

Fig. 1. Schematic section of the sun, showing the location of the transition zone which connects the convection zone with the more rapidly rotating core. Centrifugal pumping in the Ekman layer establishes the angular velocity gradient in the shear region.

lar momentum by these currents is probably too slow to be of importance in the sun.

We conclude that the removal of angular momentum from the surface layers will produce a state of differential rotation. The general nature of the shearing region has been established from Holton's work. The detailed structure of the shearing region must await an analysis of a continuous spin-down process for a stratified fluid in a sphere.

Having found that an Ekman spin-down process establishes a thin region of shear beneath the convection zone, we now consider the stability of this layer. We find, as did Dicke (2) for his model, that the stratification is sufficient to prevent the occurrence of a Kelvin-Helmholtz instability. Goldreich and Schubert (7) have found an instability which occurs whenever the angular momentum per unit mass decreases outward (which is the case in the shear region in the present situation). This is a centrifugally driven instability which occurs only for disturbances small enough in one dimension to allow thermal diffusion to relax the strongly stabilizing influence of stratification. The requirement of small scale in one dimension makes this mode of instability highly susceptible to interference from other motions. In this respect, the location of the shear layer immediately below the convection zone is significant, for we find that oscillating fluid motions, associated with internal gravity waves produced by penetrative convection, are sufficient to prevent the growth of disturbances.

Goldreich and Schubert estimate that a steady meridional circulation with velocity greater than 13 cm/sec will prevent the instability. This velocity is

based on the length scale and growth time of the most unstable disturbance. For oscillatory motions, we must also require that the amplitude of the motions exceed the smallest dimension of the most unstable disturbance (about 3 km).

Convective parcels of fluid which move radially inward and strike the outer surface of the stably stratified radiative core will penetrate and induce internal gravity waves in the core. Waves induced by a single impact will spread both horizontally and vertically, and a statistical superposition of waves induced by the entire spectrum of penetrative convection will lead to a random field of gravity waves extending into the radiative core. The radial extent of this field will be limited by damping of the waves, which for the sun is primarily due to radiative relaxation of the buoyancy force.

Internal gravity waves produced by a convective layer have been studied both experimentally and theoretically by Townsend (14). For the solar gravity waves, compressibility effects are small and Townsend's analysis, modified to account for thermal diffusion, may be applied. For any reasonable assumption about the distribution of velocity and length scales of convective blobs striking the edge of the stable region, the velocities and amplitudes of the induced motions are, throughout the shear region, more than an order of magnitude larger than the critical values necessary to prevent the centrifugal instability.

We conclude that the interior of the sun is rotating more rapidly than the surface layers. This state is a natural consequence of the continuing solar wind torque and the weak rotational coupling between the core and the convection zone. We agree with Dicke's suggestion that the measured solar oblateness may be due to differential rotation.

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## Soap Bubbles: Two Years Old and Sixty Centimeters in Diameter

*Abstract. Soap bubbles of long life (over 2 years) and large size (over 60 centimeters in diameter, 100 liters volume) have been produced from bubble solutions improved by the addition of water-soluble synthetic organic polymers such as polyvinyl alcohol or polyoxyethylene. The natural life can be defined as the time it takes for the bubble, if left undisturbed, to contract from the original size to a flat film.*

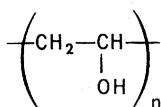
The study of soap bubbles has fascinated some of the most outstanding scientists for the last few centuries. Dewar was able to blow bubbles 40 to 50 cm in diameter, the most durable of which lasted about 90 days (1). Since then, the production by polymer chemistry of molecules of reproducible molecular weight (2) and highly effective fluorocarbon surface-active agents (3)

have made possible the production of very large, long-lived soap bubbles.

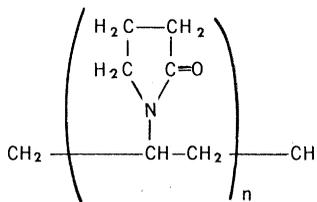
Every soap bubble contracts because of the surface film pressure and becomes smaller and smaller until it is finally converted into a flat film in the blow tube which produced it. Dewar (1) has measured contraction curves of soap bubbles in hydrogen. Lawrence found that, in air, the rate of effusion of

air was so much slower that a complete contraction curve could not be obtained (4) because the life of soap bubbles could not be made long enough. However, now we define the *natural life of soap bubbles as the time it takes for the bubble, if left undisturbed, to contract from its original size to a flat film.* This definition applies, regardless of the gas in the bubble so long as it does not react chemically with the bubble. The natural lives of bubbles vary greatly; an ammonia bubble in  $\text{NH}_3$  contracts very rapidly (a few seconds), whereas a bubble of the same size of sulfur hexafluoride in  $\text{SF}_6$  contracts about a million times slower (many months).

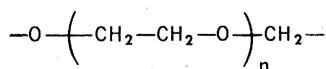
I have blown bubbles from common wetting agents, both cationic and anionic, used in investigations of film properties (Table 1), and from bubble solutions to which have been added water-soluble synthetic organic polymers (5). The polymers used were polyvinyl alcohol, Plasdane C, and Polyox. Polyvinyl alcohol (6) was used as a 5.0-percent solution (by weight) and has a viscosity at  $20^\circ\text{C}$  of 4 to 6 cp. It has an average molecular weight of 14,000 and the following chemical structure, where  $n$  is about 300



Plasdane C (7) was used as a 5.0-percent (by weight) solution. Plasdane has little effect on the viscosity of water if used in concentrations below 10 percent. My solution has a viscosity of 2.0 centistokes. The average molecular weight is 40,000, and the structural formula, where  $n$  is 360, is



Polyoxyethylene (Polyox) (5), used as a friction-reducing agent, has a molecular weight of about 7 million and the following structural formula, where  $n$  is 160,000



Polyox was used as a 0.25-percent (by weight) solution in distilled water. The viscosity of 2.0-percent Polyox solution in water is high in view of the very high molecular weight. This molecular weight

Table 1. Life of soap bubbles formed with common wetting agents (5, 16).

Solution	Percentage (by wt.) in $\text{H}_2\text{O}$	Average time before bursting (min)
Sodium lauryl sulfate	4.0	5
<i>n</i> -Cetyldimethylbenzylammonium chloride	4.0	10
3-Cetyltrimethylammonium bromide	2.0	50
FC-128, anionic wetting agent	2.0	$\approx 240$
FC-134, cationic wetting agent	2.0	$\approx 120$

does not stay constant but gradually decreases, as a result of oxidation, at a rate depending on the presence of various catalytic agents. Thus, unless their activity is inhibited and the inhibitors are specified, the reproduction and description of the behavior of Polyox solutions are not complete.

The surface-active solution we used is the Kuehner solution (8) which consists of 1 volume of 4.49-percent (by weight) sodium dibromostearate in water plus 1 volume of glycerol.

Solutions A to F, from which the large bubbles were blown, had the following compositions:

Solution A contained 2 volumes of Kuehner solution (8), 1 volume of 5.0-percent polyvinyl alcohol, and 3 volumes of glycerol. This solution is identical with my "double-bubble" solution (9, 10), so-called because each constituent forms bubbles. Bubbles made from polyvinyl alcohol solution are quite different from those blown from Kuehner solution. They are much more difficult to blow because the film is much thicker and at the same time much tougher. After a few days, both the solution and the film "gel." The gelled bubble has a highly resilient structure that so far has not shrunk with time. It is probably this property that gives the specially beneficial properties to the *double-bubble solution*.

Solution B contained 1 volume of 5.0-percent Plasdane C in water, 1 volume of Kuehner solution, and 2 volumes of glycerol.

Solution C consisted of 2 volumes of a 3.30-percent (by weight) solution of the cesium salt of perfluoro acid from FC-128 (5). The FC-128, which is the potassium salt, was converted into the free acid by addition of sulfuric acid, extraction with ether, evaporation, and then neutralization with cesium hydroxide. (At that time it was of interest to

see what effect the large cesium cation would have on the properties of the film.) This cesium salt was mixed with 1 volume of 5.0-percent polyvinyl alcohol, 1 volume of 0.25-percent Polyox, and 4 volumes of glycerol.

Solution D was made of 1 volume of solution A and 1 volume of solution C, in which 2 volumes of the 2.0-percent solution of the potassium salt (FC-128) were used instead of the cesium salt.

Solution E contained 2 volumes of 2.0-percent FC-128, 1 volume of 0.25-percent Polyox, 1 volume of 4.0-percent polyvinyl alcohol, and 4 volumes of glycerol.

Solution F consisted of equal volumes of solutions A and E.

Most of these solutions gel on standing for a few days at room temperature, and the *gel forms even better bubbles than the fresh solution*.

The apparatus was very similar to that shown in fig. 1 of (9), except that the blow tube was placed inside a glass sleeve, inserted into a rubber stopper, and sealed in the usual way with a rubber tube lubricated with glycerol; thus the blow tube could be dipped into the solution and pulled up, and a bubble could be blown without admitting any air with dust from the outside. Soap solution (10 to 20 ml) was placed at the bottom of the flask. After the bubble was blown, both the inlet tube and the exit tube were closed with rubber stoppers (in special cases regular vacuum stopcocks were used). An advantage of this procedure is that the gas in the flask is being continually purified from dust—the archenemy of soap bubbles—because the air is used over and over again. Thus, the life of bubbles, blown consecutively in the same flask, increases. If the bubbles survive the first few hours after "birth," most of them (well over 80 percent) last over 100 days, provided their diameter is big enough. This procedure is particularly important for the big bubbles. In addition, the extra solution available in the flask prevents any change in the composition of the soap bubble due to evaporation or condensation of moisture. Thus, unless otherwise specified, the bubbles were always blown at *constant composition*.

The air was purified by passage over laboratory cleaning brushes wetted with pure glycerol to free the air from dust particles, one of the main reasons for bubbles' bursting. The air was freed of carbon dioxide by being bubbled

Table 2. Results on new long-life soap bubbles. In addition to these bubbles, *flat films* were blown in spherical flasks and left vertically. The results were: (i) Solution A: the film lasted 385 days; it was flat for about half a year and contracted to zero because it started curving. (ii) Solution E: after 610 days the film was still flat; it started curving about the 625th day and now is over 808 days old (28 March 1969). [Lifetimes reported here have been updated since the acceptance date of the manuscript.]

Bubble No.	Solution	Natural life (days)	Remarks
1	A	370	Formed a flat film
2	A	555	Formed a flat film
3	A	346	Formed a flat film
4	A	730	On 728th day, drop (like No. 8) fell and became a flat film on 730th day
5	A	679	On 28 March 1969 still a big bubble
6	B	578	Formed a flat film
7	C	374 +	Burst due to hammering during carpentry work
8	D	705	Gelled with heavy drop at bottom

through sodium hydroxide solution and dried. Dewar's (I) procedure, whereby the Tyndall cone is used to check the optical properties of the air in the flask, was also used. Fresh air, not purified of dust, showed a very pronounced Tyndall cone; in contrast, air passed over the brushes wetted with glycerol showed no Tyndall cone (I).

Bubbles are preserved best if blown in standard spherical 1- to 20-liter flasks. Evidently the geometry of a sphere produces symmetrical air currents around the bubble and thus does not subject it to any strain. A good substitute, for many purposes, is the standard glass 5- to 10-gallon (19- to 38-liter) distilled-water jug (diameter, 28 cm; height, about 36 cm).

The flasks were clasped in ordinary laboratory stands placed on foam rubber to prevent any harm from vibrations. In most cases the bubbles were blown with compressed air from a standard cylinder regulated by a needle valve. The bubbles were kept in a basement where the temperature was practically constant over the whole year ( $20^{\circ} \pm 1^{\circ}\text{C}$ ). Using Dewar's (I) precautions and Plateau's (II) solution of 3 volumes of pure 2.50-percent (by weight) sodium oleate in water (plus one to three drops of concentrated  $\text{NH}_3$ ) mixed with 1 volume of glycerol, we obtained soap bubbles which lasted 40 to 90 days.

Bubbles, made from solutions A to F (Table 2), were beautifully colored, although rather thick at the beginning. The thickness of a bubble film was determined by letting the bubble contract slowly through an efflux tube (10), collecting the drops at the bottom, and weighing them. Thus, a fresh golden-orange bubble film, made from solution A, had a thickness of  $1.68 \mu$  (12).

After bubbles made from these solutions had lasted over 200 days, I became interested in blowing larger bubbles. Big bubbles were blown in a Plexiglas box (55.9-cm edge cube) (13), but they did not last more than a week. Then, Corning Glass Works provided first one and later three 72-liter Pyrex flasks (14), the largest spherical flasks produced in this country. They were perfect for my purposes, and many bubbles were blown to a diameter of 40 to 50 cm out of solutions A to F.

In addition to the surface-active compounds FC-128 and 172 (5), I also used, as substitutes, the pure perfluoro acids—perfluorodecanoic acid ( $\text{C}_9\text{F}_{19}\text{CO}_2\text{H}$ ), perfluorooctane sulfonic acid ( $\text{C}_8\text{F}_{17}\text{SO}_3\text{H}$ ), and perfluorocaprylic or -octanoic acid ( $\text{C}_8\text{F}_{15}\text{COOH}$ ); they were converted into sodium, potassium, and cesium salts and proved excellent wetting agents.

A 200-liter sphere was then made available (15), and bubbles of the following horizontal diameters, in centimeters, were consecutively blown from solution A [(-- indicates that the bubble burst while being blown): 45.0, 50.8(--), 51.1(--), 54.4, 55.7(--), 54.4, 58.0(--), (--), 55.9, 52.3(--)(visitor!), 50.1, and 60.6

The last one, the largest bubble blown to date, took approximately 6 hours to blow. This is about the largest bubble that could be blown in the flask, because it contained about 200 ml of solution A at its bottom. As the bubble drains, the drop hanging at the bottom extends its vertical diameter, and adequate space, about 2 to 3 cm, has to be provided above the level of the liquid to prevent contact.

Bubbles in several giant Pyrex flasks were set aside for observation over a long period. One bubble with an original

diameter of 50 cm was left standing over 675 days (bubble No. 5 of Table 2). The very slow formation of a transparent, colorless upper segment (usually referred to as black fall) and a horizontal silver edge with a beautiful rainbow-colored curtain underneath was noticed shortly after the bubble was blown. The very slow descent of this "horizon" was observed for 500 days; on or about this day the silvery edge reached the "south pole" and disappeared from the bubble. The vertical diameter of this bubble after 528 days was 37.73 cm. In view of its large size, it is likely that it will be the longest-living soap bubble on record.

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- The sources of our chemicals for the new bubbles were: oleic acid, USP, Merck & Co., Inc.; glycerol, Matheson, Coleman & Bell (spectroquality reagent); polyvinyl alcohol, E. I. du Pont de Nemours & Co., Inc. and J. T. Baker Chemical Co.; polyoxyethylene (Polyox), Chemicals Division of Union Carbide Corporation; polyvinylpyrrolidone, General Aniline & Film Company; FC-128, 134, and 172 wetting agents, Minnesota Mining & Manufacturing Company; and  $\text{Br}_2$ , Baker's Analyzed Reagent Grade. Infrared spectra of all starting materials were taken and are kept on record.
- Polyvinyl alcohol, which is isomeric with polyoxyethylene, was obtained from the J. T. Baker Chemical Co. Identical with it is Elvanol (Grade 70-05) of E. I. du Pont de Nemours & Co., Inc.
- Plasdone C was obtained from General Aniline and Film Company.
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- The Plexiglas cube box was made by the Dynalab Corporation, Rochester, New York.
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- W. H. Tombs informed me of the flask, being manufactured by Corning in England under the trade name "Q.V.F." (quick-fit visible flow). The flask was donated by Corning Glass Works.
- Sources of wetting agents: sodium lauryl sulfate, Fisher Scientific Company; *n*-cetyltrimethylammonium chloride, J. T. Baker Chemical Company; 3-cetyltrimethylammonium bromide, Eastman Organic Chemicals.

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