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Some Three-Body Atomic Systems

Scattering experiments dealing with resonance and threshold behavior provide tests of theory.

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Since the Nobel-prizewinning experiments of Lamb (*1*), which dealt with the detailed understanding of the two-body atomic system, the hydrogen atom, considerable effort has gone into the study of the next more complicated case—the three-body atomic system, such as the helium atom, He, and the atomic hydrogen negative ion, H^-, and the three-body molecular system, such as the hydrogen molecular ion, H_2^+. Although Lamb, from his study of the spectroscopic hyperfine structure of atomic hydrogen, demonstrated that quantum mechanics can accurately describe the two-body problem, no one has yet been able to exactly solve the three-body problem.

Within the last decade a large number of experiments have been performed on three-body systems in order to generate sufficiently detailed data to test various approximations to the exact theory. Within the last few years considerable theoretical and experimental progress has been made, and although one cannot yet arrive at an exact solution, theoreticians have devised a number of approximations which describe most of the details thus far found experimentally. But none of the work thus far reported for the three-body system is of equal sophistication to that reported for the two-body system. This is yet to come.

Although we tend to think of the study of the three-body system as purely an intellectual endeavor—and indeed this is one of its purposes—it is important to realize that most of the experimental and theoretical techniques developed have been applied to more complex cases. Furthermore, since the interstellar plasma is made up largely of electrons, protons, and hydrogen and helium atoms, most of the results of these studies find direct application in astrophysics.

In this short review I discuss the atom He and the ion H^- and show some of the similarities and differences between them as found through atomic collision physics. In each case we are dealing with a two-electron system and a positively charged nucleus. In the first case the nucleus is the singly charged proton, and in the second it is a doubly charged alpha particle. The major differences that exist must, then, result from the extra charge on the He^{++} nucleus.

Essentially, all the physics I discuss is summarized in Fig. 1, which is a partial energy-level diagram for the hydrogen atom, the hydrogen negative ion, and the helium atom. Referring to this diagram, I discuss what happens when an electron is thrown into the field of a hydrogen atom and into that of a helium positive ion. Also, I discuss this second system (He^+, *e*) in yet another way, describing a technique in which, by means of vacuum ultraviolet photons (that is, photons with energies in excess of 7 electron volts), one is able to photo-ionize the helium atom through auto-ionizing channels. Throughout the discussion the main emphasis is placed on the formation of electron scattering resonances and upon the details of ionization and excitation in the energy interval near threshold.

Hydrogen Negative Ion: General Considerations

Let us consider first what happens when an electron is brought into the field of the hydrogen atom. In Fig. 1, the energy scale refers to the situation in which the electron and the hydrogen atom in the ground state are infinitely separated and at rest. As the electron is allowed to move toward the hydrogen atom there is sufficient force between the electron and the atom to bind the second electron to the atom. The binding energy is approximately 0.7 electron volt. To stabilize the H^-, a third body has to carry off the excess energy. To the best of our knowledge, there is only one bound state of the hydrogen negative ion, which has the configuration $(1s)^2\ ^1S$, the same configuration as the ground-state helium atom.

As the electron energy is increased through the energy interval below 9.65 electron volts, primarily elastic scattering of the electron from the hydrogen atom occurs (*2*). No state of H^- corresponding to a bound excited state of He has yet been observed. However, above 9.65 electron volts there are a number of series of temporarily formed negative ion states which have a helium counterpart and which are associated with two electrons both of which are excited. These states are well above the level where the electron quickly separates ("autodetaches") from the H atom after a short life of approximately 10^{-13} second. Corresponding to the interval on the energy scale below and just above the $n = 2$ level are a number of possible series of short-lived resonances

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with the configurations ^{1,3}S, P, D, Whereas the resonances which are associated with, but are energetically just below, the $n = 2$ level can decay only into the elastic scattering channel, those which are just above have a choice. The electron can leave the hydrogen atom either in the ground state (that is, through the elastic scattering channel) or in the first excited state (that is, through the inelastic scattering channel).

The resonances corresponding to poorly defined energy levels just above the threshold can be thought of as "open-channel" resonances, since they can decay, leaving the atom in the excited $2s$ or $2p$ state, the parent level for the temporary negative ion. From analogy with nuclear physics, they are also known as "shape" or "potential" resonances, since they are bound in the same potential well as those resonances which appear below the $n = 2$ level.

Below and above each of the subsequent principal energy levels there appear a number of families of resonances. For the $n = 3$ level, ^{1,3}F resonance series are expected, and now the configurations can be ^{1,3}S, P, D, F. The process continues up to the ionization limit, where two electrons are thrown off into the continuum, leaving a proton behind.

High-Resolution Electron Impact Studies

Since the hydrogen atom prefers to combine with a second atom to form a molecule, the investigator must fashion a beam of free-flight hydrogen atoms in which the electron scattering takes place. The method of forming such a beam is essentially that used by Lamb (*1*), wherein hydrogen molecules are thermally dissociated in a high-temperature oven and effuse from the oven as atoms in free flight. However, in atomic-hydrogen-beam experiments the density of the hydrogen atoms is normally of the same order as the density of the background gases in the vacuum chamber. This is approximately 10^{-7} torr, or 10^9 atoms per cubic centimeter. In order to weed out the signal associated with scatter from hydrogen atoms from that associated with scatter from background gases, W. L. Fite (*3*) at General Atomic chose to modulate his beam at a given frequency and to synchronously detect reaction products at this frequency. At the same time, the modulated crossed-beam technique was independently used for other collision studies by L. M. Branscomb (*4*) at the National Bureau of Standards and S. Foner (*5*) at Johns Hopkins University.

Fifteen years ago this approach to the atomic-physics collision experiment was new and unique. However, by that time synchronous detection, which had been developed largely through the study of radar, had become commonplace in nuclear-magnetic-resonance and electron-spin-resonance studies in solids. In fact, in atomic physics, Polykarp Kusch (*6*) had used field modulation in his Nobel-prizewinning studies on the electron magnetic moment—another atomic-physics experiment which had profound effects upon our theoretical understanding of atomic structure. It is estimated that, through the use of modulation and synchronous detection techniques, a gain in signal over background in excess of 10^3 can be obtained, enough to make many otherwise marginal experiments easy and the nearly impossible experiments feasible within a reasonable length of time.

For the experiments on the (e, H) scattering systems described in this article, it was necessary to develop a source of electrons with high enough energy resolution so that the structure associated with compound-state formation and other threshold processes could be detected and analyzed (*7*). Figure 2 shows schematically two electrostatic analyzers, one of which is used to produce a beam of electrons with an energy resolution of approximately 0.05 electron volt. The second analyzer is mounted on a rotating table and can be used to detect electrons which have been scattered either elastically or inelastically as a function of energy and angle.

Also shown schematically in Fig. 2 is a photomultiplier which has been used to detect Lyman-alpha radiation resulting from collisions between electrons and hydrogen atoms (*8*). Between the vacuum ultraviolet-sensitive photomultiplier and the collision region is a gas filter. When oxygen is used as the filtering gas, Lyman-alpha radiation readily passes through one of nine narrow transmission bands which characterize molecular oxygen, while most of the molecular and background radiation is absorbed in the oxygen-gas filter.

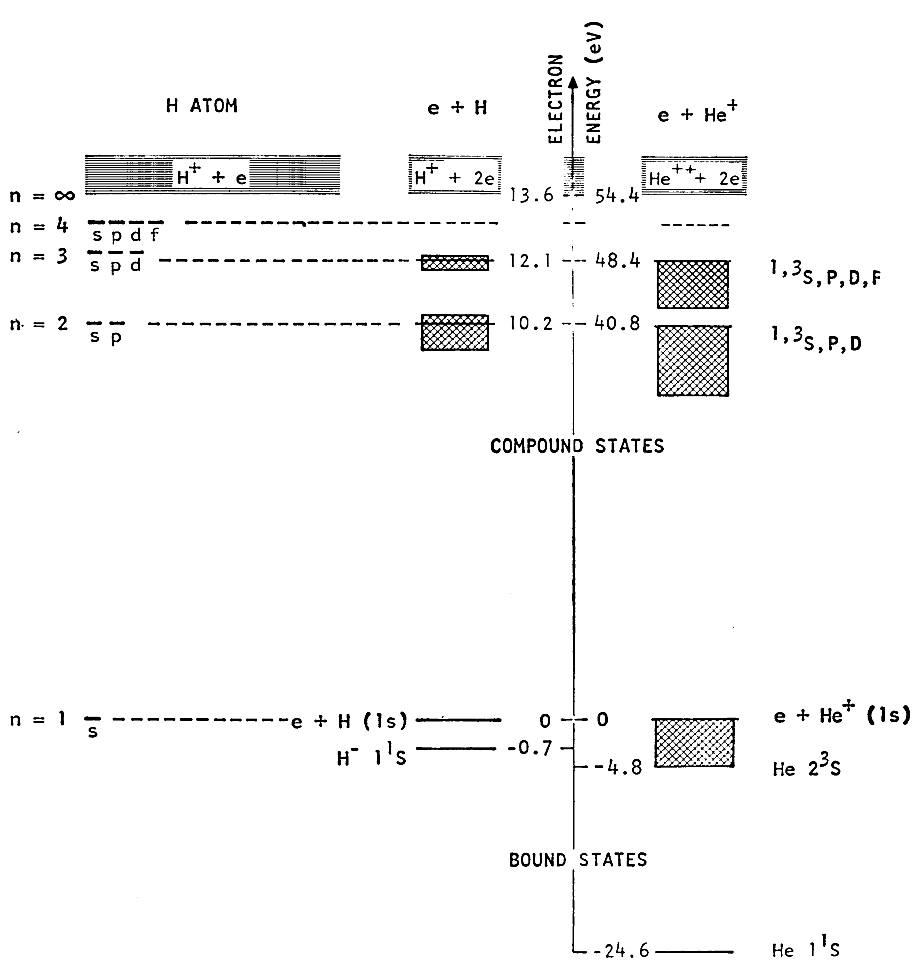


Fig. 1. Energy-level diagram for the hydrogen atom, the hydrogen negative ion H⁻, and the helium atom.

The electron beam crosses the modulated beam of hydrogen atoms, which, in Fig. 2, runs perpendicular to the diagram. Mounted along the axis of the beam of hydrogen atoms are an ion lens and a mass spectrometer which are used to focus and analyze protons formed in collisions between electrons and hydrogen atoms.

In high-resolution studies of such collisions, three parameters must be determined: (i) the magnitude of the cross section under study; (ii) the energy resolutions of the electrons used in collisions; and (iii) the energy of the bombarding electrons. Thus far, no completely satisfactory method of determining an absolute cross section has been found, since the problems associated with measuring the density of the hydrogen atoms in the beam are extensive. Therefore, the experimental cross sections reported in the literature have been related to Born calculations at high electron energies—energies in excess of 200 electron volts, where, according to all indications, the Born approximation describes in detail the total cross section (*9*).

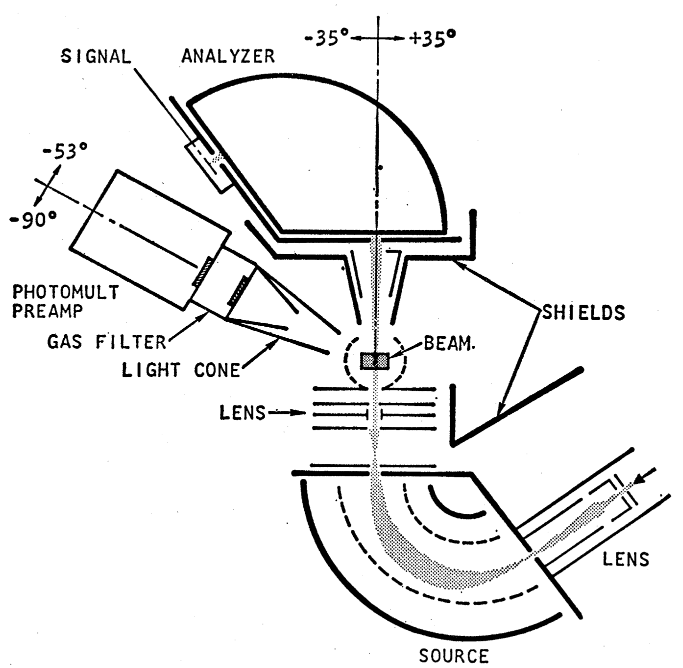
The resolution of the electrons is usually determined through the use of a second analyzer which measures not only the spatial distribution but the energy distribution of the bombarding electrons (*7*). One may also determine the energy resolution by measuring the energy distribution of elastically scattered electrons (*10*).

With the apparatus now available, it is very difficult to determine the energy of electrons with the accuracy needed for these detailed experiments. Consequently, some standard must be used as the energy reference. The standard normally used is the threshold onset of some spectroscopic phenomena, either ionization or excitation. In many of the experiments involving the collision between an electron and a hydrogen atom, the reference commonly used is the ionization threshold for atomic hydrogen—1 Rydberg, or 13.605 electron volts in the laboratory system (*7*). With this energy reference it has been possible to determine the positions of structure in elastic and inelastic scattering channels within ±0.01 electron volt.

Electron, H-Atom Elastic Scattering

Let me now consider in more detail the interaction of the electron with the hydrogen atom in the energy range below the first excitation threshold. At still lower energies, below the onset of the known negative-ion resonance states, the agreement between theory and experiment is reasonably good, although no experimental result is of sufficient accuracy for testing the theory in detail (*11*).

Since the Schrödinger equation describing the three-body system cannot be solved exactly, the equation is expanded and truncated in various ways and then compared with experimental results. The first and simplest expansion is referred to as the Born approximation. This approximation implies a structureless scattering center. Within the limits of the Born approximation there is no hint that the elastic scattering resonances just below $n = 2$ exist. However, when Smith, McEachran, and Fraser (*12*) first considered, through a pair of coupled equations, a more complex scattering center than is implied



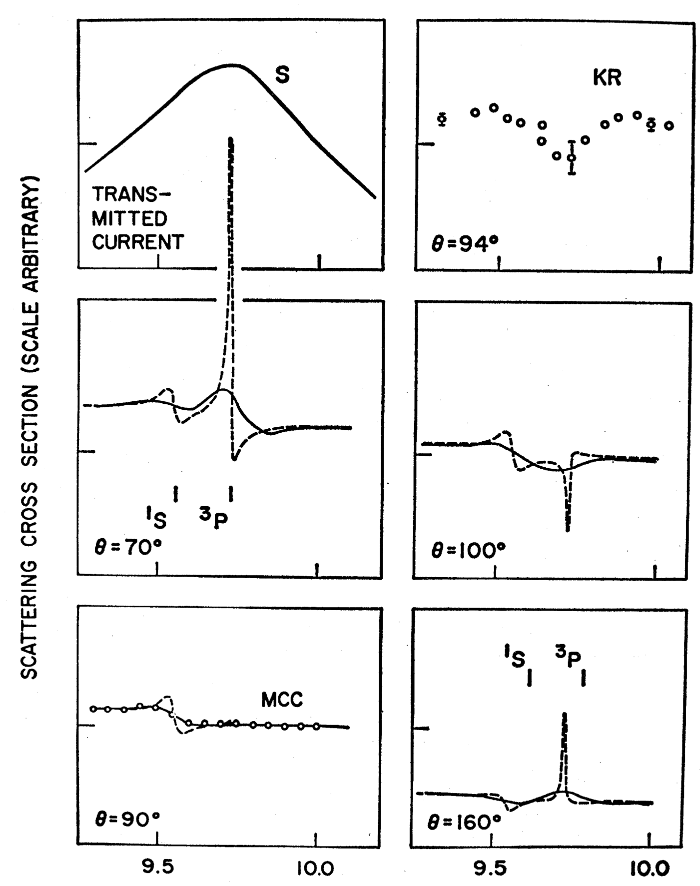


Fig. 2 (above left). Schematic representation of two electrostatic electron energy analyzers, one used as a source of monoenergetic electrons, the other as a detector of scattered electrons. Between them is the atom beam along which are mounted the lenses for the Paul mass filter. Also mounted on a rotating table is a Lyman-alpha detector (see *7*). Fig. 3 (right). The experimental results of Schultz (*S*) (*17*), Kleinpoppen and Raible (*KR*) (*18*), and McGowan, Clarke, and Curley (*MCC*) (*19*) compared with the differential scattering calculations of McGowan (*20*) for different scattering angles θ. The dashed line corresponds to the calculated cross section for an angular window of 15 degrees. The solid line represents the same calculation with the experimental value for electron energy distribution incorporated.

by the Born approximation, they observed resonance structure. However, at that time they did not interpret it as resonance structure. A short time later Burke and Schey (*13*), using the close-coupling scheme, identified the structure as temporary negative ion states.

Following the discovery of these states, a number of theoretical studies involving several other approximate solutions to the exact problem verified the existence of resonance states (*14*). Gailitis and Damburg (*15*) suggested that below $n = 2$ there is not one but a series of resonances for each configuration, due to the abnormally large dipole potential in this case. O'Malley and Geltman (*16*) obtained further verification for this suggestion through a detailed variational calculation for the ^{1,3}S, P channels, using the projection operator technique first developed by Feshback.

Only three experiments which investigate the elastic scattering resonances have been performed. The first was reported by Schulz (*17*). In his experiment, a beam of electrons with a broad energy distribution was fired across a tube partially filled with hydrogen atoms. Shown in Fig. 3 is the current of electrons transmitted through hydrogen-atom gas in the tube. Subsequent to this first measurement, two differential scattering experiments were reported in rapid succession, both having the geometry shown in Fig. 2. In the first of these, reported by Kleinpoppen and Raible (*18*), the angle of observation of the scattered electrons was between 94 and 96 degrees relative to the direction of propagation of the electrons. These results are also displayed in Fig. 3. The other is the experiment reported by McGowan Clarke, and Curley (*19*), wherein the angle of observation was 90 degrees relative to the direction of the primary electron beam. The results of this experiment are also shown in Fig. 3, and shown in greater detail in Fig. 4.

At first sight, the results for the three experiments appear to be markedly different. However, it was later demonstrated (*20*) that they are consistent within the limits imposed by the rapid change in character of the differential electron elastic scattering cross section in the vicinity of the lowest ^{1}S and the ^{3}P scattering resonances. Shown also in Fig. 3 is the calculated differential cross section based upon the Breit-Wigner resonance formula so common in nuclear physics. The solid curve is the calculated cross section obtained with the experimental resolution of the electrons folded in. The broken curve is the same curve before the electron energy resolution is included.

It becomes clear now that the structure observed by Schulz is due to the transmission of electrons through the large peak of the ^{3}P resonance at small angles. The dispersion curve observed by McGowan, Clarke, and Curley reflects primarily the ^{1}S resonance, while the large dip in the cross section at higher energies observed by Kleinpoppen and Raible is associated primarily with the ^{3}P resonance as seen at angles slightly larger than 90 degrees.

Later, Chen (*21*) demonstrated theoretically that the position of the higher member resonances is insensitive to the type of wave function used in regions close to the nucleus. This reflects the fact that the incident electrons are captured in large orbits. Utilizing this observation, Chen was able to show that the series of ^{1}S and ^{3}S resonances have spectral widths and level spacings that decrease exponentially as they approach the threshold of the $n = 2$ level of the hydrogen atom. The width always remains smaller than the spacing, so all resonances of the same series are nonoverlapping. Furthermore, because of the Lamb shift, the small spacing between the $2s$ and $2p$ atomic levels—the theoretically would-be infinite sequences of compound states—become finite.

The approximation now most often used to describe electron scattering resonances is the close-coupling approximation, wherein three or more states of the hydrogen atom are considered in the expansion of the wave function describing the interaction. However, in the usual set of coupled equations, polarization of the excited states and the complete electron-electron interaction are not considered.

Does the addition of more states change the calculated position and widths of resonances, or the magnitude of the cross section? For the S and P resonances below the $n = 2$ level, the addition of more than three states seems to have no or little effect upon position, but some effect upon width and cross section (*22*). Also, in the finite expansion of the scattering wave function the inclusion of terms which account more fully for the interaction of one electron with the other in the field of the atom or ion (that is, the inclusion of electron-electron correlation terms) makes a great difference (*22*). It has been recently demonstrated by Ormonde, McEwen, and McGowan (*23*) that, just below the $n = 2$ level, the position of the ^{1}D resonance which dominates the electron scattering spectrum there (Fig. 4) is strongly dependent upon the number of states of the hydrogen atom that are used to describe the resonance. This implies that, in order to theoretically describe states of higher angular momentum, much more information is needed about the scattering potential between the electron and the hydrogen atom than is contained in just a few states. In effect, this dependence may well be associated with the polarization of the excited target state (*24*).

Recently, the Russian theoretician Faddeev formulated the three-body problem in a more rigorous form than had been previously available. This formulation represents an interesting and exciting approach. The Faddeev equations have been successfully applied by Ball, Chen, and Wong (*25*) to the study of elastic scattering resonances.

Excitation of H Atom to n = 2

Chronology often complicates the description of a scientific finding. However, the study of electron impact excitation of the $2p$ state of the hydrogen atom is a classic case, in which we repeatedly thought the problem was understood, only to find later that our momentary understanding of the problem fell far short of the true picture. The history is outlined schematically in Fig. 5. The main features of the threshold region as it changed are shown in the diagram at left. At right are given the names of the principal researchers and the dates when the work (designated either theoretical or experimental) was reported.

In 1948, Wigner (*26*) predicted that, within the context of a polarization potential, the excitation threshold should be dependent upon the excess energy (the energy above that needed for excitation) raised to the one-half power. Subsequently, Stebbings, Fite, Hummer, and Brackmann (*27*) verified this finding, within the limitations of low-resolution electron impact experiments. Some years later, however, while using the close-coupling scheme to describe the excitation threshold, and recognizing that the dominant potential in the limit of large distances R was a $1/R^2$ potential. Damburg and Gailitis (*28*) predicted that the excitation cross section should be finite at threshold within the limits of the Lamb shift. Shortly thereafter, Chamberlain, Smith, and Heddle (*29*), also in a low-resolution crossed-beam experiment, verified this finding and showed experimentally that, in the threshold region, there may be some further structure.

In 1968, using the three-state close-coupling scheme and allowing for electron-electron interaction (correlation), Burke *et al.* showed (*30*) that the threshold for $2p$ excitation is indeed finite but that, following immediately after the threshold, is an open channel or "shape" resonance which appears in the ^1P channel of the H^- temporary ion. At the same time, Williams and I (*31*) verified this conclusion experimentally and demonstrated that further resonance structure existed in this region of the excitation threshold. Accompanying our report was a theoretical study by Marriott and Rotenberg (*32*) demonstrating that within the ^1P channel there is structure which may be equivalent to that found by Williams and me. However, the exact nature of this structure is not yet completely understood.

A comparison of theory and experiment is shown in Fig. 6. The agreement between the theoretical curve with the experimental energy distribution incorporated and the experimental results is good, and verifies the theoretical predictions of finite cross section near threshold with a large open-channel resonance following immediately after threshold.

As the electron energy is further increased below the $n = 3$ level, more resonances are found, both theoretically and experimentally. The agreement between predicted (*33*) and observed (*34*) positions of the S and P resonances is good. However, as in the case of the elastic scattering channel, the predicted position of the ^1D resonance does not agree with the position found experimentally; the disagreement suggests, again, that more states of the hydrogen atom within the close-coupling scheme must be included in the calculation if the position, and probably the width, of resonances of states of higher angular momentum, such as the D state, are to be adequately described. It is observed in the experimental results (*34*) that resonance structure continues above the $n = 3$ state and almost to the thresholds of excitation for higher states. However, the state of the art today does not allow us to study the H^- scattering resonances in detail, for lack of energy resolution.

Recently, experiments (*35*) and calculations (*36*) have been made on the polarization of the Lyman-alpha radiation emitted from the hydrogen atom as a result of electron bombardment. Within the limits of the resolution of the experiment, the agreement between theory and experiment is good. Furthermore, the calculated cross section for excitation lies within 15 percent of the measured cross section in the case not only of $2p$ excitation (*34*) but of $2s$ excitation (*37*) as well. Furthermore, the predicted shape of the $2s$ excitation cross section agrees quite well, within the limitations of the attainable resolution, with the shape found experimentally.

Electron, H-Atom Ionization

The most difficult problem associated with three-body systems, from the point of view both of experiment and of theory, is the study of ionization. The

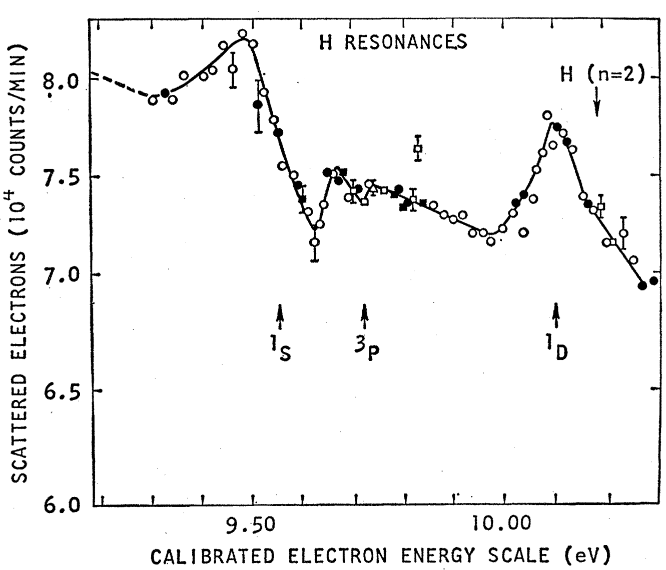


Fig. 4 (above). The differential scattering results observed at an angle of 90 degrees with respect to the direction of the bombarding electrons, showing the lowest three resonances, ^1S, ^3P, and ^1D (see *19*).

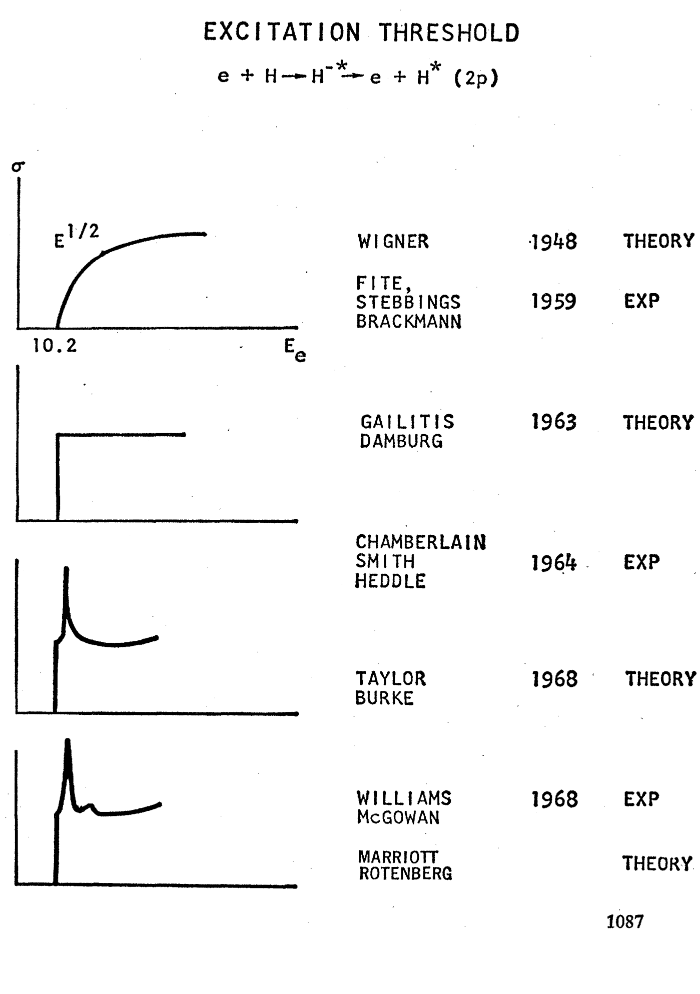


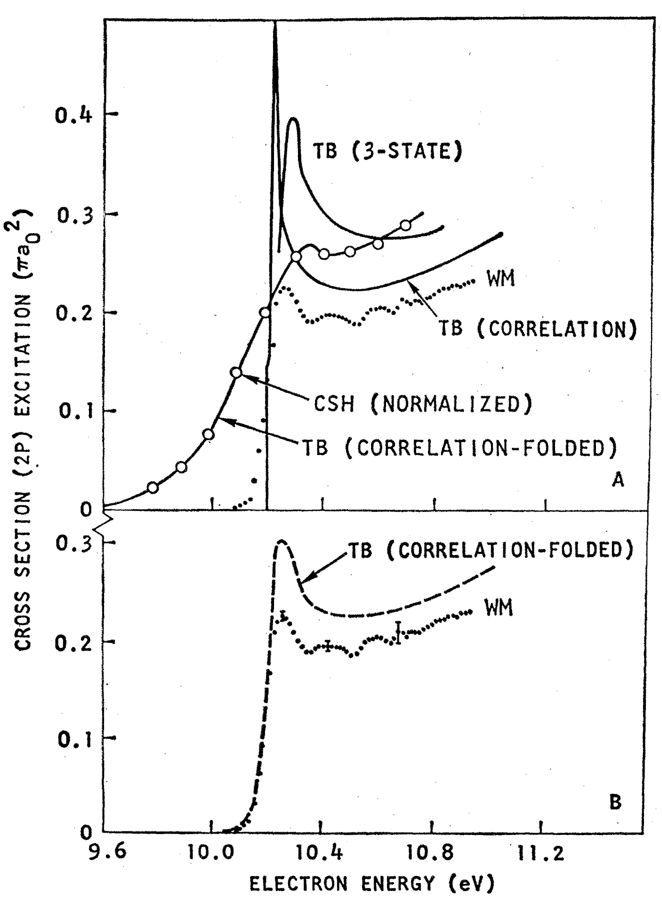
Fig. 5 (right). Schematic representation of progress that has has been made in the study of the $2p$ excitation threshold $[e + H \rightarrow H^{-*} \rightarrow e + H^* \ (2p)]$, showing what has been predicted and what has been found experimentally, the authors, and the year of discovery.

problem is particularly difficult in the threshold region, where the three charged particles are just beginning to break up. Within the context of the Born approximation, if one assumes the artificial and unrealistic case where both electrons leave the proton as plane waves, it follows that the cross section is dependent on the square of the electron energy E_e in excess of the ionization potential IP—that is, $(E_e - IP)^2$. Also, within the context of the Born approximation, the next possibility is that one electron completely shields the other electron from the proton and that, consequently, the one electron leaves as the plane wave and the other electron leaves as a Coulomb wave. Within this scheme, then, one derives a law that the threshold depends upon the excess energy to the 1.5 power (*38*). This is considered by most people to be the upper limit for the power-law dependence of the cross section.

In 1953, Wannier (*39*), using classical phase-space arguments, arrived at an energy dependence for the electron-impact-ionization cross section near threshold. The electron energy dependence derived by him is $(E_e - IP)^{1.127}$; for a number of years, experimental verification was sought but never achieved. (A summary of the history is given in Fig. 7.) Several years later, Geltman (*40*) demonstrated that, within the context of the Born approximation, if one considers both of the electrons that leave near threshold to be *s*-wave electrons, then the cross section would have a linear dependence upon the excess electron energy.

Two experiments followed which gave information on the threshold region for ionization of atomic hydrogen, one by Fite and Brackmann (*3*) and the other by Boksenburg (*41*). Both were crossed-beam experiments involving electrons and atomic hydrogen. In both cases the energy distribution of the electrons used was poor; within the limitations of the experiments it was concluded that, over an interval of approximately 3 electron volts (Fig. 8A), the cross section for ionization near threshold was indeed a linear function of the excess energy. Subsequent to publication of these experimental results, Peterkop (*42*) and Rudge and Seaton (*43*) published papers defining a linear threshold law.

Further discussion of the upper limit for the dependence of the ionization threshold on the excess energy was initiated by Omidvar (*38*), within the context of the Born approximation; this discussion was followed by a demonstration on the part of Temkin (*44*) that the asymptotic form of the wave function used by Rudge and Seaton (*43*) was not necessarily correct. About this time, Clarke and I (*7*) demonstrated experimentally that, for the case of electron ionization of hydrogen within the first 0.4 of an electron volt just above threshold, the ionization cross section is a nonlinear function of the excess electron energy and, to a first approximation, the power-law dependence is $(E_e - IP)^{1.13 \pm 0.03}$ (see Fig. 8B). It is interesting to note that this experimental finding agrees with the early classical predictions of Wannier and with the more recent extension of the work by Vinkalns and Gailitis (*45*),



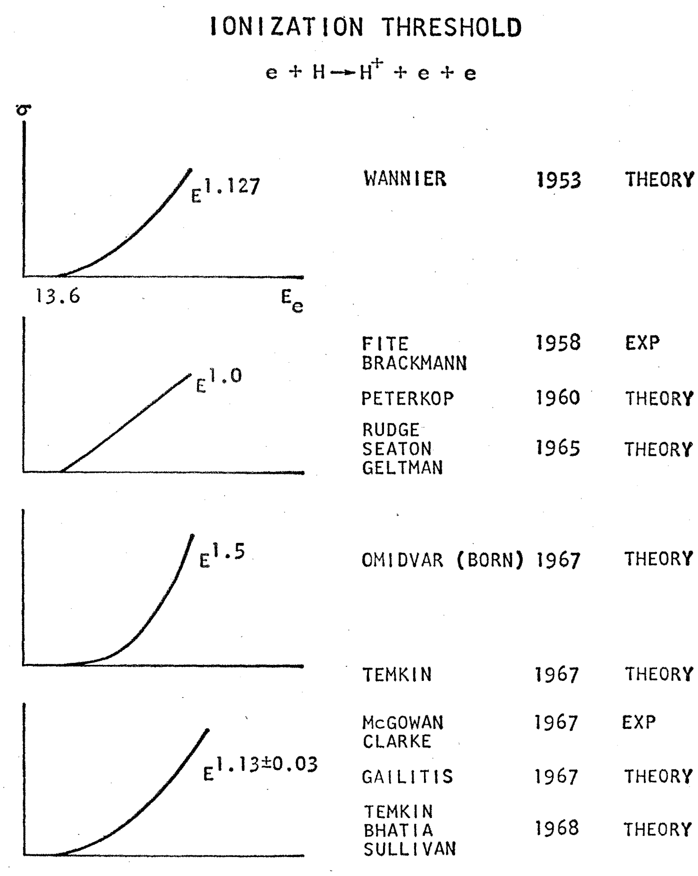


Fig. 6 (left). A comparison of the two experimental results of Chamberlain *et al.* (*CSH*) normalized with respect to theory and the absolute measurements of Williams and McGowan (*WM*) with two different theoretical results of Taylor and Burke (*TB*) (*22*). The theoretical curve is shown with and without the experimental value for electron energy distribution incorporated, respectively: *TB* (*correlation*) and *TB* (*correlation-folded*). Fig. 7 (right). Schematic representation of progress that has been made in the study of hydrogen atom ionization, showing what has been predicted and what has been found experimentally. Ionization threshold, $e + H \rightarrow H^+ + e + e$.

but it does not necessarily support all assumptions given in that theory.

Temkin, Bhatia, and Sullivan (*46*) have gained further insight into the threshold region of electron-atom ionization by examining the nature of doubly excited negative ion states below this threshold. To me, an experimentalist, this approach is esthetically pleasing, since I am greatly impressed by the effect resonances have upon the excitation thresholds. It appears to be implicit in the calculation that the autodetachment which occurs below the beginning of the ionization continuum can be carried over into autoionization above the onset of the continuum. Under these circumstances, the cross section is found to depend on the excess energy as $(E_e - IP)^{(3-\gamma)/2}$, where gamma lies between 0 and 0.5, a range which defines a power-law dependence between 1.5 and 1.25. Unfortunately, available electron energy resolution does not now permit us to study the region within 0.01 electron volt of threshold, where a power-law dependence higher than 1.13 may be applicable (*7*).

It is clear that, before we can completely understand the ionization phenomena, more sophisticated experiments and theoretical calculations will have to be carried out. The experiments will have to include angular and energy correlation measurements for the new electrons released near the ionization threshold (*47*), while the theory will have to describe more realistically the breakup of the temporary H^-.

The Helium Atom

The situation in the (e, He^+) system is similar in many ways to that in the (e, H) case. Consider an electron brought into the strong Coulomb field of the He^+ ion. We know from spectroscopy that the electron can be bound to the He^+ ion with energy of 24.6 electron volts, and that, as in the case of all strong Coulomb fields, there can be many bound levels of the helium atom between its ground state and its ionization limit.

Above this limit, however, the level structure is similar to that of H^-: there are a number of series of resonances or auto-ionizing levels in the (e, He^+) system, just as there are autodetaching levels in the (e, H) system. However, when we represent the helium system at the scale of the H^- system, as has been done in Fig. 1, we find that, proportionately, the resonance structure below the $n=2$ of He^+ occupies a much larger energy interval than that below the $n=2$ of the hydrogen atom. Furthermore, no open-channel potential resonances have thus far been found immediately above the $n=2$ inelastic threshold, such as were found in the case of H^-.

Each of these effects is no doubt a result of the stronger Coulomb force exerted on the electron by the alpha particle than by the proton. Finally, in our consideration of the (e, He^+) system, the electron energy can be increased to the point where the second electron is removed from the helium nucleus. Only one experiment has been performed on this system; in that, the electron energy resolution and the experimental uncertainties were large enough to disguise the threshold results.

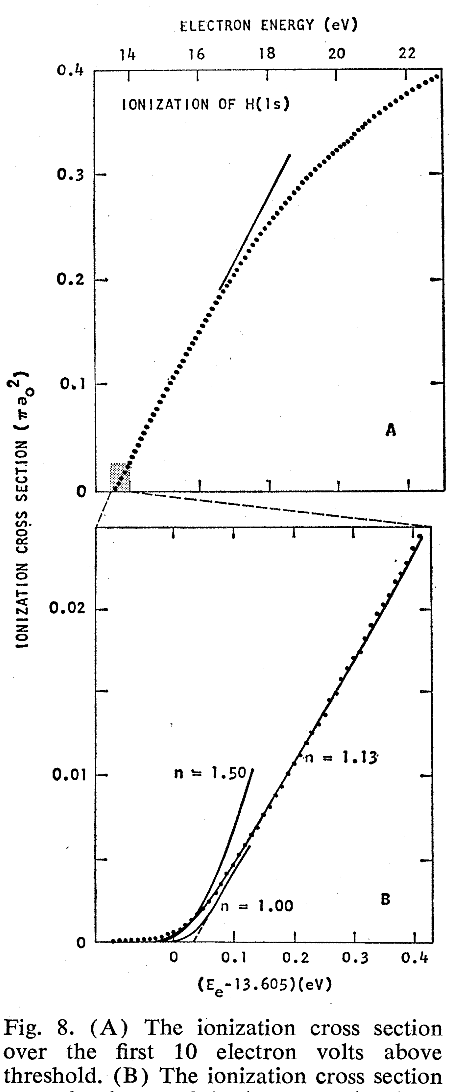


Fig. 8. (A) The ionization cross section over the first 10 electron volts above threshold. (B) The ionization cross section over the lowest 0.4 electron volt above threshold, calculated with a power-law dependence of 1.13, compared with cross sections calculated with two other power-law dependences (*7*).

Electron, Helium Ion Scattering

The application of high-resolution electron impact techniques to a study of resonances of helium is greatly simplified by the fact that helium does not have to be handled in a low-density atom beam but, rather, can be kept in a pot through which the electrons or photons are fired. The density of the atoms within this pot of gas can be higher by 2 to 3 orders of magnitude than the density of the atoms in the hydrogen atom beam, thus the signal is greatly increased and all aspects of the experiment are simplified. High-resolution electron impact studies have been made, by Silverman and Lassettre and by Simpson *et al.* (*48*), of the autoionizing states which appear just below the $n=2$ level of He^+ (see Fig. 1). These same resonances have been excited by ion impact, as reported by Rudd (*49*). However, the (e, He) and (ion, He) problems are problems of even greater complexity than the three-body case and consequently are not considered here.

The most powerful tool thus far used to study any atomic scattering resonance structure is the National Bureau of Standards' 180-million-electron-volt electron synchrotron. The synchrotron is used as a continuum light source for absorption spectroscopy in the region 180 to 470 angstroms, which corresponds to electron energies of 69 to 26.4 electron volts. The radial acceleration of the electrons in this machine gives rise to radiation commonly called "synchrotron light," which is relativistically restricted to a small angular zone in the plane of the electron orbit, the radiation being observed in the near-forward direction of the orbiting electrons. Some details of the apparatus are shown in Fig. 9.

With this experimental configuration, as many as nine members of the ^1P resonances have been observed (*50*), whereas, with electron impact techniques, because of low resolution only one member—the first and broadest—of the series is usually observable. The use of photoabsorption further simplifies the problem in that only ^1P resonances can be excited if, as required for photoexcitation, the angular momentum changes by 1 and the spin remains fixed. In Fig. 10 are shown the absorption resonances just below the $n=2$ level of He^+. Not seen in Fig. 10 is the second series of weaker resonances which were first predicted by Cooper, Fano, and Prats (*51*) and later observed by Madden and Codling (*52*). The agreement between the position and width of these resonances as measured and as calculated within the three-state close-coupling scheme (including electron-electron correlation) is very good (*53*) and gives further confidence in the close-coupling scheme. Other descriptions of these resonances have also been supported by experimental results (*54*).

As I pointed out above, the spacing between the resonance states of H^- within a particular series is an exponential function. In the case of helium, because of the stronger Coulomb force exerted on the two electrons by the alpha-particle core, the spacing between the auto-ionizing levels is hydrogenic (*55*), so their spacing depends upon the inverse square of the principal quantum number of the resonance states within each resonance series.

Excitation of Helium Positive Ion

Our understanding of the resonances above the $n = 2$ level of He^+ is not as complete as our understanding of those below that level. There have been few calculations, and thus far only two experiments to give us any information on the nature of the excitation threshold. The first experiment reported was that by Dance, Harrison, and Smith (*56*), wherein these investigators crossed a beam of He^+ ions with a low-resolution beam of electrons, excited the He^+ ions to the 2*s* metastable state, and allowed the ions in the metastable state to be transported to another low-noise region in the instrument, where they were caused to radiate. The photon of approximately 41 electron volts was then detected by an x-ray counter. In these results (Fig. 11), no resonance structure was detected; however, a broad peak at approximately 47 electron volts can now be correlated with known resonance structure that had been predicted by Ormonde *et al.* (*57*) below the $n = 3$ level of He^+. A similar peak is seen for the excitation of H atoms to the 2*s* state (*37*).

In the second experiment, Daly and Powell (*58*) used an approach quite different from any yet described. In their experiment, He^+ ions were formed by electron impact and were trapped for a considerable length of time in the space-charge region of a magnetically confined electron beam. While these He^+ ions were oscillating back and forth in a space-charge well around the electron beam (*59*) they were struck a second time, the He^+ ions being thus excited to the 40.8-electron-volt metastable state. The metastable ions were again retained in the source, long enough for a third electron to ionize the metastable ions. A high-resolution mass spectrometer was used to separate the He^{++} ions formed from the residual H_2^+ found in the background gas in the mass spectrometer. Even though one could resolve H_2^+ from He^{++}, enough hydrogen molecular ions were scattered to the He^{++} position to completely obscure the signal. However, by a clever arrangement of retarding grids and the use of a secondary emission detector, Daly and Powell were able to determine unequivocally the energy profile for the excitation cross section of He^+ to the 2*s* level.

The results are shown in Fig. 11, along with the early predictions of Ormonde, Whittaker, and Lipsky (*57*). In view of the difficulty of the experiment and the fact that the theoretical predictions were at the time only tentative, the agreement is remarkable. The sharp rise observed experimentally in the excitation cross section at the threshold is qualitatively in agreement with what is expected for the excitation of ions under electron impact. However, because of the complexity of

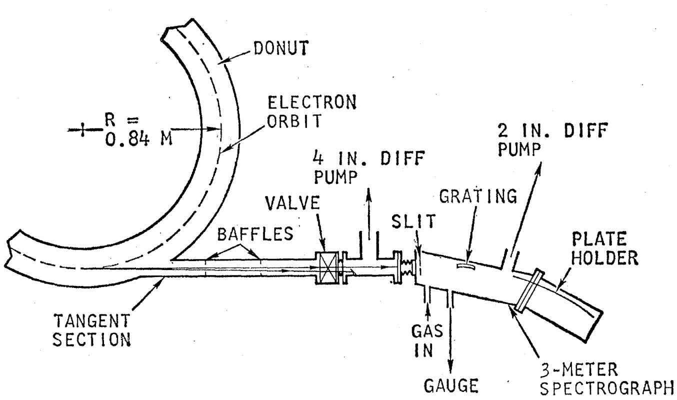
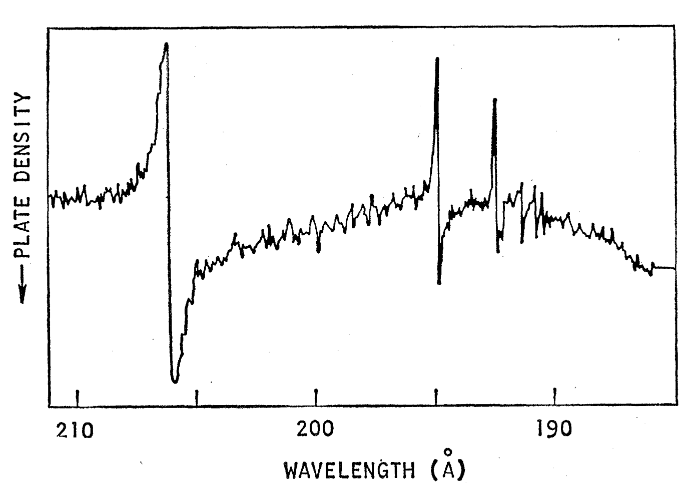
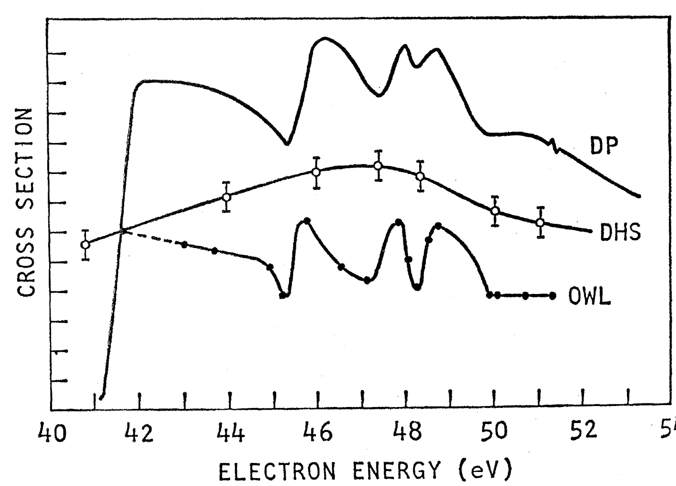


Fig. 9 (left). Schematic diagram of the synchrotron and other apparatus used to study photoabsorption in helium (*52*).

Fig. 10 (left below). The photoabsorption of helium as measured by Madden and Codling (*52*).

Fig. 11 (right below). The 2*s* excitation function for He^+ as measured experimentally by Daly and Powell (*PD*) (*58*) and by Dance, Harrison, and Smith (*DHS*) (*56*) compared with the theoretical predictions of Ormonde, Whittaker, and Lipsky (*OWL*) (*57*).





this experiment, details such as the exact nature of the threshold and of the resonances are subject to some question.

As of today, there is no indication of an "open channel" or "shape" resonance near the excitation threshold, as there was in the case of electron-hydrogen atom scattering. Again, this may follow from the fact that the Coulomb interaction with a doubly charged nucleus is the dominant force, which moves the resonances to an energy below the excitation threshold, thus transferring it to the closed channel.

Electron, Helium Ionizing Collisions

If we were to look at the case for helium that is analogous to the electron impact ionization of the hydrogen atom, we would have to discuss electron impact ionization of He+, but the ionization of He+ would be dominated by the Coulomb interaction. Unfortunately, this experiment has not yet been carried out with sufficient detail to shed light on the threshold process (*60*). Neither has the photoabsorption experiment for helium been carried out, in which the helium atom absorbs one or more photons, thus releasing two electrons at threshold.

However, several experiments on the electron impact ionization of the helium atom have recently been reported (*61*). As in the case of electron impact ionization of the hydrogen atom, it is found that, near the ionization threshold, the cross section is a nonlinear function of the excess electron energy. One surprising feature is the observation that the nonlinearity extends over an interval greater than 40 electron volts, whereas in the case of electron impact ionization of atomic hydrogen, nonlinearity was observed only over an interval of approximately 0.4 electron volt. Once again, the power-law dependence is 1.13, or slightly higher, at threshold (*61*). It would seem that the marked change in polarization between the hydrogen atom and the helium atom has not greatly affected the threshold behavior, although it has affected the cross section over the extended range of electron energies.

Concluding Comment

It is still impossible to solve the three-body problem exactly; however, it can be described well in various approximations. The (*e*, H) and the (*e*, He+) atomic systems in particular have received extensive theoretical and experimental attention. As a result, it is found that the close-coupling and the Faddeev approximations describe most of the details of the elastic and inelastic scattering resonances and inelastic scattering thresholds that have thus far been experimentally examined. However, we still do not understand electron impact excitation of high-lying states of H and He+, and our understanding of the three-body breakup problem (ionization) must still be classed as rudimentary.

The next generation of experiments necessary to further develop our understanding of these systems will be more difficult by an order of magnitude. Included in any list must be experiments which measure the absolute cross section for elastic and inelastic scattering with an accuracy of 1 or 2 percent. The scattering of polarized electrons from polarized H atoms will make it possible to separate singlet from triplet contributions to the cross section. Coincidence experiments will be necessary to study upper states of excitation of H and He+ under electron impact. Electron exchange will have to be deeply investigated. The replacing of one electron with a positron will make it possible for us to eliminate the effect of electron exchange in scattering experiments but will add the complication of positronium formation (an e^+, e^- hydrogen-like atom). Angular and energy correlation experiments will help us untangle the difficult problem of ionization. Studies of photon impact at higher resolution will give us the spectral resolution which will eventually allow us to test theoretical approximations with an accuracy approaching that of Lamb's experiment. Single and double photon processes will be investigated.

The climb to the first major plateau in our understanding of the three-body atomic problem has been very exciting, but the climb yet to come promises to be even more thrilling.

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Moon Illusion Explained on the Basis of Relative Size

The moon looks small overhead not because it seems close but because of the broad extent to the horizon.

Frank Restle

The most remarkable natural illusion is the size of the moon. The moon appears larger at horizon, approximately 1.2 to 1.5 times the apparent diameter at zenith. Since the visual angle of the moon is always 0.5°, there is no physical basis for the illusion; it is therefore a perceptual phenomenon.

Of the known illusions, the moon illusion is notably large and reliable. The only artificial illusions of comparable magnitude involve repeated use of the same simple inducing principle or depend heavily on perspective drawing. The moon illusion, however, changes the size of a reasonably simple white circle (there is nothing to indicate that the shadows on the face of the moon affect the illusion) in a plain sky. The usual lines and swirls used to generate artificial illusions are all absent in the case of the moon illusion, which nevertheless is among the greatest in magnitude.

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One unusual characteristic of the moon illusion, as observed in nature, is that the two "apparent sizes" must be viewed at different times, because the moon takes several hours in transit from horizon to zenith. It is possible, however, by use of mirrors or other artificial devices, to obtain more conventional psychophysical comparisons; and the magnitude of the illusion is measured at about 1.3 to 1. The magnitude of the illusion does not depend on the fact that measurement is not ordinarily by direct comparison.

A second unusual characteristic of the moon illusion is that one of the objects viewed (the zenith moon) is overhead and hence is viewed with the eyes turned upward, with the head turned upward, or both. The hypothesis that the moon illusion depends on these factors was given some support by Boring and Holway, but in their psychophysical method the subject matched the apparent size of the moon with the apparent size of a disk that was visible only a few feet away and well enough illuminated to have a visible texture.

Rock and Kaufman (*1, 2*) reported a series of studies that effectively contradicted the Boring and Holway experiments. They established that a moon looks large near the horizon (wherever the horizon is, even if displaced overhead) and looks small when it is far from the horizon and in empty space (even if that space is straight ahead).

Apparent Distance Hypothesis

How do Rock and Kaufman propose to explain the moon illusion? They note that common observation and some experimental studies indicate that the sky appears to have the shape not of a hemisphere but of a flattened soup bowl, so that the horizon seems farther away from an observer than does the sky overhead. How this apparent distance would produce the moon illusion is shown in Fig. 1.

The visual angle subtended by the moon is fixed at approximately 0.5°. To the observer, the moon seems to be on the surface of the sky, and it appears more distant near the horizon than at zenith. When two actual objects subtend identical visual angles but are at different distances from the observer, the more distant object can be calculated to be the larger object. The observer performs this calculation and deduces that the horizon moon must be the larger object; hence, it appears larger.

This apparent distance hypothesis suffers from a number of difficulties. The first objection is that no calcula-