SCIENCE

Hydroxyl and Water Masers in Protostars

Properties of these maser molecules are related to the earliest evolution of a star.

M. M. Litvak

Proposals have been made that protostar conditions may be responsible for the anomalous microwave emission of the OH radical (1, 2). The intense maser emission from OH and H2O molecules usually comes from what may be young, condensed regions (2, 3) either (i) close to, or embedded in, HII regions, those areas ionized by early-type hot stars; or (ii) close to infrared stars, some of which may be protostars themselves; or (iii) close to infrared nebulas (like that in Orion), which probably contain young stars still encased in a "cocoon" of dust; or (iv) close to remnants of supernovas, for which the association with protostars is less obvious. Each of these types of OH emitters has a characteristic strong emission in a different one of three of the four hyperfine-split transitions of the ground state, an indication of the important differences in the conditions of excitation for these cases.

Perhaps the strongest support for the protostar hypothesis for these masers has come from measurements made by very long base-line interferometry on the OH emission points at 1665 megahertz (4). These observations have yielded the following picture for W3 OH: There are about six points with apparent diameters ranging from 3 to

 10×10^{14} centimeters (or about 0.01 second of arc). Some points are a little elongated or are resolved into pairs or triplets of very close points. The apparent sizes of any of the $\rm H_2O$ points, which usually lie within 1 minute of arc of the OH points, are not known yet, since they are unresolved by single antennas

In approximately 10¹³ seconds a cloud about 1017 cm in diameter with a mass of about 1 $M \odot$ ($M \odot$ represents sun mass) and with a density of about 10⁵ per cubic centimeter will develop a small condensation within it having a central density of approximately 1011 cm⁻³, while the temperature everywhere remains between 10° and 100°K (5). The diameter of this central region is about 3×10^{14} cm which is approximately the value obtained from the interferometer studies of W3. The larger sizes obtained for W49, a more distant source, may be resolvable into a cluster of smaller points of about the same dimensions as those in W3. According to Larson (5), a dense opaque core of $10^{-2} M_{\odot}$ and radius $r \lesssim 10^{14}$ cm then develops inside this in about 300 years with an initial temperature of approximately 200°K and a density of about 10^{14} cm⁻³. The core is bounded by a stationary shock which slows the infall of mass from outside, thus leaving nearly hydrostatic equilibrium inside. The core rapidly heats up to about 2000°K, doubles its mass by continuing infall of material, and reaches a density of 1017 cm⁻³ in about a year. A second stellar core, with the approximate dimensions 1 Ro, 10^{-3} Mo, 10^{22} cm⁻³, and 20,000°K at its center, then forms within the first core. In a year, the parts of the cloud outside this core but within $r \sim 10^{13}$ cm may be heated to greater than 1000°K, while the stellar core has expanded to about 10 Ro and has acquired a surface temperature of about 4000°K. Until this stage is reached the remainder of the cloud has a density profile, varying as r^{-2} , from 10^{11} cm⁻³ at r $\sim 10^{14}$ cm to 10^5 cm⁻³ at $r \sim 10^{17}$ cm. This outer radius of the cloud is kept fixed in the calculation.

We must take into account two important and basic observations in order to put the OH and H₂O emission into this picture. First, the observed brightness temperatures $T_{\rm B}$ are 10^{12} to 1013 °K. Therefore, the maser amplification is equal to approximately a factor of 1011 and not much more. For unsaturated amplification the microwave optical depth $\tau_{\rm m}$ must be about In 10¹¹ or 25. Second, for the same emission feature with this brightness, the observed width $\delta \nu_m$ is less than or approximately equal to 3 khz (for example, the 1665-Mhz Doppler feature in W3 at -43.7 km/sec) (3). At best, maser action can narrow the line by the factor $\tau_{\rm m}^{-\frac{1}{2}}$ because of unsaturated amplification. Then, the line width before amplification, $\delta \nu$, cannot be greater than about 15 khz. Because of Dopplerbroadening, the equivalent kinetic temperature must be less than approximately 2500°K. Because of pressurebroadening, the hydrogen density must be less than 3 \times 10¹³ cm⁻³ and the

The author is assistant leader of the Solid State Theory Group at Lincoln Laboratory, Massachusetts Institute of Technology, Lexing-

ion density (of C^+ and other ions) must be less than about 10^{10} cm $^{-3}$. The lines that are narrower than 1 khz require temperatures less than 300° K and hydrogen densities less than 10^{13} cm $^{-3}$.

We thus can conclude that only the part of the cloud outside the cores might be responsible for maser action, that the second core is too hot and too dense, and that the first core is too dense. An examination, given below, of the difficulties of chemical and collisional pumping for the OH maser and the advantages of optical pumping indicates that the most likely OH maser conditions are to be found in the cold, outer region of radius 1017 cm and densities a little less than 10^6 cm⁻³. The strongest OH sources emit as many as $\sim 10^{46}$ photon/sec and the associated H₂O sources emit about 10⁴⁸ photon/ sec, if we assume nearly isotropic emission. A shock-heated layer of OH and H₂O with a large diameter of about 10¹⁶ cm would produce sufficient infrared H₂O and OH resonance lines and ultraviolet OH lines to pump these masers. This shock would converge toward the center for the more general cases of a moving cloud boundary, one that is pushed inward by either gas pressure or stellar winds from the surrounding region. The shock would move outward after reflecting from the center, while initiating a rapid series of events leading to a stellar core. In addition, for those cases of an OH region that is embedded in an HII region, a very slowly converging ionization front, with a still larger radius, would separate the neutral and the photo-ionized regions.

Failure of Chemical Pumping of OH

Let us now consider how chemical pumping must fail at densities greater than 10^6 cm⁻³. The difference in population of the upper (u) and lower (l) maser states is written as

$$\triangle n_j \equiv (n_u - n_l)_j$$

for the Λ -doublet of the jth rotational state. Although we are mainly interested in the ground state (${}^2\Pi_{3/2}J = 3/2$) emission, emission from excited state Λ -doublets must be quite probable under conditions of high excitation ($n>10^8~{\rm cm}^{-3}$ and $T>1000^{\circ}{\rm K}$). Such emission has been observed in the $2\Pi_{1/2}$ (J=1/2 and 5/2) and ${}^2\Pi_{3/2}$ (J=5/2) states (6), but only in certain

hyperfine components and in very few sources.

In particular, for the ground state (j = 1)

$$\Delta n_1 \simeq [R\eta_1 - \bar{n}_1 \Delta W_1]/(W_{11} + W_D)$$
 (1)

where \bar{n}_1 is the sum of the populations in the upper and lower maser states, η_1 is the fraction of the total OH production rate R that gives population inversion in the ground Λ -doublet, W_D is the destruction rate per second per molecule for either state of the Λ doublet, ΔW_1 is the difference in the collision rates between the upper and lower states, and W_{11} is the sum of these rates per second per molecule. Because the approach to a steady state between the production and destruction rates is a matter of only an hour or less at densities of about 1011 cm⁻³, we may safely set

$$n_{\rm OH} = R/W_{\rm D}$$

Then

 $\Delta n_1 \simeq$

$$[\eta_1 - \bar{n}_1 W_{11} h \nu_1 / (2n_{\text{OH}} W_{\text{D}} kT)] R / W_{11}$$
 (2)

when we neglect W_D with respect to W_{11} in the denominator and use

$$\Delta W_1 \sim h \nu_1 W_{11}/(2kT)$$

where $h\nu_1$ is the energy difference between the upper and lower states, k is Boltzmann's constant, and T is the temperature of the collision particles (7).

To obtain inversion, we must satisfy $\Delta n_1 > 0$ or

$$\eta_1 > \bar{n}_1 W_{11} h \nu_1 / (2 n_{\text{OH}} W_{\text{D}} k T)$$
(3)

When the destruction process

$$OH + O \rightarrow O_2 + H$$

is dominant, as is likely at temperatures below 700°K, where $W_{\rm D}$ is approximately equal to 10^{-10} exp $(-600^{\circ}\text{K}/T)n_{\rm O}$ sec⁻¹ (8). We now assume that $n_{\rm O}/n_{\rm H}$ is about 10^{-3} (cosmic abundance) and $n_{\rm H}$ is approximately equal to n, the total density. However, much of the O and H might be bound in the grains or in various molecules. We estimate that W_{11} is approximately equal to 10^{-9} n sec⁻¹. For $T > 100^{\circ}\text{K}$, we estimate that $\bar{n}_{1}/n_{\rm OH}$ is approximately $10^{2}/T$ (9). Then, according to Eq. 3, for these temperatures

$$\eta_1 \gtrsim 10^2 \, n/(n_0 T^2)$$

Accordingly, for $T \lesssim 700^{\circ}$ K, we need an efficiency η_1 that is greater than or approximately equal to 1, which is im-

possible. For $2000^{\circ}K > T > 700^{\circ}K$, we may take the dominant destruction process (8) to be

$$OH + H \rightarrow O + H_2$$

with $W_{\rm D}$ approximately equal to 10^{-11} exp $(-3700^{\circ}\text{K}/T)$ $n_{\rm H}$ sec⁻¹. For $T\sim 1000^{\circ}\text{K}$, we find that η_1 is greater than 10^{-2} , but η_1 increases rapidly to unity as T decreases to 700°K . When $n_{\rm H_2}$ is greater than 0.1 $n_{\rm H}$, then we must consider the destruction mechanism (8)

$$OH + H_2 \rightarrow H_2O + H$$

with W_D approximately equal to 10^{-10} exp $(-3000^{\circ}\text{K}/T)$ n_{H_2} sec⁻¹.

To establish what total densities, n, allow the amplifier to be unsaturated, as we have assumed, the microwave transition rate (10)

$$W_{\rm m} = kT_{\rm B}A\Omega_{\rm m}/(4\pi h \nu_j)$$

must be sufficiently small so that

$$2W_m < W_{11} \tag{4}$$

for the ground state and

$$2W_{\rm m} < W_{jj} (1 + W_{\rm R} Q_{\rm R}/A_j) + A_j/Q_{\rm R}$$

for excited rotational states (11), where A_j is the far-infrared decay rate and Q_R is the trapping factor for this resonance radiation. The rate W_R (per second) is for collisional de-excitation of rotation. The solid angle Ω_m is the local angle over which appreciable microwave intensity is emitted at a point on the amplifier output surface. For unsaturated amplification

$$\Omega_{\rm m} \sim { au_{\rm max}}^{-1}$$

where au_{max} is the maximum value of au_{m} along a ray which emerges at the output point in question (10). Since $W_{\rm m}$ is approximately 10 sec⁻¹ in this case, n would be greater than about 10¹⁰ cm^{−3} according to the condition expressed in Eq. 4, but less than 1013 cm⁻³ because of the restriction on pressure-broadening of the line width discussed earlier. For a model protostar of approximately 1 M_{\odot} , a radius of 10^{14} cm corresponds to $n \sim 10^{11}$ cm⁻³. This material, if not quite opaque, has a free-fall time of about 50 years (12). Densities of approximately 1013 cm⁻³, which, of necessity, lie within an opaque core, would more than double in 5 years as a result of the infall of material and would exceed the density limit imposed by pressure-broadening. This seems hardly consistent with the constancy of

most of the strong sources for as many years (3).

Let us first suppose that η_1 is approximately 10^{-2} and n is about 10^{11} cm⁻³ and an amplifier length l is approximately 3×10^{14} cm. We shall consider later the question of whether this efficiency is possible at such high densities. To obtain

$$au_{\rm m}=5 imes 10^{-19}$$
 $\int \Delta n_1 dl \nu/\delta \nu=25$ (5) for $l\sim 3 imes 10^{14}$ cm and $\delta \nu/\nu\sim 10^{-5}$, we need Δn_1 to be about 1 cm⁻³. From Eq. 2

$$\Delta n_1 \sim \eta_1 R/W_{11} \tag{6}$$

thus R is approximately 10^4 cm⁻³ sec⁻¹ for the total OH production rate. For the moment let us consider the values of R (per cubic centimeter per second) suggested by Solomon (13), reduced by a factor of 80 because of various errors, and those suggested by Stecher and Williams (14). These are, respectively

R (Solomon) ∼

$$4\times10^{-17}\exp{(-3700\,^{\circ}{\rm K}/T)}\,n_0n_{\rm H}$$
 R (S. and W.) \sim

$$10^{-17} \sqrt{T} \exp(-2040^{\circ} \text{K/T}) n_0 n_{\text{H}}$$

For $T \sim 1000$ °K, $n_0/n_{\rm H} \sim 10^{-3}$, and $n_{\rm H} \sim n \sim 10^{11}$ cm⁻³, R (Solomon) is approximately 10 cm⁻³ sec⁻¹ and R (S. and W.) is approximately 300 cm⁻³ sec⁻¹, which are both too weak.

Now we show that the efficiency η_1 for these and similar mechanisms cannot be as great as 10^{-2} at these densities. Both of these produce OH almost entirely in vibrationally excited states. The cascading from such states (j) to the ground state (j = 1) contributes to Δn_1 (Eq. 6) at a rate proportional to

$$\eta_1 \equiv \sum \eta_{vj} (A_j/Q_v)/[(A_j/Q_v) + W_{jj}]$$

where A_i is approximately equal to 10^2 sec-1, a typical radiative decay constant for vibration, Q_v is the trapping factor for the near-infrared resonance radiation, W_{ij} , which is about 10^{-9} n sec-1, is the sum of the rates for collisional transitions across the Λ -doublets in the vibrational states, and η_{vi} is the pump efficiency when W_{jj} is negligible. For the first vibrational state, Q_v is approximately 10 when $\tau_{\rm m}$ is about 25 and η_{vj} is approximately 10^{-2} . For the second vibrational state, Q_v is about 1. Because the contribution to η_1 from the first vibrational state decreases with n when n is greater than 10^{10} cm⁻³, we conclude that the inversion is to be carried directly to the ground vibrational state by the 1.4-µ transition from the second excited vibrational state. However, this decay occurs only 1/10 of the time, compared to decay to the first vibrational state. Hence η_{vj} must be less than 10^{-2} for this process, if we take into account that the pumping into any particular rotational state is approximately 10^{-1} R.

These difficulties arise from the fact that the OH inversion is first produced in a vibrationally excited state and is not dependent on any specific mechanism: The situation is much worse if most of the OH inversion is first produced in rotationally excited states, without excitation of vibration. Then, according to similar arguments, with A_j/Q_R approximately equal to 10^{-3} sec⁻¹, n must be less than 10^6 cm⁻³ for $\eta_1 \sim 10^{-2}$.

The remaining cases invert the ground state Λ -doublet either directly or by cascading directly to the ground state from an excited electronic state. The only process of the latter type is the very weak preassociation pumping ($R \sim 3 \times 10^{-21} \ n_0 n_{\rm H} \ {\rm cm}^{-3} \ {\rm sec}^{-1}$) suggested by Hughes (15). Unacceptable densities greater than 10^{16} cm⁻³ would be needed.

Calculations for $\sim 1000^{\circ}\text{K}$ indicate that the O is mainly contained in CO and H_2O and the H is contained in H_2 (16). Atom exchange reactions requiring the breaking of two bonds

$$WX + YZ \rightarrow WY + XZ$$

have activation energies

$$E \sim 0.28 [D(WX) + D(YZ)]$$

where D is the bond energy (17). Thus, E is greater than or approximately equal to $30,000^{\circ}$ K, a value too large to allow such reactions to be important. This is aside from the question of which reactions are exothermic—none with CO as reactant are likely to be, with 11.09-electron volt binding energy of CO. Thus the only alternative explanation left is atom exchange of H, the most plentiful atom, with water, the next most plentiful molecule containing O, as in

$$H + H_2O \rightarrow OH + H_2$$

which is a slightly endothermic reaction. The rate, obtained from that for the inverse reaction, is $R \simeq 10^{-10}$ exp (-11,000°K/T) $n_{\rm H}n_{\rm H_2O}$ cm⁻³ sec⁻¹. This rate is very sensitive to temperature, especially with $n_{\rm H} \sim \sqrt{n}_{\rm H_2} \times 10^{(10.75-11,250/T)}$ (18). For $n_{\rm H_2} \sim n \sim 10^{11}$ cm⁻³, and $n_{\rm H_2O} \sim 10^{-3}$ $n \sim 10^8$ cm⁻³, the value of R is the required

 10^4 cm⁻³ sec⁻¹ when T is approximately 1600° K, which is too high for line widths as narrow as those observed for many lines. Somewhat higher densities do not lower this temperature significantly. No other reactions can be found with higher production rates of OH than this one at these densities and temperatures.

Hyperfine-Split Transitions

The dominance of emission in hyperfine-split transitions that do not have the largest line strengths indicates that certain hyperfine state populations are preferred by the maser pumping or by effects that accompany the pumping, such as resonance radiation fluorescence and trapping (11, 19). Different OH sources have different characteristics. The 1665-Mhz line is dominant near H II regions, the 1720-Mhz line near remnants of supernovas, and the 1612-Mhz line near infrared stars. The above discussion indicates that when n is greater than 10^{10} cm⁻³, far- and near-infrared resonance radiations are quenched by collisions. Therefore, this cannot be cause for preference of certain hyperfine states at high densities. We may dispense with the hyperfine effects of ultraviolet resonance radiation at all temperatures well above 20°K because the Doppler widths of the ultraviolet lines are greater than the hyperfine splittings. Similarly, temperatures above 60°K nullify the hyperfine effects of the near-infrared resonance radiation because the Doppler widths of the infrared lines overlap the hyperfine splittings.

We now show that a chemical reaction also cannot prefer hyperfine states (20). A particular OH hyperfine state involves the addition of the hydrogen nuclear angular momentum I ($I = \frac{1}{2}$) to the molecular momentum J, so that the total angular momentum

$$F = I + J$$

and both I and J precess about F. Consider the results in a single chemical collision. The H atom in the reactants that will appear in the OH product is originally found with equal probability for parallel and antiparallel orientations of I with respect to any axis, say J. The collision process lasts only $t \sim 10^{-12}$ second. In this short time, no molecular forces can act appreciably on the H nucleus during the collision to upset this equal distribution of paral-

lel and antiparallel nuclear spin states. This is the case because the typical hyperfine splittings (10^7 to 10^9 hz), which represent h^{-1} times the energy of interaction with such forces, are much smaller than $(\pi t)^{-1}$.

Densities between 106 and 1010 cm⁻³ and temperatures below 60°K allow both pump efficiencies greater than 10-2 and hyperfine-dependent effects. However, such low temperatures involve too strong an anti-inversion for the criterion expressed in Eq. 3 to be satisfied with chemical pumping. The maser amplifier would be saturated, and $\delta \nu_m$ would become comparable to $\delta \nu$. In this case, unless pumping inverts the OH population only near line-center, or radial velocity gradients or nonlinear mode competition affects the amplification, or unless there is cross-relaxation of the molecular velocity distribution that is more rapid than the maser rate, the temperatures must be less than 100°K. High temperatures, with line-narrowing despite saturation, is a remote possibility, but no preference for particular hyperfine states is possible.

This brings us back to conditions of $n \approx 10^6$ cm⁻³, for which far-infrared resonance radiation is not quenched by collisions. Thus certain hyperfine states can be over- or underpopulated because of unequal radiation trapping effects (19).

Energy Balance

There are greater difficulties with chemical pumping at the lower densities $(n < 10^{10} \text{ cm}^{-3})$ for which the maser is saturated. These lower densities exist in the major portion of the cloud outside the first core. Upon infegrating the emission rate over this volume, we find that

$$(1/16)\eta_1 R l^3 \sim L_{\rm m}/4\pi$$
 (7)

is required by the output of a single OH protostar of luminosity $L_{\rm m} \sim 10^{45}$ to 10^{46} microwave photons per second (21). With the density n(r) proportional to r^{-2} , the mass M increases linearly with the radius. From Eq. 5 and the condition that $\tau_{\rm m}$ is approximately 25, we find that $\eta_1 n_{\rm OH} l$ is approximately equal to 3×10^{14} cm⁻². Since it is still reasonable to consider nearly steady-state conditions even for $n\sim 10^6$ cm⁻³, we also have $n_{\rm OH}=R/W_{\rm D}$. Reasonable values of $W_{\rm D}/n$ are approximately 10^{-11} to 10^{-13} cm³

 sec^{-1} for any of the destruction processes mentioned above for T > 700°K. From Eq. 7 and our estimates of $\eta_1 n_{\text{OH}} l$ and W_{D}/n , we find that M/M_{\odot} is approximately equal to $10^6 \times (n/10^6)$ cm⁻³)^{-1/2}. We would require, for n < 10^{10} cm⁻³, that M be greater than 10^4 M_{\odot} , which is too large for a protostar. Furthermore, to maintain $T > 700^{\circ}$ K, this region must be opaque or nl must be greater than 10^{25} cm⁻² and yet radiate in the infrared region with a luminosity $L \sim \sigma T^4 \pi l^2/4$ that is much less than approximately 10⁴ L_o, according to observations with negative results (22). The latter condition implies that l is less than 10^{15} cm. Then the opacity condition cannot be satisfied for n <10¹⁰ cm^{−3}. Thus, chemical pumping poses difficulties, whether there is maser action in the inner or outer parts of the protostar.

Collisional Pumping

The special collisions of H with OH which excite rotation mainly in the upper A-doublet states have already been mentioned above (7). When we use $\tau_{\rm m} \sim 25$ and assume that there is mainly H rather than H_2 and that T is less than or approximately equal to 100°K, we find that $\eta_1 n_{\text{OH}} l$ is approximately 10^{14} cm⁻². For $\eta_1 < 10^{-1}$, we find a projected OH density for which the far-infrared decay of the first excited rotational state ${}^{2}\Pi_{3/2}$ (J=5/2) is sizably slowed by radiation trapping. We need n to be less than 3×10^6 cm⁻³ to prevent excessive thermalization of the Λ -doublets by collisions. From Eq. 7 and the above value of $\eta_1 n_{OH} l$, and a pump rate by means of rotational excitation of the ${}^{2}\Pi_{3/2}$ (J=5/2) state given by $R < 10^{-9}$ exp $(-120^{\circ}\text{K}/T)n_{\text{H}}\bar{n}_{1}$ $cm^{-3} sec^{-1}$, we obtain $(n/10^5 cm^{-3})$ $(l/10^{17} \text{ cm})^2 > 10^2$; this implies that l is greater than 10^{17} cm and that freefall velocities are in excess of 5 km/ sec. If H₂ is as plentiful as H, these three quantities would have to be even larger than these large values. Collisions with H₂ do not cause population inversion. The narrowest observed lines correspond to $T \sim 10^{\circ} \text{K}$, this being a case of saturated emission; R is then too small and sensitive to temperature to account for the emission. Furthermore, with this model, maser emission from the $^2\Pi_{3/2}$ (J=5/2) state and not the $^2\Pi_{1/2}$ (J=5/2) state should be common, which is contrary to observations.

Optical Pumping

From the discussion to follow, we find that the outer part of the cloud can be pumped fast enough for the weaker OH sources by radiation from the surface of Larson's first core. But radiation from a shock front with a much larger area must be present for pumping the stronger OH, and H₂O, sources, unless the emission is fairly directional. Also, nearby O5 stars would then suffice (10). In any case, we consider that n is less than or approximately equal to 106 cm⁻³ for the maser region. We again examine the saturated case. To estimate the saturation rate $W_{\rm m}$, we may use the local solid angle $\Omega_{\rm m} \sim \pi l_*^2/l^2$, where $l_* \sim$ 3×10^{14} cm is the diameter of the apparent source according to interferometric measurements (19, 23). In a saturated maser, the signal, which is strong in a particular direction, may intensify itself at the expense of the weaker signal that is propagating in a different direction but must share the population inversion. This nonlinear competition greatly decreases $\Omega_{\rm m}$ from its values for an unsaturated maser. Equation 4 then implies

$$(T_{\rm B}/10^{10} {\rm ^{\circ}K})(l_{*}/3 \times 10^{14} {\rm cm})^{2} >$$

 $(n/10^{5} {\rm cm}^{-3})(l/10^{17} {\rm cm})^{2}$ (8)

where nl^2 is nearly a constant determined by the given velocity of collapse of the outer boundary. Data on several sources give a value of approximately 10^3 for the left side of the above inequality. The right side of Eq. 8 must be greater than T for the collapse of a uniform cloud having a temperature T (5).

We use an equation similar to Eq. 7, except that the optical pump rate $R_{\rm p}$ replaces R. If we account for the absorption of the pump radiation by the OH maser and the number of separate pump lines s, we find that

$$2\pi\eta_1 I_p(r) r^2 s \delta \nu_p / (h\nu_p) \sim L_m \tag{9}$$

where $I_{\rm p}(r)$ is the intensity emitted by the surface of the pump region at a distance r from the center, and $\delta v_{\rm p}/v_{\rm p}$ is the fractional line width of a pump line. The efficiency η_1 applies to the optical pumping now, that is, $\Delta n_1 \sim \eta_1 R_{\rm p}/W_{11}$. Instead of the criterion expressed in Eq. 3, we have

$$\eta_1 > \bar{n}_1 W_{11} h \nu_1 / (2R_p kT)$$
(10)

We have chosen

$$R_{\rm p} \equiv n_{\rm OH}(r)B'I_{\rm p}(r)$$

where B' is the Einstein coefficient for absorption. Upon using Eq. 9 and T <100°K, we see that the condition expressed in Eq. 10 is also satisfied when $(n/10^5 \text{ cm}^{-3})$ $(r/10^{17} \text{ cm})^2$ is less than 10 T, a value noted in connection with the condition expressed in Eq. 8. The free-fall velocity is less than 15 km sec⁻¹. With $\eta_1 \gtrsim 10^{-2}$ and $r \sim 10^{16}$ cm as the radius of the hot OH layer that pumps cooler OH molecules in the outer layers, and with blackbody radiation in the OH resonance lines, we find that near-infrared or ultraviolet pumping is indeed possible at 1000° to 4000°K (11). The hot layer, although not opaque except in certain molecular lines, may be heated by a shock front usually converging toward the center. Conditions on the OH opacity and the cooling rate for the gas just passed over by the shock require that the layer thickness be approximately 1019 OH molecules per square centimeter and that n be approximately 10^{11} cm⁻³ ahead of the shock. The grains would have evaporated enough to permit 2.8and $0.3-\mu$ radiation to penetrate to $r \ge 10^{17}$ cm. Although far-infrared pumping from the shocked layer is not strong enough at any reasonable temperature, a larger diameter (1018 cm) shell containing grains heated to approximately 100°K (for example, for 126-cm⁻¹ pumping) could be a strong enough pump. The region with n greater than 1011 cm-3 would be a disk having a thickness of about 1014 cm if the mass is approximately $1M_0$. Since the flow would roughly follow the magnetic field, it is not unreasonable that the disk axis coincides with the average field direction. The OH emission might be directed along slender filaments of condensation in this direction because the larger column densities of OH and the smaller gradients of radial velocity optimize the amplification. With such directional emission, the radius of the shock front may be smaller and still produce adequate pumping.

Time Variations

The transit time l/c over the length l of the amplifier might determine the characteristic time for an appreciable change of intensity. Small changes in $\tau_{\rm m}$ for the unsaturated maser, for high densities and l approximately equal to 3×10^{14} cm, could mean large changes over a time of about 10^4 seconds (3 hours). These are not observed (24).

The maser output would be very sensitive to fluctuations in molecule and electron densities. For the saturated maser, at lower densities where n is approximately 10^6 cm⁻³ and l is approximately 10¹⁷ cm, the intensity might change somewhat over a time l/c approximately equal to 3×10^6 seconds (1 month). These are observed in several sources (24). The time $(W_{11})^{-1} \sim 10^9/n \sim 10^3$ seconds (20) minutes) needed for response to changing pumping conditions is very short so that the emission can follow changes in the optical pump intensity. These may occur in times like 1 year. Then, as the gas and grains near a shocked layer are heated up, the intensities in the different hyperfine transitions could change their ratios (25). Changes in the saturated microwave intensity due to changing fluid flow traveling across approximately 1016 cm might take as long as about 102 years. The saturated maser output would be insensitive to density fluctuations, which provides further support for maser action in the outer part of the protostar (26).

Zeeman Splitting

Because of the high conductivity of an HI region even with a fairly small degree of ionization, the formation of protostars should be accompanied by a compression of the magnetic field lines, in a manner consistent with a constant flux per unit mass $2\pi Brdr/dM(r)$, where B is the azimuthal field at radius r and M(r) is the mass contained within this radius. If the collapse of a cloud has spherical symmetry and if the original magnetic field configuration has components perpendicular to the radial direction equal to $B \sim 10^{-5}$ gauss for $n \sim 10$ cm⁻³ and $r \sim 10^{19}$ cm for a uniform cloud of 102 Mo, then we might expect that about 10^{-2} gauss cm² g⁻¹ is a typical value for the flux per mass (27). For an expected density profile n proportional to r^{-2} in the collapsing cloud of $1 M_{\odot}$, we calculate from this constant that B is about 10^{-3} gauss at $r \sim 10^{17}$ cm and $n \sim 10^5$ cm⁻³, and $B \sim 1$ gauss at $r \sim 10^{14}$ cm and $n \sim 10^{11}$ cm⁻³; higher values of B occur in the cores if there is no excessive dissipation of the magnetic energy into heat. For clouds of larger mass, the fields at these radii would be proportionately larger. The OH data, although not consistent with regular Zeeman patterns, can nevertheless rule out $B > 3 \times 10^{-3}$ gauss because the range of frequency of the spectral features around the zero field frequency is less than 5 khz at the same location according to the very long base-line interferometers (3, 4). Can the field have leaked out during the compression? The characteristic time for this phenomenon in the cloud as a whole is (28)

$$t_{\rm B} \sim \frac{10^5 \, {\rm yr} \times n_{\rm e} \, \left(\frac{n}{10^5 \, {\rm cm}^{-3}}\right) \left(\frac{r}{10^{17} \, {\rm cm}}\right)^2}{\left(\frac{B}{10^{-3} \, {\rm gauss}}\right)^2}$$

For the cloud properties we have been discussing and for $n_{\rm e} \sim 1$ ion or electron per cubic centimeter (due mostly to Na+ and K+ at $\sim 1000^{\circ}$ K and possibly to ionization by cosmic rays in the cooler outer region), $t_{\rm B}$ is approximately 10^{5} years, which about equals the free-fall time for the outer radius. We conclude then that the magnetic field strength is probably less than or approximately equal to 10^{-3} gauss near the edge of the protostar cloud but reaches much higher values inside the protostar cloud which cannot be reconciled with the OH data.

Polarization

For laboratory lasers, it is known that opposite senses of circular polarization compete for amplification for $\Delta F = 0$ transitions such that a high degree of circular polarization results (29). The OH emission is approximately 3 khz wide. Unless the OH level widths are equal to or greater than this bandwidth, requiring that n be greater than or approximately equal to 1013 cm⁻³, the OH case is not similar to the laboratory one (30). Since saturation is not possible at these densities for the observed signal strengths, the mode competition found in the laboratory is negligible for the OH maser. In addition, since the Zeeman splittings are likely to exceed the OH level widths for young regions, and those of high density, the competition for circular polarization at the same frequency is greatly reduced or absent, an effect also observed for laboratory lasers in magnetic fields.

The lower density condition, that n be less than 10^6 cm⁻³, would be consistent with sufficiently low magnetic splittings, but the ratio of level width to the emission bandwidth is much smaller than 1. This is the factor by which the competition between modes

of opposite circular polarization is reduced. However, an additional mechanism for obtaining circular polarization has been found in the parametric down-conversion of the higher frequency Zeeman component to the lower frequency one, if we assume that the Zeeman splitting is comparable to the Doppler width (19). This parametric process involves the coupling of these modes and electron cyclotron waves through the nonlinear properties of the magnetoplasma in the OH region (31). There is circular polarization when the microwave propagation is at small angles to the magnetic fields, elliptical polarization at moderate angles, and linear polarization at right angles.

Water Vapor Maser at 22.2 Gigahertz

Pumping of the strong H₂O maser (32) in OH regions appears possible in the rapidly cooling layer that follows the shock front itself. A general H₂O rotational level has a lifetime determined by collisions, by trapped far-infrared radiation to lower levels, and by a pumping mechanism. Let us consider relatively low temperatures so that the magnetic sublevel populations roughly decrease monotonically from the ground energy level to higher-lying ones. The amount of trapping of the far-infrared radiation thus also decreases for the higher levels. If the hydrogen density is less than 109 cm⁻³, the collisional de-excitation may become less rapid than far-infrared fluorescence for all rotational levels above a certain energy. There is then a demarcation above which levels are significantly depleted of population, but below which the levels are well populated and nearly in thermal equilibrium. When this demarcation occurs near the $5_{2,3}$ and $6_{1,6}$ levels, then population inversion may be obtained, with external pumping replenishing the excitation that is lost as microwave emission. Collisions between the two maser levels would only reduce the population inversion. In order to obtain significant maser amplification for the transition from the $6_{1,6}$ level to the $5_{2,3}$ level, the projected densities for the population difference must be $\sim 10^{16}$ cm⁻². The time variations of several features in W49 would suggest path lengths of $\sim 10^{16}$ cm. Thus, the population in either maser level is ~ 1 cm⁻³. At rotational temperatures T of about 40°K, the popu-

lation of the $4_{3,2}$ level, which mainly determines the trapped lifetime of the $5_{2.3}$ level, is ~ 0.15 times that of the $5_{0.5}$ level, which mainly determines the trapped lifetime of the $6_{1,6}$ level. Then the trapped lifetime of the $6_{1,6}$ level would be comparable to, rather than less than, that of the $5_{2,3}$ level in the absence of collisions. For a kinetic temperature of about 100°K, the total H₂O density is greater than 10³ cm^{−3} for the above maser level densities, provided the hydrogen density is 108 cm⁻³ so that the rotational ladder is roughly in collisional equilibrium up to the $5_{0,5}$ level but not above it. The resonance radiation from the H2O molecules at temperatures of 2000° to 4000°K in the shock front pumps the infrared transitions at ~ 2000 and 4000 cm⁻¹ in the cool molecules farther downstream. By absorbing this radiation, a molecule is excited from the $5_{2,3}$ level in the ground vibrational state mainly to the 51,4 levels in the v_1 and v_2 vibrational states (33). This rapidly decays to the $6_{2,5}$ level in the ground state. Then this decays by far-infrared radiation to the 6_{1.6} level. Stimulated emission from the $6_{1,6}$ to the $5_{2,3}$ level completes the pump cycle. An equally important pump cycle is

$$5_{2,3} \rightarrow 6_{3,4} * \rightarrow 6_{2,5} \rightarrow 6_{1,6}$$

where the asterisk indicates a state that is vibrationally excited in either the ν_1 band or the more easily excited ν_2 band. Weaker pump routes are

$$5_{2,3} \rightarrow 6_{1,6}^* \rightarrow 6_{2,5} \rightarrow 6_{1,6}$$

 $5_{2,3} \rightarrow 5_{1,4}^* \rightarrow 6_{2,5} \rightarrow 6_{1,0}$

and

$$5_{2,3} \rightarrow 5_{3,2}^* \rightarrow 6_{2,5} \rightarrow 6_{1,6}$$

The rate of anti-inverting by

$$\begin{aligned} 6_{1,6} &\to 6_{2,5} ^* \to 6_{3,4} \to 5_{2,3} \\ 6_{1,6} &\to 7_{0,7} ^* \to 6_{3,4} \to 5_{2,3} \end{aligned}$$

and the like can be shown to be somewhat weaker than the rate of inverting the population. A little less than 10^{48} microwave photons per second can be emitted isotropically with this pumping. We would expect observable fluorescence of about this amount in the infrared lines at ~ 2000 and ~ 4000 cm⁻¹ in the ν_2 and ν_1 bands, respectively, and 106 and 203 cm⁻¹ for the transition from the $6_{2,5}$ level to the $6_{1,6}$ level and that from the $6_{3,4}$ level to the $5_{2,3}$ level. Strong 4000-cm⁻¹ pumping can also occur by way of the ν_3 vibrational state as follows: $5_{2,3} \rightarrow$

 $6_{2,4}^*$ (or $5_{2,4}^*$) \rightarrow $6_{2,5}$ \rightarrow $6_{1,6}$. The transition from the $6_{2,5}$ level to the $5_{3,2}$ level at 44 cm⁻¹ might be a maser at somewhat higher rotational temperatures, if we assume the same amplifier length. At similar densities but lower rotational temperatures, masers might be found between lower levels such as the transition from the $3_{1,3}$ level to the $2_{2,0}$ level at 183 Ghz with the pump route

$$2_{2,0}\!\to 2_{1,1}^*\!\!\!\!*\to 3_{2,2}^{}\!\to 3_{1,3}^{}$$

Another maser might be the transition from the $5_{1,5}$ level to the $4_{2,2}$ level at 324 Ghz, with the main pump routes

$$4_{2,2}\!\to 5_{3,3}^*\!\to 6_{2,4}\!\to 5_{1,5}$$

and

$$4_{2,2} \! \to 5_{3,3} \! * \to 5_{2,4} \! \to 5_{1,5}$$

For saturation to set in, the pump rate per molecule of $B'I_{\rm p} \sim 10^{-1}~{\rm sec^{-1}}$ would be just less than the observed maser rate $W_{\rm m} \sim kT_{\rm B}\Omega_{\rm m}A/(4\pi h\nu)$. This happens when $T_{\rm B}\Omega_{\rm m}$ is about 10^9 °K-steradian. (Note that A is approximately $2\times 10^{-9}~{\rm sec^{-1}}$, and $h\nu/k$ is about $1^{\circ}{\rm K}$.) For features in W49 that are rapidly time-varying over several days perhaps, the amplifier is not quite saturated. Then $\Omega_{\rm m}$ is approximately $\tau_{\rm m}^{-1}$ or 1/25 steradian and $T_{\rm B}$ is approximately $3\times 10^{10}~{\rm ^{\circ}K}$, which is similar to the OH brightness temperatures for W49.

Summary

There are stringent conditions on any chemical process that might be responsible for the population inversion and the observed properties of the OH maser. So stringent are they that no such process is likely to be found. On general grounds, the high densities and narrow line widths would imply unsaturated amplification. This is most sensitive to small differences in protostar conditions. Yet the intensities are comparable for the different emitters. despite different line widths and polarizations, which attests to a variety of conditions. The emissions from the excited rotational states become progressively weaker at higher positions in the rotational ladder, not, as expected, for excitation temperatures greater than 700°K and for the unsaturated amplification that is required by a total density greater than 1010 cm⁻³. The time variations, polarization, and preference for

certain hyperfine components are inexplicable at high densities. Furthermore, Zeeman splittings would be much larger than observed. However, in the outer parts of the cloud, with optical pumping from a shocked OH layer and heated grains, the conditions are favorable for maser emission with the observed properties.

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$$n_J/n_{
m OH} = f_J/\sum_{m J} f_{m J}$$

where

$$f_J = (2J+1) \exp (-E_J/kT)$$

$$E_J \sim E_0 + BJ (J+1)$$

The energy E_0 is zero for the $\Pi_{3/2}$ rotational ladder and equivalent to 126 cm $^{-1}$ for the $\Pi_{1/2}$ ladder. The rotational constant B is 18.5 cm $^{-1}$ \times hc. For $kT > E_0$

$$\sum_{J} f_{J} \sim 2kT/B$$

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$$CH^+ + O \rightarrow OH + C^+$$

are fast even at low temperatures and in-

- are fast even at low temperatures and insensitive to temperature, but, because of the Coulomb effect of the ions, the populations of the \$\text{\$\text{\$A\$}}\$ doublet states will be thermalized rather than inverted in the collisions.

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 26. The observed sizes of ~0.01 second of arc
- The observed sizes of ~ 0.01 second of arc can result from the diffraction and refraction of the signal from "hot spots" by large-scale spatial fluctuations of the electron density. This condition plus that of no appreciable intensity changes in times like 10° to 10° seconds, imply that the scale length a for a turbule is less than 10° cm (perhaps, $a \le 10^{\circ}/\sqrt{n_e}$ cm if the ion plasma length characterizes the turbulence) and that the rootmean-square electron density fluctuation $<\delta n_e >$ is much less than $10^{11}/\sqrt{r_e}$ cm⁻² to obtain mean-square electron density fluctuation $<\delta n_e>$ is much less than $10^{11}/\sqrt{(al)}$ cm⁻³ to obtain small phase modulation for 18-cm radiation [see P. A. G. Scheuer, Nature 218, 920 (1968)]. With $n_e<1$ cm⁻³ and $l\sim10^{17}$ cm, $<\delta n_e>$ must be $<<3\times10^{-2}$ cm⁻³, which appears very reasonable. G. L. Verschuur, Phys. Rev. Lett. 21, 775 (1968); Astrophys. J. Lett. 155, L155 (1969). The collapse of a piece of this cloud requires that there be sufficiently small magnetic energy, in fact, such that the ratio of flux to mass is
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