see also (*49*)].

42. For example, statistical analyses of Abell's cluster catalog have led to the following conflicting conclusions: definite superclustering on a 50-megaparsec scale for at least a fraction (but not the totality) of the cluster population, according to Abell (*20*); some superclustering on scales of 50 to 200 megaparsecs, according to Kiang and Saslaw (*32*); no significant superclustering, according to Yü and Peebles (*43*). [However, after reading a preliminary version of this paper, Dr. Abell informed me (personal communication) that the results of Yü and Peebles "followed only by not counting those clusters in distance group 5 in the southern hemisphere, where the superclustering appears most obvious. In fact, the superclustering was so pronounced in that part of the catalog that they felt it was not representative and so discounted it."]. Similarly, analyses of the Zwicky catalogs have led to the widely diverging conclusions of Zwicky and his collaborators (*13–18*) and of Karachentsev (*22*). One additional remark should be made here; in the study of superclustering it may not be strictly equivalent to study the distribution of galaxies in general (as was done in the analysis of the Local Supercluster and of the Lick Observatory counts), on the one hand, or the distribution of large or rich clusters (as listed in the Mount Palomar surveys) on the other. It is entirely possible that superclustering of large clusters is much less pronounced and prevalent than superclustering of groups and small clusters which are automatically excluded in the Abell and Zwicky catalogs by the very definition and method of selection of "rich clusters." For example, only one cluster—and not a particularly rich one—the Virgo cluster, is known in the Local Supercluster, and if this supercluster were seen from a great distance, it would not be recognized as such from cluster counts because it would be represented by only one cluster (and perhaps none). To use again the human population analogy, it is doubtful that the obvious "superclustering" of the general population indicated by statistics of agglomerations of all sizes (that is, complete "counts") would be readily detected by an analysis restricted to the worldwide distribution of the great capital cities only (the "rich clusters"). I suspect that the apparent disagreement in the conclusion reached by various investigators arises, at least in part, from a failure to recognize that only a small fraction of the total galaxy population is concentrated in the relatively rare "rich clusters."
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45. This concept was foreshadowed as long ago as 1928 when Jeans wrote in *Astronomy and Cosmogony* (Cambridge Univ. Press, Cambridge, p. 352) his well-known, possibly prophetic speculation: "The type of conjecture which presents itself, somewhat insistently, is that the centers of the nebulae are of the nature of 'singular points,' at which matter is poured into our universe from some other, and entirely extraneous, spatial dimension, so that to a denizen of our universe, they appear as points at which matter is being continually created."
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49. In recent years A. G. Wilson has given much thought to the concept of a hierarchical cosmos and has searched for signs of a possible "discretization" in "modular" structures (*51*) that might perhaps relate cosmic and atomic constants through quantized relations. Although the numerological aspects of this approach—reminiscent of Eddington's brilliant but futile *Fundamental Theory*—are admittedly highly speculative, the basic concepts and evidence of a hierarchical world structure discussed by Wilson are very much the same as those presented here. I am indebted to Dr. A. Wilson for calling my attention to his own extensive work in this area and for a preprint of his chapter *Hierarchical Structure in the Cosmos* (*52*).
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The Borax Lake Site Revisited

Reanalysis of the geology and artifacts gives evidence of an early man location in California.

Clement W. Meighan and C. Vance Haynes

In 1948, Harrington published the results of his archeological studies under the title "An ancient site at Borax Lake, California" (*1*). From this location in northern California (Fig. 1) was recovered a large and diverse collection of stone implements, including a group of fluted points comparable to similar specimens from widely scattered locations where they were found in association with the bones of extinct animals such as mammoth and giant bison (*2*). Fluted points have been recognized for some time as diagnostic traces of the early Indian hunters of about 10,000 to 12,000 years ago in North America, hence the title of Harrington's report and the importance of the site as one more location where an early assemblage was found. However, for reasons discussed below, there arose an immediate and continuing controversy over the age of the site, the nature of the artifacts, and the interpretation to be drawn from the Borax Lake collection. The result was that the Borax Lake material was put to one side and treated as uncertain in meaning. We now reexamine the evidence from Borax Lake and attempt to define its importance to studies of early man in the New World. The site has more than local significance for a number of reasons.

1) The collection has remained for

Dr. Meighan is professor of anthropology in the department of anthropology, University of California, Los Angeles; Dr. Haynes is associate professor of geology in the department of geological sciences, Southern Methodist University, Dallas, Texas.