Reports

Martian Mass and Earth-Moon Mass Ratio from Coherent S-Band Tracking of Mariners 6 and 7

Abstract. *Range and Doppler tracking data from Mariners 6 and 7 have been used to obtain values for the ratio of the mass of the earth to that of the moon which are in substantial agreement with those determined from other Mariner and Pioneer spacecraft. There is an inconsistency of about 0.004 percent in values for the mass of the moon determined from lunar trajectories. A gravitational constant for Mars of 42,828.48 ± 1.38 cubic kilometers per second per second, obtained on the basis of data collected during the 5 days prior to the closest approach of Mariner 6 to Mars, is in excellent agreement with the result obtained by Null from tracking data of Mariner 4.*

The accurate navigation of Mariners 6 and 7 to the vicinity of Mars required the use of two-way, phase-coherent range and Doppler tracking data from the National Aeronautics and Space Administration/Jet Propulsion Laboratory (NASA/JPL) Deep Space Network. Because of the importance of these data in celestial mechanics, JPL scheduled tracking coverage of both Mariners 6 and 7 from launch to encounter with Mars, the closest approach to Mars, and beyond in such a way that new information could be obtained on the ratio of the mass of the earth to that of the moon, the gravity field of Mars (in particular, the mass of the planet), and the ephemerides of Mars and the earth. In addition, it was recognized that these data complemented Doppler data from Mariner 4; however, range measurements were not obtained from that spacecraft. Furthermore, a combination of data from the three flyby trajectories and direct radar range measurements to the planet itself during the 1969 opposition would not only result in significant improvements in values for the ephemeris of Mars, but, because of these improvements, would also permit an exploration of the size, shape, and gross topography of Mars by utilization of the radar range measurements to its surface. In this regard, it was realized that the times of immersion and emersion of the Mariner spacecraft as they were occulted by Mars would provide important calibration points for the size of the planet (*1*).

Tracking data from Mariners 6 and 7 and the radar range measurements to Mars are still being collected; meaningful analysis will require many months. Therefore, this report deals with the use of the tracking data alone to determine the earth-moon mass ratio and the mass of Mars.

As the earth revolves about the center of mass of the earth-moon system at a distance of about 4671 km and at a speed of 12.4 m/sec, it impresses a sinusoidal curve on range and Doppler tracking data with a frequency equal to the sidereal mean motion of the moon and with an amplitude inversely proportional to $(1 + \mu^{-1})$, where μ^{-1}, the ratio of the mass of the earth to that of the moon, is approximately equal to 81.3. The principle involved in the determination of μ^{-1} is that there is a value which eliminates the monthly cycle in the tracking data residuals in a least-squares sense. No other unknown parameter in the representation of the tracking data has a frequency anywhere near that of the moon's orbit; the determination of μ^{-1} is direct and reliable. The lunar ephemeris is known to an accuracy of seven or eight figures, and the use of this value does not introduce any noticeable error into μ^{-1}, which, by comparison, can be determined to the order of 10 to 20 parts in 10^6.

In our search for possible sources of systematic error in the data, it has been very difficult to think of anything important that has a monthly cycle, although a significant S-band propagation effect correlated with the rotation of the sun would be close enough to cause difficulty. However, periodic variations in the interplanetary medium, the only reasonable possibility, if present, must be very small for Mariners 6 and 7; the total delay for an inverse square distribution of electrons with a density of 6 per cubic centimeter at the earth's distance is at most only 4 m for the Mariner trajectories from the earth to Mars. Melbourne (*2*) has suggested that a 28-day sinusoidal variation of solar flux of 0.1 percent with the appropriate phase could produce an error of about 0.001 in μ^{-1} because of a similar variation in the solar radiation pressure on the spacecraft. The agreement of the several interplanetary spacecraft, however, does not seem to indicate that this sort of systematic error is present unless the phase of the flux variation is the same for each mission, which does not seem likely.

There is an instrument on both Mariners 6 and 7 which measures relative variations in solar flux to an accuracy of better than 0.1 percent. When these data are reduced, it should be possible to determine whether a variation is present that could significantly bias the solutions for μ^{-1}.

Values for the earth-moon mass ratio as determined from Mariner 6 Doppler

Table 1. Determinations of the ratio of the mass of the earth to that of the moon, μ^{-1}, from Mariner, Pioneer, and Ranger tracking data, and from radar bounce data to the planets. [Deviation $(\mu^{-1} - \bar{\mu}^{-1})$ from the arithmetic mean $\bar{\mu}^{-1}$ of the Mariner and Pioneer values is tabulated for each determination. The root-mean-square deviation from a mean of 81.3008 is 0.0008 for the Mariner and Pioneer determinations.]

Spacecraft	μ^{-1}	$\mu^{-1} - \bar{\mu}^{-1}$
Mariner 2 (Venus)	81.3001 ± 0.0013	−0.0007
Mariner 4 (Mars)	81.3015 ± 0.0017	0.0007
Mariner 5 (Venus)	81.3006 ± 0.0008	−0.0002
Mariner 6 (Mars)	81.3011 ± 0.0015	0.0003
Mariner 7 (Mars)	81.2997 ± 0.0015	−0.0011
Pioneer 6	81.3005 ± 0.0007	−0.0003
Pioneer 7	81.3021 ± 0.0004	0.0013
Combined Rangers	81.3035 ± 0.0012	0.0027
	Radar data	
	81.302 ± 0.002	0.0012

Table 2. Statistical properties of the fit to preencounter Doppler data. (Units are hertz at S-band frequency which can be converted approximately to millimeters per second if the values are multiplied by 67.)

Tracking station	Number of points	Data interval (UTC)	Mean residual (hz)	Root-mean-square residual (hz)
Australia	200	7/26, 06:49 to 7/30, 15:41	0.000006	0.0028
South Africa	26	7/27, 16:28 to 7/27, 23:02	0.000164	0.0019
Spain	80	7/26, 17:29 to 7/30, 22:31	−0.000086	0.0025
California	311	7/26, 01:23 to 7/31, 04:40	−0.000105	0.0043
California	53	7/26, 00:35 to 7/30, 06:38	−0.000098	0.0024

data measured over a period of 12 weeks (4 May to 28 July 1969) and Mariner 7 Doppler data measured over a period of 11 weeks (8 May to 22 July 1969) are shown in Table 1 along with results from Mariner 2 (*3*), Mariner 4 (*4*), Mariner 5 (*5*), Pioneers 6 and 7 (*6*), and reductions of passive radar reflection measurements to the planets (*7*).

A solution obtained by a combination of data from Rangers 6, 7, 8, and 9 (*8*) is also shown, although the selenocentric gravitational constant *GM* for the moon was determined directly by the Ranger impact trajectories; μ^{-1} must be computed with an assumed value for the geocentric gravitational constant *GE* ($\mu^{-1} = GE/GM$). The error in *GE*, however, is on the order of 1 part in 10^6. Therefore, the percentage error in the value of μ^{-1} determined from Ranger measurements is practically equal to the percentage error in *GM*. The fundamentally different method for the determination of μ^{-1} from Ranger impact trajectories, as opposed to the use of interplanetary trajectories such as those from the Mariner and Pioneer flights, is reflected in a significant difference in the values. The arithmetic mean of the seven Mariner and Pioneer values of μ^{-1} is 81.3008 with a deviation from the mean of 0.0008. On the other hand, the Ranger value differs by 0.0027 from this mean value.

We can see no reason why one should suspect that the direct determination of μ^{-1} from measurements of the Mariner and Pioneer spacecraft is subject to systematic errors of a size that would adjust the value to that given by Ranger. We have even processed the Mariner data with two computer programs, one in a single precision code (about eight figures) and the other in double precision (about 16 figures) and with both heliocentric and geocentric formulations of the equations of condition in the method of weighted least squares. The value of μ^{-1} is essentially the same in all cases. In our opinion, therefore, it is necessary to perform new reductions of data from the lunar spacecraft.

Several conditions have changed since the Ranger solution for μ^{-1} was obtained about 2½ years ago. First of all, values for the gravity field and the ephemeris of the moon have been improved significantly by analyses of Lunar Orbiter data (*9*). In addition, it is now possible to perform the necessary computations in double precision, whereas those of 2½ years ago were processed in single precision. In any case, the question of reconciling the lunar and interplanetary values of μ^{-1} is receiving increased attention because of the results of Mariners 6 and 7 which support a value of μ^{-1} closer to 81.300 or 81.301 than to 81.303 or 81.304 as suggested by the combined Ranger flights.

The determination of the mass of Mars based on measurements of both Mariners 6 and 7 is complicated by nongravitational forces that act on the spacecraft. In particular, one channel of the infrared spectrometer (IRS) operates in a cryogenic environment produced by the expulsion of hydrogen and nitrogen gas from a pressure-regulated system. This expulsion of gases imparts a force of 100 dynes or more to the spacecraft and produces a velocity change in its trajectory on the order of 0.1 m/sec. In normal operation, the system starts to expel gases about 35 minutes before encounter and continues at a constant pressure through encounter for a period of about 80 minutes. After this, the gas is allowed to escape into space without pressure regulation; about 5 hours are required before the decay reaches an insignificant level at approximately an exponential rate. On Mariner 6, the system did not operate normally, and, as a result, the force on the spacecraft acted over a period of 4 or 5 days after encounter instead of over a few hours as intended. The system operated normally on Mariner 7. This spacecraft, however, was affected 6 days before encounter by an unknown "event" which imparted a velocity increment to the trajectory of 1 m/sec or less, or a change of less than 4×10^7 g cm/sec in momentum.

Table 3. Range residuals from the fit to preencounter range and Doppler data. (Units are 10^{-9} second which can be converted to residuals in meters for the topocentric distance to the spacecraft if the values are multiplied by 0.15. The mean residual is −0.30, and the root-mean-square value of the nine residuals is 26.9, or 4.0 m.)

Time of reception 27 July 1969 (UTC)	Residual
01:12:02	−42.1
01:59:02	62.7
02:29:02	5.9
02:59:02	6.2
03:29:02	2.9
03:59:02	−16.2
04:29:02	5.5
06:06:02	−8.2
06:36:02	−19.5

Telemetry records indicate that the spacecraft attitude was disrupted by the event, whatever the source. The nature of the unknown forces acting is not yet clear. One possibility is that a meteorite struck the spacecraft. Another possibility is that the on-board battery (which gave indication of malfunction after the time of the event) was punctured, either by the meteorite impact or as a result of an internal pressure buildup, and that the leaking electrolyte imparted a low-thrust force for a day or so. Because of uncertainties over the nature of the event and the resulting forces on the spacecraft in the few days prior to the encounter, we have not been able to obtain any results for Mariner 7 at this time.

The data recorded prior to the initiation of the blowdown of the IRS cryogenic system on Mariner 6 are not influenced by unknown forces and can be used to obtain a value for the mass of Mars. However, data from about 35 minutes prior to the encounter and beyond cannot be used until, in a fashion similar to that for Mariner 7, engineering telemetry data can be combined with the tracking data in an analysis of the nature of the nongravitational forces acting on the spacecraft.

To obtain the mass of Mars, we fitted Doppler data (670 points) from 26 July, 5 days before encounter, to 31 July, 04:39:57 (Universal Time Coordinated), a time approximately 7 minutes before the start of the IRS cooling.

The parameters in the least-squares solution were the position and velocity of the spacecraft at the epoch, the gravitational constant $GM_♂$ for Mars, the distance off the earth's axis of rotation, and the longitude of each of the five NASA/JPL Deep Space Network tracking stations for which data were available. The statistical properties of the fit, which were good, are summarized in Table 2. The solution for $GM_♂$ is $42,828.22 \pm 1.83$ km^3/sec^2. A second solution was performed in which the Doppler data were fitted along with nine range measurements taken on 27 July from the NASA/JPL Deep Space Network (Mars tracking facility) at Goldstone, California, with the experimental ranging system used for Mariner 5. The statistical properties of the Doppler fit were not changed appreciably by the addition of the range data. The nine range residuals are shown in Table 3. The value of $GM_♂$ for the range and Doppler fit is $42,828.48 \pm 1.38$ km^3/sec^2, which is not significantly different, either in size or in estimated standard error, from the value based on Doppler measurements alone. This estimate was computed with a standard error of 62×10^{-9} second on each range point and a standard error of 0.05 hz on Doppler data sampled at 1-minute intervals.

Although the introduction of range data into the fit does not appreciably affect the solution for the mass of Mars, it is of value in the determination of the orbit of the Mariner spacecraft. This has important implications for other Mariner experiments which will require good orbital data for final analysis. Precise knowledge of the orbit is important in relation to information on the ephemeris of Mars and will be significant in later analyses of the tracking data when the nongravitational forces on Mariners 6 and 7 are better understood. At present, we believe that the areocentric orbit for Mariner 6 from the fit to range and Doppler data can be used to predict events along the trajectory to better than 1 second in time or to better than 8 km along the flight path. For example, our best estimate of the time of closest approach to Mars is 31 July 1969, 05:19:06.4 (± 1 second) (UTC).

In addition to Mariners 6 and 7, the only other source for an accurate determination of the mass of Mars is Mariner 4. A recent analysis (*10*) of Doppler data, taken over a 10-day interval during which the spacecraft was centered about a closest approach, has yielded a value for $GM_♂$ of $42,828.32 \pm 0.13$ km^3/sec^2. Therefore, the masses determined from Mariners 4 and 6 are in agreement, and there is good reason to use the value determined from spacecraft measurements in calculations with other planetary data.

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Silicon: A Possible Factor in Bone Calcification

Abstract. *Silicon, a relatively unknown trace element in nutritional research, has been uniquely localized in active calcification sites in young bone. Silicon increases directly with calcium at relatively low calcium concentrations and falls below the detection limit at compositions approaching hydroxyapatite. It is suggested that silicon is associated with calcium in an early stage of calcification.*

Approximately 5000 quantitative, electron probe microanalyses for calcium, phosphorus, and silicon were made on 50 specimens of normal tibia from young mice and rats (0 to 28 days old) with five different sample preparation techniques. As a result, silicon, a relatively unknown trace element in nutritional research, has been shown to be localized in active calcification sites in young mouse and rat bone. The amount of silicon present in specific regions within the active areas appears to be uniquely related to "maturity" of the bone mineral. In the earliest stages of calcification in these regions, when the calcium content of the preosseous tissue is very low, both silicon and calcium contents rise congruently. In more advanced stages the amount of silicon falls markedly, and, as calcium approaches the proportions present in hydroxyapatite, silicon is present only at the detection limit; the more "mature" the bone mineral the smaller the amount of measurable silicon. Concomitantly maximum amounts of silicon are present at molar ratios of calcium to phosphorus of approximately 0.7, but at ratios of calcium to phosphorus approaching that of hydroxyapatite silicon again falls below the detection limit.

To carry out a study involving quantitative analysis for and precise location of trace elements in biological tissue sections, unusual precautions must be taken to avoid contamination, redistribution, or removal of the elements. This prerequisite cannot be emphasized too strongly in the case of an element such as silicon which is so abundant in the environment (*1*). Modified histological specimen procedures used were (i) freeze-drying and embedding in polymer, (ii) vacuum drying and embedding in polymer, (iii) hand polishing of freeze- and vacuum-dried embedded slices with materials free of silicon, (iv) cryostat cutting with subsequent freeze-drying, and (v) fixation in absolute alcohol and embedding in paraffin. The comparison standard for quantitative electron microprobe analysis was a natural apatite for which a distinctly superior analysis of major and minor elements is available (*2*). There was no measurable wavelength shift between the specimen and the standard. Sequential quantitative analyses