

to the specialist. It places phycology on a global stage and may help the investigator to orient himself. The contributions are for the most part from the pens of distinguished authorities in the field. However, it will surely be confusing to the nonspecialist who expects to learn much about algae, man, or the environment.

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Advances in Algebra

The Great Art. Or the Rules of Algebra. GIROLAMO CARDANO. Translated from the Latin edition (Nuremberg, 1545) and edited by T. RICHARD WITMER. M.I.T. Press, Cambridge, Mass., 1968. xxvi + 270 pp., illus. \$10.

In a sense Girolamo Cardano's *Ars magna* in 1545 was to mathematics what the *De revolutionibus* of Copernicus and the *De fabrica* of Vesalius had been two years earlier to astronomy and anatomy. Indeed, the *Ars magna* was perhaps the most revolutionary of the three. Copernicus had applied the mechanism of Ptolemy to the view of Aristarchus, and Vesalius had corrected details in Galen; but the work of Cardan disclosed the greatest step in the algebraic solution of equations since the days of Hammurabi. Quadratic equations had been solved by the pre-Hellenic Mesopotamians, but cubics had resisted the best efforts of ancient and medieval mathematicians. Today the solution is well known, yet the volume in which it was first made public has been as little read as it has been much praised. Even mathematicians who use the familiar "Cardan rule" frequently are unaware that in the *Ars magna* the author three times candidly wrote that he had obtained the key to the solution from Tartaglia and that the formula originally had been discovered in about 1515 by one Scipione del Ferro, professor of mathematics at Bologna. With an English translation available, perhaps it is not too much to hope that Cardan will be more widely read and that "his" rule will before long become known as "del Ferro's rule."

Past neglect of the *Ars magna* is easily understood. Quadratic equations today are represented by a universal notation, $ax^2 + bx + c = 0$, and solved by a single formula. From Babylonian days to the time of Cardan there were three distinct types of quadratic equa-

tion: square and thing equals number; square equals thing and number; and square and number equals thing. (A fourth type, square and thing and number equals zero, was excluded as having no solution.) For cubic equations there are 13 cases instead of three; and Cardan rhetorically and laboriously worked through each one, giving numerical illustrations and geometrically based demonstrations, all in the tradition of Mohammed ibn Musa al-Khowarizmi. In Witmer's translation the repetition of cases is unavoidable, but the tedium is mitigated through the liberal use of modern notations. A critic can argue that such modernization misrepresents the thought of the original, but such an indefatigable scholar can check the symbolic translation against the original rhetorical version in extant Latin editions. Less demanding English readers will welcome the fact that now they have a less challenging entree not only to Cardan's solution of the cubic but also to the surprisingly ample store of algebraic methods to be found in the *Ars magna*.

At one point in the book Cardan wrote that he would do little with equations beyond the cubic. "For as the first power refers to a line, the square to a surface, and the cube to a solid body, it would be very foolish for us to go beyond this point. Nature does not permit it." The author fortunately did indeed go beyond three dimensions, and in the next-to-last chapter he divulged a method, discovered by his amanuensis Ferrari, for solving equations of the fourth degree; and even quintic equations come in for some consideration. In one of the most rewarding portions of the volume, the chapter "On the transformation of equations," Cardan converts an equation of the form $x^5 + ax^3 = N$ to one of the form $x^5 = bx^2 + N$. He evidently was less timid about dimensionality than about imaginary numbers, which he stigmatized as "useless." At one point he framed and solved a problem leading to a quadratic equation with imaginary roots; but he missed the significant relation between imaginary numbers and cubic equations. It was later disclosed by Bombelli that when the three roots of a cubic are real, del Ferro's rule fails—that is, unless one follows a hazardous path through the realm of imaginaries. Following this discovery, one might well ask the very pertinent question, are "imaginary" numbers really imaginary?

There are algebraic novelties in the

Ars magna which in part support Cardan's boast that the book is "so replete with new discoveries . . . that its forerunners are of little account." To the translator we owe a debt of gratitude for making so readily accessible this rich store of renaissance algebra, and it may be ungenerous to suggest that a more adequate index would have increased somewhat our debt. (Mention might incidentally be made of the confusion resulting when footnote indices are attached like exponents to numbers and unknowns: a casual glance on page 58, for example, might lead one to believe that a 16th-century mathematician was working with such things as x^{55} and 80^6 .)

In addition to the text of the *Ars magna*, the reader will welcome the translator's preface in which attention is called to Cardan's numerous contributions to algebra. There is also a foreword, dated July 1968, in which, just a month before his untimely death, Oystein Ore left a perceptive evaluation of the place of the *Ars magna* in the history of mathematics and an account without rancor of the roles of Cardan and Tartaglia in their notorious feud. The *Ars magna* closes with the words, "Written in five years, may it last as many thousands." As Latin is becoming distressingly little read, it is only through translations such as this that Cardan's wish may come true.

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For Accelerator Users

Particle Acceleration. J. ROSENBLATT. Methuen, London, 1968 (U.S. distributor, Barnes and Noble, New York). viii + 183 pp., illus. \$5.50. Methuen's Monographs on Physical Subjects.

In the preface to this volume the author states that it is intended primarily for those who are concerned with machines mainly as research instruments but must nevertheless learn the principles of their operation. The book deals with a wide variety of subjects, including cascade generators, insulating-core transformers, tandem Van de Graaff accelerators, and linear and circular accelerators of both low and high energy. It includes discussions of focusing, phase stability, beam extraction, and strong focusing as well as a very brief mention of "meson factories."