conclusions, even for August 1965 where a doubled rate would still be negligible relative to other months of the year.
3. M. B. Davis, *Bull. Geol. Soc. Amer.* **78**, 849 (1967). Experiments with traps of different sizes showed that the number of grains captured was directly proportional to the cross-sectional area of the trap opening. This result justifies expressing the results as deposition per unit area. At Frains Lake the sum of pollen deposition at one trapping station during two successive sampling intervals (a total of 3 weeks) was the same as deposition into a single trap left in place during the entire 3-week period at a replicate station. This result argues against gain or loss of pollen from traps during collection, and supports the contention that traps measure deposition continuously during sampling.
4. W. Tutin, *Mem. Ist. Ital. Idrobiol.* **8**, 467 (1955); E. Gorham, *Limnol. Oceanogr.* **3**, 291 (1958); W. P. Mueller, *Invest. Indiana Lakes Streams* **6**, 1 (1964).
5. I thank Dr. W. R. Solomon for information on the kinds of pollen present in the air during this period at the University Hospital in Ann Arbor, 12 km from Frains Lake.
6. In the subtraction method pollen is assumed to be available to enter the lake only within the flowering season. Massive input of pollen deposited onto vegetation surfaces earlier in the year has been observed in a small pond in Denmark [H. Tauber, *Rev. Paleobot. Palynol.* **3**, 277 (1967)], but this phenomenon does not appear important at Frains Lake, which is surrounded by meadows and therefore exposed to the wind. Twigs collected from willow shrubs growing along the shores were devoid of pollen in March 1968 and therefore could not have served as a source for ragweed pollen during the spring season of heavy pollen deposition. Another source for out-of-season pollen is surface run-off from the surrounding meadows during periods of heavy rainfall. I assume this source to be relatively unimportant since the surrounding slopes are gentle, grass-covered, and without a drainage network of streams.
7. Surface samples of shallow-water sediment in Frains Lake contain higher percentages of ragweed (31 percent), willow (12 percent), and aquatic plant pollen (6 percent of terrestrial plant pollen) than deep-water sediment (20, 4, and 2 percent, respectively).
8. Written results of experiments in which traps have been placed at various levels between the water surface and the sediment surface at Frains Lake are in preparation. The mixing of pollen and sediment throughout the water column that they indicate shows that turbidity currents moving close to the lake bottom are not primarily responsible for the redeposition described here.
9. M. B. Davis and E. S. Deevey, *Science* **145**, 1293 (1964); M. B. Davis, *Rev. Paleobot. Palynol.* **2**, 219 (1967); M. Tsukada, *Botan. Mag. Tokyo* **79**, 179 (1966); D. R. Whitehead, in *Quaternary Paleoecology*, E. J. Cushing and H. E. Wright, Jr., Eds. (Yale Univ. Press, New Haven, 1967), p. 237.
10. R. B. Davis, in *Quaternary Paleoecology*, E. J. Cushing and H. E. Wright, Jr., Eds. (Yale Univ. Press, New Haven, 1967), p. 143, has emphasized burrowing by benthic animals as a mechanism for mixing sediment. There seems little question that benthos also mix the sediment at Frains Lake. But at Frains Lake, and presumably many other lakes, physical mixing appears to be more important, especially as physical processes can cause lateral as well as vertical movement of sediment.
11. Contribution No. 94 from the Great Lakes Research Division, University of Michigan. Supported by NSF grants GB 2377 and GB 5320. I thank J. M. Beiswenger and L. B. Brubaker for technical assistance and W. C. Kerfoot for reading the manuscript.

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Wedge Dislocation as the Elastic Counterpart of a Crystal Deformation Twin

Abstract. *A crystal deformation twin may be visualized to form and grow by the movement of partial dislocations only if the twinning dislocations are especially distributed to give an invariant shear. One consequence of this requirement is that, if a critical resolved twinning stress exists in the same sense for twinning as for slip, this stress depends on the reciprocal thickness of the twin. This type of model for twinning may be developed through the use of relatively unknown disclinations, in particular, the wedge dislocation.*

There are six general types of elastic dislocations which may be introduced into any doubly connected body, such as a hollow cylinder (*1*). The relative displacements Δu_i of the cut surfaces when

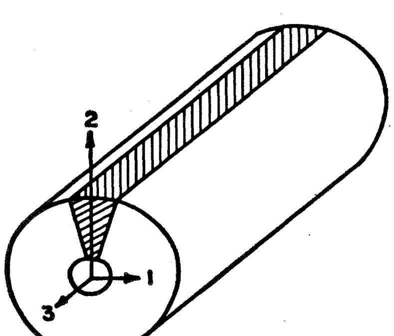

Fig. 1. Screw disclination or wedge dislocation.

these dislocations are produced may be expressed as

$$\Delta u_i = b_i + d_{ij} x_j \qquad (1)$$

where $i,j = 1,2,3$; b_i is a polar vector which specifies the relative translation of the surfaces; d_{ij} is an axial vector which specifies their relative rotation; and x_j is the position vector. Only two of the three dislocations characterized by b_i, their Burgers vector, are unique, and these, which are termed edge and screw dislocations, are widely used in theories of crystal plasticity. Only two of the three dislocations specified by d_{ij}, which are now called disclinations corresponding to a rotation parallel or normal to the dislocation line, are also unique. The screw disclination, or wedge dislocation, is shown in Fig. 1. These dislocations have not been used very

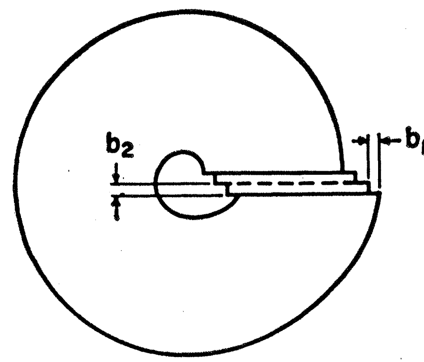

Fig. 2. Model of a crystal deformation twin utilizing partial dislocations.

much in crystal deformation theory. In their case, the residue of the displacement integrated around a circuit containing the dislocation axis is proportional to the radius of the circuit. This report is a description of the procedure by which dislocations of type b_i may be used to model a crystal deformation twin. Because of the invariant rotation which characterizes disclinations, they may be used also to develop a model for twinning.

A number of the features of a crystal deformation twin (*2*) have been modeled as in Fig. 2. The twinned volume is enclosed within the solid horizontal lines of the cylindrical section shown on the right side of the figure. The twin is composed of successively aligned dislocations, each having a partial lattice-displacement vector b_1 separated in the vertical direction by the interplanar spacing b_2. The twinning shear, γ_{12}, is given by the ratio (b_1/b_2). The stress-concentrating properties of this model for a twin have been described in terms of a multiple Burgers vector dislocation (*3*). The work W_e done by an external shear stress σ_{12}, when the twin is produced on the set of surfaces Σ_S by displacements u_{1S}, is given by

$$W_e \approx \int\int_{\Sigma_S} u_{1S}\, \sigma_{12}\, d\Sigma_{2S} \qquad (2)$$

in which $d\Sigma_{2S}$ is an elemental vector component of the set of cut surfaces,

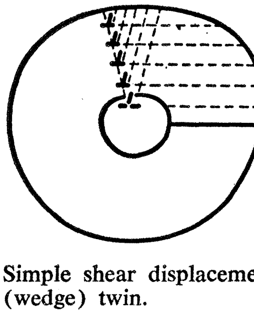

Fig. 3. Simple shear displacements which form a (wedge) twin.

and the incoherent surface energy of the interfaces of the twin is neglected. The incremental work done by the external stress on altering the twin dimension in the shear direction χ_1 may be expressed in terms of the number of partial dislocations n composing the twin as

$$\delta W_e \approx b_1 \sigma_{12} n (d\chi_1 \times dl_3) \quad (3)$$

From Eq. 2, the force dF_1 acting to produce this displacement is given by

$$dF_1 \approx n b_1 \sigma_{12} \times dl_3 \quad (4)$$

Since

$$n b_1 = n \gamma_{12} b_2 = \gamma_{12} t$$

where t is the twin thickness, the force per unit length acting on the twin is

$$(dF_1/dl_3) \approx \gamma_{12} t \, \sigma_{12} \quad (5)$$

If it is presumed that a critical resolved shear stress for deformation twinning τ_{CRTS} exists by virtue of a constant force per unit length acting on the twin, as has been argued for the slip process (*4*), then τ_{CRTS} is inversely proportional to the twin thickness.

Thus the invariant twinning shear is the essential feature of a deformation twin. This feature makes the second type of dislocation (Eq. 1 and Fig. 1) useful for developing a more complete model for deformation twinning. The simple shear displacements associated with the wedge dislocation (Fig. 1) may be derived from the equations of equilibrium for this body, because a simple shear is composed of a rotation plus a pure shear (*5*). If the terms resulting from the conditions at the surface boundary are neglected

$$u_1 = (d_{12}/2\pi) \{ -\chi_2 \tan^{-1} (\chi_1/\chi_2) - [(1 - 2\nu)/4(1 - \nu)] \chi_1 \ln (\chi_1^2 + \chi_2^2) \} \quad (6)$$

where ν is Poissons ratio. The terms in Eq. 6 are a multiple combination of several of those in the complete solutions of the displacements for a single edge dislocation (*6*). Figure 3 shows the simple shear displacements which produce the twin. Further properties of a deformation twin may be directly determined through a complete analysis of these wedge dislocations.

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References and Notes

1. F. R. N. Nabarro, *Theory of Crystal Dislocations* (Clarendon Press, Oxford, 1967).
2. R. W. Armstrong, *Iron and Its Dilute Solid Solutions* (American Institute of Mining, Metallurgical, and Petroleum Engineers, and Interscience, New York, 1963), p. 263.
3. ———, *Deformation Twinning* (Gordon and Breach, New York, 1964), p. 356; R. Lagneborg, *Trans. Met. Soc. AIME* **239**, 1661 (1967).
4. F. R. N. Nabarro, *Phil. Mag.* **42**, 213 (1951).
5. A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity* (University Press, Cambridge 1927), p. 227; J. F. Nye, *Physical Properties of Crystals* (Clarendon Press, Oxford, 1957), p. 104.
6. J. D. Eshelby, *Brit. J. Appl. Phys.* **17**, 1131 (1966).

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18 September 1968

Mass of Pluto

Abstract. *Analysis of the observations of Neptune indicates a reciprocal mass of Pluto of 1,812,000 (0.18 Earth masses). If the density is the same as that of Earth, the diameter would be 7200 kilometers. If 6400 kilometers is accepted (from other sources) as the upper limit of the diameter, then Pluto must be at least 1.4 times as dense as Earth.*

One of the outstanding discordances among the solar system constants is the inconsistency between the physically measured diameter of Pluto and the dynamical determination of the mass of Pluto from its perturbation of the motions of Neptune and Uranus. Measurement of the disk of Pluto by Kuiper (*1*), using the 200-inch telescope, revealed an apparent semidiameter of 0.23 arc sec with an internal consistency of ± 0.01 arc sec. Use of the adopted value of the astronomical unit in kilometers, leads to a value of the diameter of Pluto of 5928 km. More recently, from a near occultation of a 15th-magnitude star by Pluto, Halliday *et al.* (*2*) determined an upper limit to the diameter of Pluto of 6400 km. If we use the direct measurement, or the upper limit, for the diameter of Pluto and then assume that the density of Pluto does not exceed the density of Earth, the corresponding values for the mass of Pluto would be 0.10 or 0.13 Earth masses, respectively. If, on the other hand, the dynamical determination of the mass of Pluto by Wylie (*3*) of 0.91 Earth masses (Sun/Pluto = 360,000), based on an analysis of the motion of Neptune, or the determination by Brouwer (*4*) of 0.82 Earth masses, based on the motion of both Uranus and Neptune, is utilized in combination with the above measurements of the diameter, the mean density of Pluto would have to be at least 40 g/cm^3.

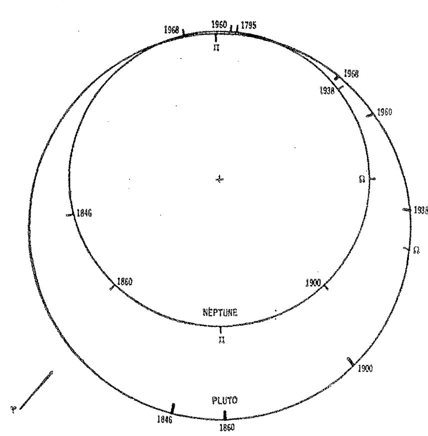

Fig. 1. Two-dimensional view of the orbits of Neptune and Pluto. Ω, node; Π, perihelion; γ, vernal equinox.

The discovery of Neptune in 1846 was one of the triumphs of celestial mechanics. Both Leverrier and Adams, on the basis of the departure of observations of Uranus from gravitational theory, were able to predict the presence and location of Neptune. Although the presence of a trans-Neptunian planet was long suspected, Wylie's analysis (*3*) has shown that its location could not be predicted gravitationally. The discovery of Pluto in 1930 must be considered as being due more to an intensive astrometric search than to any prior knowledge of position from gravitational theory.

The orbits of Neptune and Pluto form an interesting system. As shown in Fig. 1, it appears that the orbit of Pluto actually crosses the orbit of Neptune near perihelion, but, while Neptune's orbit lies principally in the plane of the ecliptic, the orbit of Pluto is inclined to this plane by 17°. An analysis of the motions of these two planets over an extended period of time (*5*) has shown that the closest approaches of the two planets librate about the aphelion of Pluto in an arc of some 76°, with a libration half-period of 10,000 years. The positions in orbit occupied by both Neptune and Pluto since discovery of Neptune are shown in Fig. 1; the point of closest approach of the two bodies occurred in 1896—at a distance of 18.9 A.U. Shown also are the nodes of the orbits of Neptune and Pluto on the ecliptic, as well as the position of Neptune in 1795 when it was observed but not recognized as a planet. These observations were later recovered and reduced by Lalande. The observations from the discovery of Neptune in 1846 to the present encompass more than 70 percent of the orbit of Neptune. Although the observations