

to the specialist. It places phycology on a global stage and may help the investigator to orient himself. The contributions are for the most part from the pens of distinguished authorities in the field. However, it will surely be confusing to the nonspecialist who expects to learn much about algae, man, or the environment.

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Advances in Algebra

The Great Art. Or the Rules of Algebra. GIROLAMO CARDANO. Translated from the Latin edition (Nuremberg, 1545) and edited by T. RICHARD WITMER. M.I.T. Press, Cambridge, Mass., 1968. xxvi + 270 pp., illus. \$10.

In a sense Girolamo Cardano's *Ars magna* in 1545 was to mathematics what the *De revolutionibus* of Copernicus and the *De fabrica* of Vesalius had been two years earlier to astronomy and anatomy. Indeed, the *Ars magna* was perhaps the most revolutionary of the three. Copernicus had applied the mechanism of Ptolemy to the view of Aristarchus, and Vesalius had corrected details in Galen; but the work of Cardan disclosed the greatest step in the algebraic solution of equations since the days of Hammurabi. Quadratic equations had been solved by the pre-Hellenic Mesopotamians, but cubics had resisted the best efforts of ancient and medieval mathematicians. Today the solution is well known, yet the volume in which it was first made public has been as little read as it has been much praised. Even mathematicians who use the familiar "Cardan rule" frequently are unaware that in the *Ars magna* the author three times candidly wrote that he had obtained the key to the solution from Tartaglia and that the formula originally had been discovered in about 1515 by one Scipione del Ferro, professor of mathematics at Bologna. With an English translation available, perhaps it is not too much to hope that Cardan will be more widely read and that "his" rule will before long become known as "del Ferro's rule."

Past neglect of the *Ars magna* is easily understood. Quadratic equations today are represented by a universal notation, $ax^2 + bx + c = 0$, and solved by a single formula. From Babylonian days to the time of Cardan there were three distinct types of quadratic equa-

tion: square and thing equals number; square equals thing and number; and square and number equals thing. (A fourth type, square and thing and number equals zero, was excluded as having no solution.) For cubic equations there are 13 cases instead of three; and Cardan rhetorically and laboriously worked through each one, giving numerical illustrations and geometrically based demonstrations, all in the tradition of Mohammed ibn Musa al-Khowarizmi. In Witmer's translation the repetition of cases is unavoidable, but the tedium is mitigated through the liberal use of modern notations. A critic can argue that such modernization misrepresents the thought of the original, but such an indefatigable scholar can check the symbolic translation against the original rhetorical version in extant Latin editions. Less demanding English readers will welcome the fact that now they have a less challenging entree not only to Cardan's solution of the cubic but also to the surprisingly ample store of algebraic methods to be found in the *Ars magna*.

At one point in the book Cardan wrote that he would do little with equations beyond the cubic. "For as the first power refers to a line, the square to a surface, and the cube to a solid body, it would be very foolish for us to go beyond this point. Nature does not permit it." The author fortunately did indeed go beyond three dimensions, and in the next-to-last chapter he divulged a method, discovered by his amanuensis Ferrari, for solving equations of the fourth degree; and even quintic equations come in for some consideration. In one of the most rewarding portions of the volume, the chapter "On the transformation of equations," Cardan converts an equation of the form $x^5 + ax^3 = N$ to one of the form $x^5 = bx^2 + N$. He evidently was less timid about dimensionality than about imaginary numbers, which he stigmatized as "useless." At one point he framed and solved a problem leading to a quadratic equation with imaginary roots; but he missed the significant relation between imaginary numbers and cubic equations. It was later disclosed by Bombelli that when the three roots of a cubic are real, del Ferro's rule fails—that is, unless one follows a hazardous path through the realm of imaginaries. Following this discovery, one might well ask the very pertinent question, are "imaginary" numbers really imaginary?

There are algebraic novelties in the

Ars magna which in part support Cardan's boast that the book is "so replete with new discoveries . . . that its forerunners are of little account." To the translator we owe a debt of gratitude for making so readily accessible this rich store of renaissance algebra, and it may be ungenerous to suggest that a more adequate index would have increased somewhat our debt. (Mention might incidentally be made of the confusion resulting when footnote indices are attached like exponents to numbers and unknowns: a casual glance on page 58, for example, might lead one to believe that a 16th-century mathematician was working with such things as x^{55} and 80^6 .)

In addition to the text of the *Ars magna*, the reader will welcome the translator's preface in which attention is called to Cardan's numerous contributions to algebra. There is also a foreword, dated July 1968, in which, just a month before his untimely death, Oystein Ore left a perceptive evaluation of the place of the *Ars magna* in the history of mathematics and an account without rancor of the roles of Cardan and Tartaglia in their notorious feud. The *Ars magna* closes with the words, "Written in five years, may it last as many thousands." As Latin is becoming distressingly little read, it is only through translations such as this that Cardan's wish may come true.

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For Accelerator Users

Particle Acceleration. J. ROSENBLATT. Methuen, London, 1968 (U.S. distributor, Barnes and Noble, New York). viii + 183 pp., illus. \$5.50. Methuen's Monographs on Physical Subjects.

In the preface to this volume the author states that it is intended primarily for those who are concerned with machines mainly as research instruments but must nevertheless learn the principles of their operation. The book deals with a wide variety of subjects, including cascade generators, insulating-core transformers, tandem Van de Graaff accelerators, and linear and circular accelerators of both low and high energy. It includes discussions of focusing, phase stability, beam extraction, and strong focusing as well as a very brief mention of "meson factories."

There are a few minor inaccuracies such as the statement that 43-million-electron-volt O^{16} ions can be obtained from a tandem with 6-million-volt terminal (the beam intensity would be miniscule) and the statement that alpha-particles are excluded from two- or three-stage (tandem) acceleration because helium cannot be produced as a negative ion. Because the design of accelerators has become a very specialized field and because it is relatively rare for those in this field also to be users of accelerators, many users, particularly biologists, chemists, solid state physicists, and physicians, should find this monograph useful. It would have been helpful if the bibliography had listed references according to the various types of accelerators described in the text for those who want more detailed information about a particular type.

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Three-Body Problem

Three-Particle Scattering in Quantum Mechanics. Proceedings of the Texas A & M Conference, College Station, 1968. J. GILLESPIE and J. NUTTALL, Eds. Benjamin, New York, 1968. x + 462 pp., illus. \$15.

This volume summarizes a conference held in April 1968. The speedy publication is commendable, but the volume has had to be printed directly from a typescript prepared by the editors. This of course means some sacrifice esthetically, but also rather few (less than 200) words to a page.

Abstracts of contributed papers occupy about 20 pages of the volume, and are useful in giving some flavor of the sort of problems which are being tackled in this area, though the abstracts are too brief to be of much more use than that. The main part of the book consists of eight invited papers plus a relatively short summary paper by R. Blankenbecler.

As was to be expected, the Faddeev formalism is the main theme running through the book, though the variational approach is also prominent. The papers by R. D. Amado on three-nucleon collisions and by D. Y. Wong on the (3α) and the $(e-H)$ systems fall into the first category, as does the longest paper in the book, by H. P. Noyes and H. Fiedeldey on the calculation of three-nucleon low-energy parameters.

The bibliography of this article contains some 150 references, around 100 to work published after 1964, giving some indication of the interest and activity in a field that might look, at first sight, rather narrow and specialistic. About half of the references in the book as a whole are related to this article.

In the second category, there are articles by L. Spruch and by L. M. Delves on the variational approach. Although, as Spruch points out, variational principles can often provide compact unifying formulations of physical laws, both these articles are concerned with the variational approach as a computational tool. In his article, Delves makes a strong plea for more systematic use of the variational approach, both with linear and nonlinear parameters. His work is concerned with the use of realistic local potentials, and he pays considerable attention to the rate of convergence of the method, as well as presenting some numerical results for the three-body problem. A novel feature is his plot of the variational estimates of the triton binding energy against the year of Our Lord, an asymptote appearing at—6.5 Mev! Delves also points out that the variational approach may afford another valuable attack on the Faddeev equation with local potentials.

The remaining three papers are more formal, though no less interesting to the present reviewer. R. L. Omnès and J. L. Basdevant discuss the limitations of the relativistic versions of the Faddeev equation as well as the use of the separable approximation in this area. Analyticity properties of nonrelativistic three-particle scattering amplitudes are discussed by R. L. Sugar, who also points out that the question of the analytic structure of the three-particle amplitudes in the complex angular momentum plane remains unsolved. In the discussion, which is also summarized in the book, it emerges that knowledge of the analytic properties of the amplitude provides a general framework for discussing the threshold laws for cross-sections. The paper by C. Schwartz on generalized Bethe-Salpeter equations for coupled two- and three-body amplitudes makes interesting reading, though it is again somewhat formal. The volume as a whole has a good balance between formal theory and practical calculation and forms a useful progress report in this field.

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Measuring Trace Elements

Atomic Absorption Spectroscopy. WALTER SLAVIN. Interscience (Wiley), New York, 1968. xviii + 310 pp., illus. \$12.95. Chemical Analysis, vol. 25.

During the past ten years, simple combustion flames have become one of the most useful devices available for the quantitative determination of trace elements in solution. Their usefulness stems from the very simple way in which flames of various types can release free atoms of the metallic elements when they are found in solution. All that is necessary is to make an aerosol or spray of the solution and introduce it into the flame. A fraction of the dissolved metal atoms is eventually converted into free atoms, which the analyst can then detect at the trace level by three simple but different spectroscopic techniques, namely, atomic absorption, atomic emission, or atomic fluorescence. This book is concerned with the atomic absorption method.

The author has been deeply involved in the popularization of this analytical technique in the United States, and his many experiences are reflected in the valuable pragmatic approach he has employed.

In the preface, three main purposes of the book are identified. First, "the book is intended to supply the spectroscopist and the analytical chemist with an understanding of *why* atomic absorption spectroscopy works as it does, as well as information on *how* to apply the technique." The author has done this task well, although I believe that topics related to flame chemistry and structure, to the effect of flame stoichiometry on free-atom formation processes, and to spatial distribution of free atoms in flames deserved more discussion.

Another stated purpose of the book is to "guide the developer of new analytical methods, the chemist who is being introduced into the discipline, and the prospective purchaser of equipment." The author has done this well.

The author also expresses the hope that the book will serve as a "critical guide through the large literature that has developed." Here he is less successful. There is a tendency throughout the book to sidestep critical commentary. Let me cite some important examples. In the preface, the author notes that "the use of flame emission spectroscopy . . . applied to the determination of metals has waned considerably as a result of the upsurge in atomic absorption spectroscopy." That statement is, in-