Reports

Galaxies as Gravitational Lenses

Abstract. *Of all the galaxies in the visible part of the universe, 500 million are seen through intervening galaxies. In some instances the foreground galaxy will act as a gravitational lens and produce distorted and (in brightness) greatly amplified images of the galaxy behind it; such images may simulate starlike superluminous objects such as quasars (quasi-stellar objects). The number of gravitational lenses is several times greater than the number of quasars yet observed. In other instances the superposition of the image upon a visible foreground galaxy may simulate morphological configurations resembling N-type, dumbbell, spiral, or barred-spiral galaxies.*

When in 1963 it became obvious that the bluish starlike objects identified with discrete radio sources are 10 to 30 times brighter than the brightest giant elliptical galaxies hitherto known (*1*), an almost insurmountable problem developed—how to solve their energy supply. This difficulty was further complicated by the short time variations in optical and radio brightness observed in many of these quasi-stellar sources, which indicated that at least the variable part of the quasars (as they are usually called now) have dimensions of a fraction of one parsec (1 pc $= 3.08 \times 10^{18}$ cm).

All these difficulties could be avoided if the large redshift of the spectral lines of quasars were not of cosmological origin. However, two other phenomena which produce a redshift and have been proposed [Doppler effect (*2*) and the gravitational redshift (*3*)] led to quasar models which are not without serious difficulties and which contradict observations.

If the redshift of quasars is of cosmological origin, as we shall assume in the following, then the principal characteristic of quasars is that they are brighter than -23 absolute magnitude. (This does not mean that the low luminosity end of their luminosity function could not overlap with the high luminosity end of the luminosity function of other known astronomical objects.) Their secondary characteristics are their small apparent diameter, their bluish color, probably their infrared excess, and the fact that most of them have spectra containing strong and wide emission lines of high excitation. These secondary characteristics are shared with nuclei of the Seyfert galaxies. Although quasars, originally called quasi-stellar radio sources (QSS), were discovered through identification of strong radio sources of the 3C catalog with optical objects, it is now evident that there are more quasars below 9 flux units (1 f.u. $= 10^{-26}$ watt m^{-2} hz^{-1}) than above this flux density. In recognition of this fact, their designation has changed gradually to quasi-stellar objects (QSO). Radio emission is thus no longer one of the main characteristics of quasars.

In 1965 we attempted to explain the tremendous output of energy, secular variations, and short-term fluctuations in the brightness of quasars by assuming that they are merely optical effects—most of them images of nuclei of Seyfert galaxies, greatly amplified in brightness by the gravitational lens effect of an intervening galaxy. We (*4–8*) have since shown that all the main observed characteristics of quasars, including their increase in spatial density with increasing redshift (*5, 7, 9*), could be inherent properties of images from gravitational lenses. Although this explanation would destroy the notion that quasars constitute entirely novel components of the universe, they would remain our most powerful tools for exploration of hitherto inaccessible depths of the universe.

The most crucial question in this solution is the number of lenses one can expect. Zwicky (*10*) showed as early as 1937 that proper alignment of two galaxies, with one acting as a gravitational lens, should not be too rare. From Wagoner's (*11*) calculations we know that 5×10^8 of the galaxies in the visible universe are seen through and within the 25-magnitude/(arc sec)^2 isophote area of a foreground galaxy. Not all these intervening galaxies are gravitational lenses.

To compute the number of galaxies that produce images which simulate intrinsic luminosities of -23 to -27 in absolute magnitude, we must first write the well known equation of the brightness amplification of gravitational lenses (A) (*12*) in the metric of relativistic cosmological models:

$$A = \Delta g_o g_d (CR_o)^{\frac{1}{2}} \rho^{-1} \quad A > 2.5 \qquad (1)$$

Similarly, we write the apparent diameter of the image in seconds of arc:

$$2\alpha_i = 2.063 \times 10^5 \, [4 g_m (C/R_o)^{\frac{1}{2}}] \qquad (2)$$

Here $C = MG/c^2$ represents one-half the gravitational radius of the deflector (the lens); M is its mass, G, the gravitational constant, and c the velocity of light; R_0 is the present value of the scale factor, which has the dimensions of length; ρ is the distance of the optical axis (the line of sight through the gravitational center of the deflector) from the center of the object, or the radius of the object disk, should the optical axis pass through the disk:

$$\Delta = \delta \, [r_o/(1 + z_o)] \qquad (3a)$$

where

$$\delta = [r_1 (1 + z_d)/r_d \, r_o]^{\frac{1}{2}} \qquad (3b)$$

The values z_o and z_d are the redshifts; r_o and r_d are the dimensionless radial coordinates of object and deflector, respectively; r_1 is the radial coordinate between deflector and object; g_o is a function of the distance of the optical axis from the center of the object and of the distribution of surface brightness within the object disk. Whenever the optical axis passes through the center of the object disk, g_o can be evaluated by

$$g_o = 2 \int_0^1 F(\rho) d\rho \Big/ \int_0^1 \rho F(\rho) d\rho \qquad (4)$$

where $F(\rho)$ is the surface brightness of the object disk as a function of ρ, the distance from its center. When the optical axis passes outside the disk, $g_o = 2$. The factor g_d is a function of the distance at which the light rays pass the gravitational center of the deflector (it depends also on the mass distribution of the deflector). For a spherically shaped deflector, homogeneous with respect to distribution of mass

$$g_d = [2 g_m - 3\chi^2 (1 - \chi^2)^{\frac{1}{2}}]^{-1} 2 g_m$$

$$g_m = [1 - (1 - \chi^2)^{3/2}]^{\frac{1}{2}} \qquad (5)$$

where χ is the distance of the light rays from the gravitational center expressed as a fraction of the total radius of the deflecting body. From Eq. 2 we see that in order to produce an image of stellar appearance [that is, one with an apparent diameter less than 1 sec of arc in a standard model ($\Lambda = 0$, $q_0 = \sigma_0 = 1$) and within a redshift range of $z_o = 0.4$ to $z_o = 2.4$, the deflector mass can never exceed 5×10^{43} g. From Eq. 1 we then infer that the maximum amplification of a gravitational lens that produces stellar images is about $8000/\rho_o$ (ρ_o in parsecs). On the other hand, the average absolute magnitude of the more than 100 quasars with known redshift and apparent magnitude is about −24. We conclude, therefore, that normal galaxies, which have diameters of several kiloparsecs, cannot be objects of gravitational lenses even if their absolute magnitude is −22, unless their surface brightness is strongly peaked toward the center and the optical axis of the lens passes close to the central area.

The proper motion of the deflector and of the object galaxy (~100 km/sec) and the motion of the solar system around the center of the galaxy (~250 km/sec) will result in a scanning motion of the optical axis across the area surrounding the object or across the disk of the object itself. If the deflector galaxy is close to the observer, the proper motion of the deflector and the motion of the solar system will become greatly amplified in the frame of the object.

During this scanning motion the value of ρ (or the value of g_o) will change, which changes the brightness amplification. For a given scanning speed, the time during which the value of ρ remains sufficiently small to ensure adequate amplification depends on the intrinsic luminosity of the light source, because the optical axis may move farther from more luminous objects before the decrease in the brightness of the image becomes critical (*12a*).

As we have shown (*4*), the limitations in the maximum amplification and the consequences of the scanning motion reduce the choice of light sources (which could serve as objects to produce quasar-like images) to astronomical bodies which are less than 100 parsecs in diameter, have an average absolute magnitude of about −18, and a lifetime of not less than 10^4 years. Such objects are nuclei of Seyfert galaxies as well as the innermost cores (100 parsecs in diameter) of elliptical galaxies, or nuclei of spiral galaxies, should their surface brightness follow a de Vaucouleurs (*13*) brightness distribution.

To obtain the number of images of given absolute magnitude M_Q per magnitude interval, we must consider that in order to produce an image of given brightness, brighter objects need deflectors of smaller mass and vice versa. We therefore count all possible object-deflector pairs which at a given object distance r_o and with a deflector situated at r_d produce an image of the required magnitude. [Kiang's (*14*) luminosity functions of galaxies, $l(x)$, give the number of galaxies for a luminosity interval of one magnitude.] We then multiply the number of objects by the number of deflectors, expressed in terms of the mass function $m(y)$ of the latter. This function can be obtained from the luminosity functions of the galaxies serving as deflectors by a reasonable average mass-to-luminosity ratio of the galaxies: $M/L = 50\ M_s/L_s$ (where M_s and L_s are the mass and luminosity of the sun). We now transform the variable y with the help of Eq. 1 into a function of ρ and x (and of r_o and r_d); we then multiply by the probability that we will find the object within the distance ρ and $\rho + d\rho$ from the optical axis and integrate over all possible ρ values. The value for ρ_{min} would actually equal the radius ρ_o of the object disk. But this would require that we integrate separately for the cases where the optical axis is outside and inside the object disk; in the latter case, $\rho = \rho_o =$ constant. The first integral gives the number of crescent-shaped images, whereas the second gives the number of ring-shaped images. In the case of quasars where morphological details cannot be determined, such a separation would be meaningless. Fortunately, the second integral gives for all surface brightness distributions which are flat or peaked toward the center approximately the same value that one would obtain by using the first integral from $\rho = 0$. The value for ρ_{max} is determined by Eqs. 1 and 2 and the requirement that the image, in order to have a starlike appearance, may not have an apparent diameter (or apparent distance between the two crescents) in excess of one second of arc. The value for ρ_{max} can thus be expressed as a function of x and M_Q (and r_d and r_o). We integrate now from $x = 0$, the lowest possible value of x, to the limit of the luminosity functions. (Above $x = 8$ the contribution to the integral is negligible.) Finally we integrate over r_d and r_o to obtain the volumes within which the deflector and object galaxies, respectively, can be located. The total integral, which gives the number of images of M_Q absolute magnitude up to a redshift z_o corresponding to $(r_o)_{max}$, is thus:

$$N(M_Q) = b_o b_d (4\pi R_o^3)^2 \int_0^{(r_o)_{max}} \left[r_o^2 dr_o / (1 - kr_o^2)^{\frac{1}{2}} \right] \int_0^{r_o} \left[r_d^2 dr_d / (1 - kr_d^2)^{\frac{1}{2}} \right] \int_0^{x_{max}} dx \int_0^{\rho_{max}} (2\pi\rho / 4\pi R_o^2)(\Delta/\delta)^2 m(y) l(x) d\rho \quad (6)$$

where k has values −1, 0, +1 depending on the cosmological models. The integrations over dr_d and dr_o were performed with the help of tables of relativistic cosmological models (*15*) and by numerical integration with the IBM-1620 model-2 computer.

To avoid overestimation of the number of images, we used in Eq. 6 Kiang's (*14*) luminosity functions:

$$n = bl(x) = bx^3 \quad \text{for } 0 < x < 2.5$$
$$n = bl(x) = 4.94b\ 10^{0.2x} \quad \text{for } 2.5 < x < 8 \quad (7)$$

(where $x = M + 22$; M is the absolute magnitude of the galaxy and $b = 1.15 \times 10^{-3}$ Mpc^{-3} magnitude^{-1} for field and cluster galaxies) for the spatial density of object and deflector galaxies alike, but with the assumption that the actual objects of the lens, whether the nucleus of a Seyfert galaxy or the nucleus of a normal galaxy, are fainter by one magnitude than the galaxy to which they belong. Thus we used −21 absolute magnitude as the upper limit of the intrinsic luminosity of the object. Sargent (*16*) found a Seyfert galaxy with a nucleus −20.8 in absolute magnitude.

The used luminosity functions become zero at an absolute magnitude of −22. In fact, the luminosity function of galaxies should gradually diminish in accordance with Kiang's luminosity functions (*17*) for galaxies of the early and late type. Our restriction of the maximum luminosity of the objects will entail an underestimation of the number of images of high luminosity. In the computations the spatial density of the deflector galaxies was taken to be $b_d = 1.15 \times 10^{-3}$ Mpc^{-3} magnitude^{-1}, whereas the spatial density of the Sey-

Table 1. Expected number of images per magnitude interval for four absolute magnitudes in the redshift range from $z_o = 0.4$ to $z_o = 2.4$, with apparent diameters of less than 1 sec of arc, produced from nuclei of Seyfert galaxies. In parentheses are the numbers of observed quasars of equivalent absolute magnitude. The quantity k represents the curvature constant; Λ, cosmological constant; q_0, deceleration parameter; σ_0, density parameter. Used as the limiting apparent magnitude was $m_{pg} = 21.0$ and a Hubble constant of $H = 100$ km sec^{-1} Mpc^{-1}. Columns 2, 3, 4, and 5 list the pertinent characteristics of the models which were chosen according to the following considerations: (i) the Hubble plot of field and cluster galaxies is a straight line within ± 0.3 magnitude up to $z = 0.46$ in all four models; (ii) the standard model is most often used by observational astronomers because of its simple luminosity-distance function; (iii) the M1-6 model (*15*) has a very low absolute magnitude-redshift slope for quasars (*20*); (iv) the Fib model (*21*) has a zero absolute magnitude-redshift slope for quasars (*22*) and gives a simple, quantitative explanation for the curve in which count is plotted against the flux density of radio sources (*23*) and for the existence of the microwave background radiation (*24*); and (v) the M1-1 (*15*) model is included as an example of a cosmological model with negative space curvature.

Model	Parameters				Quasars of different absolute magnitude (No.)			
	k	Λ	q_0	σ_0	−23	−24	−25	−26
M1-6	1	3	0	3	65 (33)	10 (37)	1.7 (18)	0.3 (2)
Standard	1	0	1	1	296 (10)	67 (32)	10.6 (34)	1.7 (13)
Fib	1	1	0	1/3	3124 (33)	494 (38)	77 (17)	13 (10)
M1-1	−1	0	0	0	814 (10)	476 (17)	290 (26)	45 (21)

fert galaxies was estimated to be 2 percent of this number (*18*), that is, $b_o = 2.3 \times 10^{-5}$ Mpc^{-3} $magnitude^{-1}$.

Table 1 lists the number of images with apparent diameters of less than 1 sec of arc, produced from the nuclei of Seyfert galaxies for four different cosmological models. Absolute magnitudes of observed quasars were computed from the observed, apparent visual magnitudes by using the luminosity-distance function of the cosmological model in question; no K-corrections were applied (*19*).

For intrinsic luminosities fainter than absolute magnitude −22.5, comparisons between the expected and observed numbers of quasars become ambiguous. Some very bright Seyfert galaxies may be mistaken for quasars. The K-correction of the bluish nucleus of the Seyfert galaxy is negligible, whereas that of the disk may be quite considerable. Thus the disk of the galaxy may fade below visibility while the compact nucleus remains visible. Adopting Kiang's luminosity function for galaxies of the early type (*17*) as the luminosity function of Seyfert galaxies, we should expect up to $m_v = 19.7$ to see about 40,000 bluish extragalactic objects of stellar appearance in the redshift range from $z = 0.4$ to $z = 1.0$ that should be indistinguishable from quasars of intrinsic luminosity fainter than absolute magnitude −22.5.

As mentioned above, elliptical galaxies and the spherical parts of spiral galaxies also are suitable objects for gravitational lenses, if their surface brightnesses follow a de Vaucouleurs function (*13*), as most do. Their number is at least 10 to 20 times greater, but their color may be more reddish than that of nuclei of Seyfert galaxies. The present two-color search technique, developed by Sandage, uses as criterion the bluish color of quasars. This technique therefore favors the detection of quasars in which a nucleus of the Seyfert galaxy is the object of the lens. This is the explanation which we propose to account for the astonishing similarities observed between the spectra of nuclei of the Seyfert galaxies and the spectra of quasars.

The computations discussed here are straightforward but not simple; thus it is not surprising that there are in the literature several expressions of doubt that our hypothesis can explain the number of quasars (*25*); without proof it has been stated that the number of lenses is insufficient to explain the number of quasars (*2, 26, 27*). Thus it has some merit that Sadeh (*28*), who wrote

> I pose the question of whether an object called a quasar is a single, intrinsically luminous entity or the result of accidental alignment, along the line of sight, of two normal galaxies, the more distant of which has its light amplified by the gravitational-lens effect of the nearer galaxy,

computed the expected number of gravitational lenses, independently from our previously published calculations.

Sadeh's starting point was recognition that the minimum distance d_m of the deflector from the observer, from which distance it can focus light rays, is

$$d_m = a^2c^2/4MG \quad (8)$$

where a is the radius and M is the mass of the deflecting galaxy. Adopting the elliptical galaxy M87 as an example, he obtains $d_m = 10^{28}$ cm. From the greatness of this minimum distance he concludes that "normal galaxies do not qualify as gravitational lenses," and "quasi-stellar sources . . . are the only galaxies capable of acting as gravitational lenses." We must assume that use of the expression "quasi-stellar sources" is here and in his sentence: "Before calculating the probability of finding a galaxy behind a quasi-stellar source . . ." in error, because postulation that these superluminous objects exist would render further discussion superfluous.

The mass of M87 may be 8×10^{45} g rather than 2×10^{45} g, and its diameter may be not 1.2×10^{23} cm (40 kpc) but merely 4×10^{22} cm (13 kpc) (*29*). These values yield

$$d_m = 1.7 \times 10^{26} \text{ cm}$$

which is much less than the distance to any known quasars. Thus normal galaxies may indeed serve as gravitational lenses. Furthermore, since galaxies are not opaque, a and M could be the diameter and mass of the nucleus of a spiral galaxy that may be ten times smaller and five times lighter than the entire galaxy. Finally, when light rays proceed within a spherical and homogeneous mass distribution (Eq. 5), the amplification is sometimes greater than when they proceed entirely outside the mass distribution. Thus the introduction of the concept of a minimum distance is unimportant practically and rather misleading.

In his first example of calculation Sadeh (*28*) postulates, as both deflectors and objects, hypothetical galaxies 2×10^{19} cm in radius and 2×10^{44} g in mass. Counting the number of galaxies in the shadow behind the deflecting galaxy, he finds, in the standard model and up to a redshift of $z = 2.0$, 40 gravitational lenses over the entire sky. In his second example the galaxies are 50 times smaller in diameter, and he finds 0.02 lens over the entire sky.

Therefore Sadeh concludes that

> . . . quasars are not demonstrations of gravitational lenses unless additional assumptions are made: for example, if the density of galaxies increases much more rapidly with increasing distance, the probability of alignment increases. But this is an arbitrary assumption lacking theoretical or observational backing; such an assumption was made by Barnothy who claims that 3000 Seyfert galaxies can be found behind each other and act as gravitational lenses.

In no publication or appearance before any meeting anywhere have we claimed that one Seyfert galaxy acts as the gravitational lens for another; moreover, in all our calculations without

Table 2. Numbers computed with Sadeh's equation up to $z_o = 2.0$. In parentheses are the numbers of observed quasars of equivalent absolute magnitude.

Model	Absolute magnitude			
	−23	−24	−25	−26
Standard	8300(9)	1340(32)	200(31)	40(12)

exception we have used a constant number density of galaxies in comoving coordinate volumes.

Sadeh writes the equation of the brightness amplification in the form

$$A \geqslant 2(d/a)\{(MG/c^2)[d_1/d(d+d_1)]\}^{\frac{1}{2}} \quad (9)$$

where d is the distance of the deflector from the observer, and d_1 is the distance of the object from the deflector. The computations were performed in a Euclidean metric, but the space density of deflector and object galaxies was taken from Sandage's tables (*30*), which list the number of galaxies up to limiting apparent magnitudes in different cosmological models with zero cosmological constant. Sandage's tables are calculated with the large K-correction usual for galaxies. This correction unnecessarily decreases the number of gravitational lenses. Bluish objects, such as quasars or nuclei of the Seyfert galaxy, have negligible K-correction (*19*), and the visibility of the deflector galaxy does not enter the calculations at all. The number of object galaxies in the truncated cone of a/d half opening behind the deflector is then computed to a distance up to ten times d. This means that 97 percent of the counted objects are at a distance at which

$$d_1/(d+d_1) > 1/2$$

and one obtains the minimum amplification

$$A \geqslant 1.4\ (d/a)(MG/c^2d)^{\frac{1}{2}} \quad (10)$$

In the first example $a = 2 \times 10^{19}$ cm, so that, for all deflector distances greater than $z = 0.1$, the amplification factor A will be greater than 250 and will thus brighten the luminosity of the object galaxy by 6 magnitudes. Since a typical galaxy of 2×10^{44} g of mass, used by Sadeh as object of the lenses, has an absolute magnitude of about −20, only lenses that produce images brighter than −26 were counted. In the second example $a = 4 \times 10^{17}$ cm, so that the minimum amplification is 12,500—equivalent to brightening by 10.2 magnitudes; here only images with absolute luminosities exceeding −30.2 were counted. Since no quasar of such luminosity has yet been observed, 2×10^{-2} (*28*) cannot be considered too low. By Sadeh's method of calculation we have computed Table 2 in a manner similar to Table 1. When properly interpreted, Sadeh's estimate proves that the number of gravitational lenses is more than adequate to explain the number of observed quasars.

Although one might say that the hypothetical galaxies used in the first example resemble very small and extremely heavy nuclei of Seyfert galaxies, the galaxies used in the second example, which have a star density of $10^{14}/pc^3$, are astrophysical monstrosities. Such fantastically compact galaxies may have been employed because Sadeh used the same letter a to denote two different quantities: in his Eq. 5 (*28*), a is the diameter of the deflecting galaxy; in his Eq. 6, a is the radius of the "minimum amplification cone" at distance d. He was therefore led to the erroneous conclusion that the objects must be counted in the "shadow cone" of the deflecting galaxy; thus the minimum amplification determines the radius of the deflecting galaxy.

Let us now turn briefly to effects of the gravitational lens where the shape of the image is important. From a circularly shaped object the lens always produces two thin crescentic images that coalesce to a ring when the optical axis passes through the object disk. The apparent distance of the crescents, or the diameter of the ring (Eq. 2) depends on the mass of the deflector. As we have seen, to produce a starlike image less than 1 sec of arc in diameter, the mass of the deflector cannot exceed 5×10^{43} g. Such a galaxy would not be very bright, and this explains why the apparent brightness of the deflector is usually fainter by 2 to 7 magnitudes than the image (*6*). Thus the deflector need not be "dark" (*27*) to be invisible in front of the quasar.

On the other hand, if the deflector is the nucleus of a bright spiral galaxy, with a disk that is well visible, the brilliant image of the nucleus of a galaxy behind it, being centered on the disk of the spiral galaxy, will change in morphological appearance and resemble an N-type galaxy (*8*).

Should the deflector be so heavy that the apparent distance of the crescents becomes equal to or even larger than the apparent diameter of the foreground galaxy, then, depending on whether the foreground galaxy is an elliptical galaxy or a spiral galaxy seen edgewise, we will observe a morphological configuration resembling a spiral galaxy or a barred-spiral galaxy. Since very heavy deflector masses are needed to produce such configurations, occurrence of such simulated morphological types should be rather rare. To ascertain whether a morphological configuration is a single object or is simulated through the superposition of an image upon a foreground galaxy, two sets of emission lines, corresponding to two different redshift values, should be found as we have shown in 3C 371 (*8*).

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References and Notes

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12a. The scanning motion of the optical axis may cause three different kinds of brightness variations: (i) secular variations in consequence of the change in ρ; (ii) short time flashes when the optical axis passes in the object of the lens in close proximity to one of its stars. For example, if the scanning speed in the frame of the object is of the order of 10^4 km/sec and the star density in the object (nucleus of a Seyfert galaxy) is 10^7 stars/pc^3, flashes will on the average follow each other in about 10-year intervals (as in 3C 273); (iii) brightness changes of a few magnitudes lasting over several months when a nova outburst occurs in the vicinity of the optical axis (as in 3C 446). Should the nova emit later radio waves, the optical brightness increase will be followed by an increase in the radio flux of the quasar [J. M. Barnothy and M. F. Barnothy, *Astron. J.* **71**, 155 (1966)].
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19 December 1967; revised 18 July 1968

Fumarolic Activity in Marie Byrd Land, Antarctica

Abstract. *Ice towers, probably formed by recent fumarolic activity, have been found around the summit calderas of two volcanoes in Marie Byrd Land. These active (?) volcanoes lie within a broad belt of Mesozoic intrusion and late Cenozoic extrusion that appears to be part of the circum-Pacific orogenic province.*

The Marie Byrd Land Survey (1967 to 1968) covered a coastal sector approximately 720 km long and extending up to 320 km inland, between longitudes 110°W and 136°W. The volcanic nature of many mountains in this region had been established by oversnow traverses from Byrd Station (1957 to 1958 and 1959 to 1960). One age determination from a Mount Sidley specimen, at the southern end of the Executive Committee Range, yielded an eruption date of 6.2 million years ago (*1*). We now report evidence for recent fumarolic activity in two central Marie Byrd Land mountain ranges.

Fumarolic activity in Antarctica characteristically produces ice towers by the condensation and freezing of vapors. These features have been described from observations on Mount Erebus, Ross Island, first by Shackelton (*2*) and then by Holdsworth and Ugolini (*3*). Those features forming over active fumaroles show open central vents; the inactive fumaroles on Mount Erebus are marked by ice towers without open vents. Groups of ice towers, similar in size and shape to those pictured by Holdsworth and Ugolini, were observed at close range from helicopters around the summit calderas of Mount Berlin, in the Flood Range (135°50′W, 76°03′S), and Mount Hampton, in the Executive Committee Range (125°54′W, 76°29′S). The ice towers on Mount Hampton, which were also examined from the ground, are approximately 10 to 20 m high. No open central vents or gaseous emissions were observed, and no fumarolic condensates or sublimates could be sampled because each ice structure was mantled by fresh snow. A very recent origin for these structures is almost certain, because they stand completely unprotected from wind erosion at elevations exceeding 3000 m.

Antarctic volcanoes, known to be active because of recent eruptions or geothermal activity, include Mount Erebus, on the southwestern margin of the Ross Sea; Deception Island in the South Shetland Islands; and Mount Melbourne on the Hallett Coast (*4*). Mount Morning, 90 km southwest of Ross Island, is suspected of being geothermally active on the basis of a recent infrared scan (*5*). These recently active Antarctic volcanoes lie within a large belt of late Cenozoic volcanism that extends down the Antarctic Peninsula, across Marie Byrd Land, and northward along the Hallett Coast to Cape Adare and the Balleny Islands. Granitic plutons of late Mesozoic age underlie the eastern part of this volcanic terrain, in the Antarctic Peninsula and in Ellsworth Land (*6*). The coupling of these volcanic and plutonic characteristics is typical of the circum-Pacific orogenic belt as described in more accessible and better exposed areas (*7*). It has yet to be established however, that there is a continuity of these orogenic characteristics along the full length of the Antarctic margin of the Pacific Ocean basin. The presence of ice towers on Mount Berlin and Mount Hampton suggests that there has been recent volcanic activity in Marie Byrd Land, and that the circum-Pacific orogenic belt may extend without interruption, from Ellsworth Land across Marie Byrd Land. Our tentative conclusions require confirmation by a program of infrared scanning over this entire sector, and by determinations of the ages of granitic plutons in Marie Byrd Land mountain ranges.

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9 August 1968

Scientific Uses of Pulsars

Abstract. *The recently discovered celestial sources of pulsed radio energy can be used to test general relativity, to study the solar corona, and to determine the earth's orbit and ephemeris time. The vector positions and transverse velocities of pulsars can be measured with radio interferometers; in combination with pulse-arrival-time data, the distance determination will yield the average interstellar electron density.*

The startling discovery (*1*) and subsequent investigations (*2, 3*) of celestial objects that emit intense bursts of radio energy at regular intervals has caused great consternation (*4*) among the theorists trying to explain this phenomenon. We have not solved this theoretical problem either but, rather, wish to point out how best advantage might be taken of the existence of pulsars. In particular, we discuss several potentially important experiments that might utilize their radiation. The interpretations of such experiments are, unfortunately, dependent to some extent on the theoretical model that is assumed to describe this radiation. We therefore postulate first that pulsar emissions are perfectly regular (*5*); we also discuss models for which this assumption is invalid and consider the consequences for the proposed experiments.

A number of applications can be based on pulsars being like "one-way" radars. From accurate measurements of the times of arrival of pulses from one, or preferably more, pulsars, the orbit of the earth can be determined with standard techniques (*6*). The orbit, determined with respect to pulsar locations, can be related to optical star