Reports

Solar Differential Rotation and Oblateness

Abstract. *An investigation of the time development of differential rotation produced by the solar wind torque indicates that the sun has a rapidly rotating core.*

Measurements (*1*) of the solar oblateness support earlier suggestions by Dicke (*2*) and Roxburgh (*3*) that the radiative interior of the sun is rotating more rapidly than the outer convection zone. The possibility of such a state is suggested by the removal of angular momentum from the surface layers by the solar wind. The question of whether such a state can exist over the solar lifetime has proved difficult to resolve (*2–8*). The difficulties have been due in part to the unknown effect of extreme solar conditions (for example, stratification and radiative transfer) on known "spin-down" mechanisms and in part to uncertainties associated with the location and structure of the shearing layer separating the regions of different angular velocity. Some of the uncertainties can be eliminated by a consideration of the time development of the differential rotation.

The starting point for our considerations is the arrival of the sun on the main sequence in a state of uniform rotation. (The theory given below could be extended to allow for initial differential rotation. However, the fact that the sun is fully convective during part of the approach to the main sequence suggests that the simpler case of initial uniform rotation may be most relevant.)

As the angular momentum is removed from the surface layers, the angular velocity of the convection zone will remain uniform in space because of the efficient turbulent exchange. The central question in the analysis is the nature of the coupling between the convection zone and the radiative core. This coupling occurs in a kind of transition zone separating the convection zone and the core (Fig. 1). Certain general features of this complicated transition zone are important. As we cross the transition zone from the convection zone toward the core, the turbulent mixing decreases and the stratification changes from slightly unstable through neutral to slightly stable. Because of the decrease in turbulent mixing, a shear (in angular velocity) will begin to develop in this region as the convection zone slows down. Howard *et al.* (*5*) pointed out that centrifugal pumping associated with an Ekman layer may be important in such a situation. The nature of the Ekman layer may be established by consideration of the imbalance by forces caused by the slowing down of the convection zone.

In the initial state of uniform rotation, there is a static balance among gravitational, pressure, and centrifugal forces. As the convection zone slows down, the balance of forces is altered. In the convection zone, the angular velocity remains uniform in space as it decreases, and the static balance can be maintained by slight deformations of the convection zone. In the transition zone, a shear in angular velocity starts to develop. The associated variable centrifugal force can be statically balanced only in a stably stratified region where slight displacements can effectively alter the mass distribution. Such a balance is unavailable in the transition zone, however, because of the essentially neutral stratification. Thus the fluid in the transition zone will undergo centrifugal pumping as a result of the shear. [Bretherton and Spiegel (*8*) have found rapid spin-down by treating the convection zone as a porous solid. Since the porous solid cannot deform, as a fluid would, in response to a changing angular velocity in the convection zone, they find that centrifugal pumping occurs through the entire convection zone. We believe that the centrifugal pumping occurs only in regions of variable angular velocity, as described above.]

This picture differs from the usual Ekman process (*5, 9*). In the present case there is no rigid wall to exert a shear stress on the fluid, and the applied shear stress is distributed throughout a finite layer in the transition zone by the turbulent, penetrative convection. This layer is the locus of the centrifugal pumping and so functions as an Ekman layer. Thus the Ekman layer is that part of the transition zone where there is appreciable turbulent mixing and where the stratification is neutrally stable. The fluid interior to the Ekman layer is stably stratified, and we must consider how it is affected by the centrifugal pumping. This can be done on the basis of Holton's analysis of the Ekman process in a stratified fluid (*10*). The relevant parameters are the angular velocity of the radiative interior $\Omega = 3 \times 10^{-5}$ sec^{-1} [a value suggested by Dicke (*6*)], a typical horizontal dimension of the Ekman layer $L = 5 \times 10^{10}$ cm, the acceleration of gravity $g = 5.6 \times 10^4$ cm^2 sec^{-1} (at $0.7R_\odot$), and the scale height of the potential density in the stable region $H = 2 \times 10^{10}$ cm [estimated from Weymann's solar model (*11*)]. Holton's analysis indicates that, as the convection zone is slowed down by the solar wind, a shearing region of thickness

$$\delta^* \sim 2\Omega L(H/g)^{1/2} = 1.8 \times 10^9 \text{ cm}$$

develops just below the Ekman layer. This shearing region provides a smooth connection between the more slowly rotating convection zone and the interior fluid, which retains its original high angular velocity. [The characteristic circulation velocities associated with the Ekman pumping are of the order of L/t_L, where t_L is the time scale for the removal of angular momentum. For $t_L \approx 10^8$ years this velocity is less than 1.6×10^{-5} cm sec^{-1}, giving a Richardson number many orders of magnitude greater than ¼. Thus the circulation will be stabilized by the stratification and will not become turbulent and enhance the spin-down process as suggested previously (*5*).]

Holton's analysis does not include the long-time effects of thermal diffusion. Howard *et al.* (*5*) have pointed out [on the basis of Pedlosky's work (*12*)] that the interior circulations associated with thermal diffusion are very similar to Eddington-Sweet currents In both cases, the state demanded by mechanical equilibrium does not quite satisfy the conditions of thermal equilibrium, and slow circulations are the result. On the basis of Schwarzchild's discussion (*13*), we estimate that for $\Omega = 3 \times 10^{-5}$ sec^{-1}, the time scale for the circulations is of the order of 10^{10} years. Thus the transport of angu-

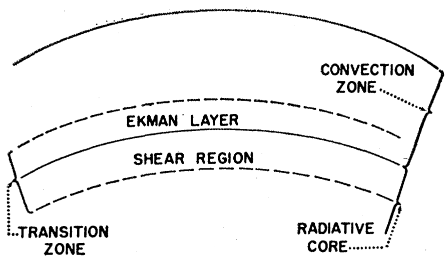

Fig. 1. Schematic section of the sun, showing the location of the transition zone which connects the convection zone with the more rapidly rotating core. Centrifugal pumping in the Ekman layer establishes the angular velocity gradient in the shear region.

lar momentum by these currents is probably too slow to be of importance in the sun.

We conclude that the removal of angular momentum from the surface layers will produce a state of differential rotation. The general nature of the shearing region has been established from Holton's work. The detailed structure of the shearing region must await an analysis of a continuous spin-down process for a stratified fluid in a sphere.

Having found that an Ekman spin-down process establishes a thin region of shear beneath the convection zone, we now consider the stability of this layer. We find, as did Dicke (*2*) for his model, that the stratification is sufficient to prevent the occurrence of a Kelvin-Helmholtz instability. Goldreich and Schubert (*7*) have found an instability which occurs whenever the angular momentum per unit mass decreases outward (which is the case in the shear region in the present situation). This is a centrifugally driven instability which occurs only for disturbances small enough in one dimension to allow thermal diffusion to relax the strongly stabilizing influence of stratification. The requirement of small scale in one dimension makes this mode of instability highly susceptible to interference from other motions. In this respect, the location of the shear layer immediately below the convection zone is significant, for we find that oscillating fluid motions, associated with internal gravity waves produced by penetrative convection, are sufficient to prevent the growth of disturbances.

Goldreich and Schubert estimate that a steady meridional circulation with velocity greater than 13 cm/sec will prevent the instability. This velocity is based on the length scale and growth time of the most unstable disturbance. For oscillatory motions, we must also require that the amplitude of the motions exceed the smallest dimension of the most unstable disturbance (about 3 km).

Convective parcels of fluid which move radially inward and strike the outer surface of the stably stratified radiative core will penetrate and induce internal gravity waves in the core. Waves induced by a single impact will spread both horizontally and vertically, and a statistical superposition of waves induced by the entire spectrum of penetrative convection will lead to a random field of gravity waves extending into the radiative core. The radial extent of this field will be limited by damping of the waves, which for the sun is primarily due to radiative relaxation of the buoyancy force.

Internal gravity waves produced by a convective layer have been studied both experimentally and theoretically by Townsend (*14*). For the solar gravity waves, compressibility effects are small and Townsend's analysis, modified to account for thermal diffusion, may be applied. For any reasonable assumption about the distribution of velocity and length scales of convective blobs striking the edge of the stable region, the velocities and amplitudes of the induced motions are, throughout the shear region, more than an order of magnitude larger than the critical values necessary to prevent the centrifugal instability.

We conclude that the interior of the sun is rotating more rapidly than the surface layers. This state is a natural consequence of the continuing solar wind torque and the weak rotational coupling between the core and the convection zone. We agree with Dicke's suggestion that the measured solar oblateness may be due to differential rotation.

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Soap Bubbles: Two Years Old and Sixty Centimeters in Diameter

Abstract. *Soap bubbles of long life (over 2 years) and large size (over 60 centimeters in diameter, 100 liters volume) have been produced from bubble solutions improved by the addition of water-soluble synthetic organic polymers such as polyvinyl alcohol or polyoxyethylene. The natural life can be defined as the time it takes for the bubble, if left undisturbed, to contract from the original size to a flat film.*

The study of soap bubbles has fascinated some of the most outstanding scientists for the last few centuries. Dewar was able to blow bubbles 40 to 50 cm in diameter, the most durable of which lasted about 90 days (*1*). Since then, the production by polymer chemistry of molecules of reproducible molecular weight (*2*) and highly effective fluorocarbon surface-active agents (*3*) have made possible the production of very large, long-lived soap bubbles.

Every soap bubble contracts because of the surface film pressure and becomes smaller and smaller until it is finally converted into a flat film in the blow tube which produced it. Dewar (*1*) has measured contraction curves of soap bubbles in hydrogen. Lawrence found that, in air, the rate of effusion of