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Low-Energy Physics from a High-Energy Standpoint

Research activities in high-energy physics that bear on nuclear structure are reviewed.

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In an article with the same title published 13 years ago (*1*) the contributions of high-energy physics to our understanding of nuclear structure were first reviewed. Principal emphasis was placed on the overwhelming importance of high-energy electrons. From a theoretical point of view, electrons were expected to be uniquely valuable for three distinct reasons. First, their interaction with atomic nuclei is, to a very good approximation, entirely electromagnetic and hence well understood. Second, this interaction is much weaker than the nuclear forces which hold the constituent protons and neutrons of nuclei together, so that electron scattering is much easier to analyze than nucleon (proton or neutron) or pion (π meson) scattering. These two factors in themselves would not favor electrons over photons as probes of nuclear structure. However, a single interaction of a photon with any material system can only result in its absorption, so that the energy, ΔE, delivered to the system is related to the momentum transfer, Δp, by the equation $\Delta E = c\Delta p$ where c is the speed of light. Thus the third reason for the utility of electrons is that the energy and momentum delivered by them to a nucleus can be varied independently. Knowledge of the direction and magnitude of the momentum vectors of the incident and scattered electrons gives both of these quantities, and any desired combination is obtained by observing the electrons scattered in a particular direction with a well-defined loss of energy.

The initial operation of the Mark III linear electron accelerator at Stanford University in the early 1950's made available a homogeneous, well collimated beam of high-energy electrons, which was exploited by Hofstadter and his colleagues for the study of both nuclear and nucleon structure. Those electrons that were scattered elastically (except for the small energy loss caused by recoil of the nucleus as a whole) provided detailed information on the static charge distributions of both spherical and nonspherical nuclei. Electrons which were scattered inelastically gave analogous data on the energies and quantum numbers of nuclear excited states as well as on the mechanism of excitation. It was only with respect to experimental availability that electrons were superior to muons (μ mesons), since muons possess all three of the properties attributed above to electrons. High-energy muon beams were not then available, and the use of muons was confined to the study of the x-ray spectrum from an "atom" in which a negative muon is bound to a nucleus and makes radiative transitions from one energy level to another. Since a muon has a mass about 207 times that of an electron, its Bohr radius is smaller by roughly the same factor; thus the muon spends a good deal of time within the nucleus. It can therefore provide very precise information on certain average properties of the nuclear charge distribution but has little to contribute to our knowledge of details on a smaller scale.

It was also pointed out in the 1955 review that there were some factors which complicate the scattering of electrons from nuclei. It is known that a charged particle radiates electromagnetic energy when it is accelerated. Thus the scattering of an electron by a nucleus is inevitably accompanied by the emission of photons, which carry off both energy and momentum and hence complicate the interpretation of the energy loss and angular distribution of the scattered electrons. While troublesome, such radiative corrections could in principle be calculated exactly, and rather good computations were available at that time. Another complicating aspect was the fact that a nucleus cannot realistically be thought of as a smooth, rigid distribution of electric charge, even when it scatters electrons elastically. There is almost certainly a granularity to the charge distribution which arises from short-range correlations between protons, although this is hard to demonstrate both theoretically and experimentally. Also, the nucleus is not a rigid object even when inelastic scattering that leads to excited states is ignored. The electron can virtually excite the nucleus while it is being scattered elastically: it can raise the nucleus to an excited state in one interaction and lower it to the ground state in another, so that the end result is the same as if the nucleus had scattered the electron elastically without changing its state at all. Such a dispersion (or polarization) effect is expected to be small since it involves at least two interactions of the

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electron with the nucleus rather than the single interaction that can cause nondispersive scattering. Only qualitative estimates of this effect were available in 1955, since a reliable calculation would require knowledge of the entire spectrum of excited states of the nucleus.

Progress since 1955

It would be impossible to survey the subject of this article in the space available with anything approaching the comprehensiveness of the 1955 review. A very large amount of work, both theoretical and experimental, has been done on most of the subjects mentioned in the preceding section and on some new ones as well. Fortunately, a number of excellent reviews of particular topics have been published, especially in the last 2 years (*2*).

Muon atom spectra have been surveyed at the 1966 Gatlinburg Conference (*3*) and the information obtainable from muon capture by nuclei was summarized at the 1966 Varenna summer school (*4*); muon beams, while now available, have not yet been used for studies of nuclear structure. Nuclear reactions may be initiated by electrons, nucleons, pions, and kaons (K mesons) at what are sometimes called medium energies (100 to 600 million electron volts) (*5*); the emphasis here is on nuclear excited states and reactions rather than on elastic scattering. Excellent progress has been achieved with regard to the agreement between the size and shape of nuclei as determined from electron scattering and from muon atom spectra. The transition probabilities between different states of the same nucleus, measured by inelastic electron scattering, have been used to test various models of nuclei, and similar information concerning closely related nuclei has been obtained from muon capture and from a variety of nuclear reactions.

The new journal *Comments on Nuclear and Particle Physics* consists of 4- or 5-page summaries of recent developments, which are written by experts for knowledgeable nonexperts and accompanied by references to both current and as yet unpublished literature. Of particular interest in the present connection are articles by Wilkinson (*6*), Bromley and Weneser (*7*), and Feshbach and Kerman (*8*). The first two references discuss new information with regard to the relative distribution of protons and neutrons in the nucleus. These are based on a study of kaon atom spectra and of the reactions that occur when negative kaons are captured in heavy nuclei. The analysis of Burhop (*9*) shows that K^- (negative kaon) absorption occurs mainly in the outer regions of the heavier nuclei and is characteristic of capture by neutrons rather than by protons. The picture that emerges is one in which both neutron and proton densities decrease with increasing distance from the center of the nucleus and attain half their central densities at about the same radius. But the rate at which the neutron density decreases beyond this point is somewhat slower than that at which the proton density decreases, so that the surface of the nucleus is rich in neutrons.

At the end of the last section, I mentioned some troublesome complications that arise in the interpretation of the scattering of high-energy electrons. The calculation of radiative corrections has greatly improved, and it is now a routine matter to "unfold" these corrections from the experimental data so that the scattering cross sections in the absence of radiation are exhibited directly. The underlying calculations have recently been reviewed (*10*). The situation with respect to the dispersion or polarization effect is, unfortunately, still far from satisfactory. As pointed out earlier, a reliable calculation would require virtually complete knowledge of the spectrum of nuclear excited states and would also be prohibitively complicated. It is not surprising, therefore, that three recent calculations (*11*) performed in somewhat different ways give divergent results. There is, however, a rather well-defined conclusion that can be drawn from all three calculations, which can be described in the following way. High-energy electron scattering from most nuclei, both elastic and inelastic, is characterized by diffraction: a general decrease in the number of scattered electrons with increasing angle or transfer of momentum, interrupted by one or more sharp dips that are called diffraction minima (Fig. 1). In the neighborhood of a diffraction minimum almost any modification of the calculation can be expected to produce large fractional changes in the scattering. This is the case with the dispersion corrections, although there is disagreement as to the magnitude of the changes. Away from the minima, all three calculations agree that the corrections are not greater than a few percent. Thus, as far as the location of the diffraction minima and the shape of the scattering curve away from the minima are concerned, dispersion corrections can safely be ignored. On the other hand, the depth of the minima cannot now be calculated reliably.

For the remainder of this article I will discuss two matters on which new information is available. These are the very precise experiments on elastic electron scattering from the calcium isotopes and the comparison of high-energy elastic electron and proton scattering from He^4.

Elastic Electron Scattering from Calcium

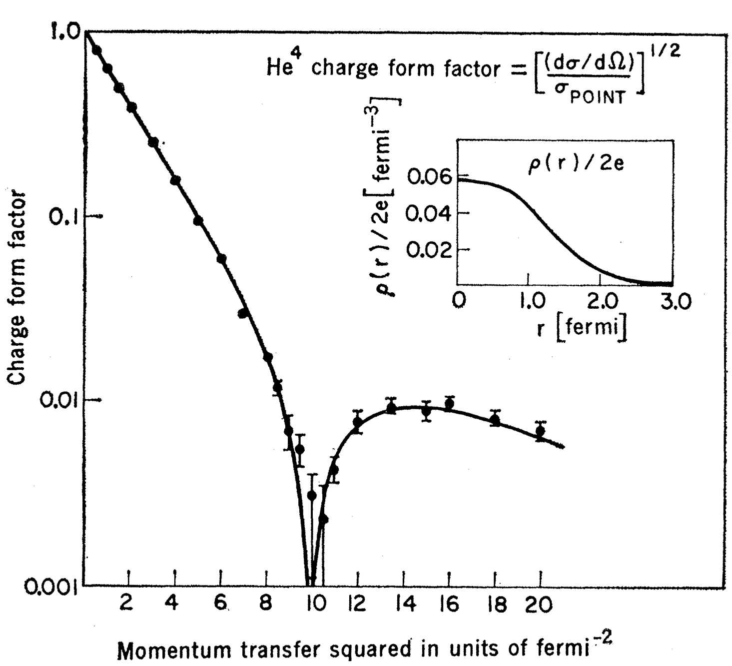
As indicated in the last section, high-energy electron scattering is generally characterized by diffraction. As in any situation in which diffraction dominates, the linear size of the object being observed is roughly proportional to the wavelength of the incident radiation divided by the angular spread of the scattering pattern. Since the wavelength of an electron is inversely proportional to its momentum, object size is inversely proportional to the momentum transferred during the scattering process. The diffraction minima serve as convenient markers of the scattering pattern, and the position of the first minimum provides a good approximate measure of the radius of the nuclear charge distribution. For example, the first diffraction minimum for calcium occurs at a scattering angle of approximately 55° when the electron energy is 250 million electron volts (Mev). This corresponds to a momentum transfer equal to twice the incident momentum multiplied by the sine of half the angle of scattering, or 230 Mev/c. Momentum transfers are often expressed in inverse length units by dividing them by $\hbar$, Planck's constant divided by 2π. The length unit used in nuclear physics is the fermi (1 F $= 10^{-13}$ cm), and the momentum transfer of 230 Mev/c corresponds to 1.17 F^{-1}. The corresponding nuclear charge radius is roughly 3.6 F. Thus a convenient, although rather rough, rule is that the product of the nuclear radius and the momentum transfer at the first diffraction minimum is equal to 4.2.

Nuclei do not have well-defined

radii. The density is roughly constant over the central part and then falls smoothly to zero. It is customary to specify a half-density radius, $r_{\frac{1}{2}}$, at which the density attains half its central value, and a skin thickness, t, over which the density drops from 90 percent to 10 percent of its central value. Approximate figures for calcium are $r_{\frac{1}{2}} = 3.6$ F, $t = 2.7$ F. These figures refer to the electric charge distribution (proton distribution) since the nuclear neutrons do not affect the electrons significantly. It has long been thought that the average density of nuclear matter is nearly constant, so that $r_{\frac{1}{2}}$ is proportional to $A^{\frac{1}{3}}$, where the atomic weight, A, is equal to the total number of protons and neutrons in the nucleus. On this basis, $r_{\frac{1}{2}}$ would be expected to increase by 3 percent in going from Ca^{40} to Ca^{44}, whereas actually it increases by about 2 percent; at the same time, t decreases by 1.6 percent. Similarly, $A^{\frac{1}{3}}$ increases by 6 percent in going from Ca^{40} to Ca^{48}, while $r_{\frac{1}{2}}$ increases by 5 percent and t decreases by 11 percent. It thus appears that the addition of neutrons to Ca^{40} increases $r_{\frac{1}{2}}$ for the charge distribution by slightly less than the increase in $A^{\frac{1}{3}}$ and makes the charge boundary substantially sharper.

The most recent measurements at 250 and 500 Mev (*12*) yield accurate comparisons between the electric charge distributions of nearby isotopes (Fig. 2). The dashed curves are plots of $4\pi r^2$ times the difference in charge density, measured in units of 0.033 proton charge per fermi, against the radius r in fermis. The top curve shows that the charge density of Ca^{40} is greater than that of Ca^{42} out to about 3.6 F, after which it is less; in other words, the same amount of electric charge (same number of protons) is spread over a larger radius in the heavier isotope, as would be expected from a slightly expanded nucleus. However, in going from Ca^{42} to Ca^{44} and then to Ca^{48}, the next two dashed curves show that the principal change is a sharpening of the edge of the charge distribution in the heavier isotope. The last curve shows what happens when two neutrons are replaced by two protons when Ca^{48} is replaced by Ti^{48}. Most of the added charge appears in the surface layer; this suggests that the corresponding neutrons in Ca^{48} are also on the nuclear surface, thus confirming the picture of a neutron-rich surface.

Detailed comparisons of charge density such as those illustrated in Fig. 2 provide a challenge to the theorist who wishes to account for them in terms of specific nuclear models. Several recent calculations have been made (*13, 14*) in an attempt to fit these charge densities on the basis of the nuclear shell model. According to this model, neutrons and protons are placed in independent-particle states in some assumed potential, which is chosen so as to yield the known shell structure and the binding energies of the last few nucleons. Proton densities are then calculated from their wave functions, and charge densities are obtained by spreading out the proton distributions in accordance with the known finite size of the proton. Refinements of these calculations permit the assumed potential to depend on the quantum numbers of the nucleon states. It would also be possible to require that the potential be that produced by the nu-



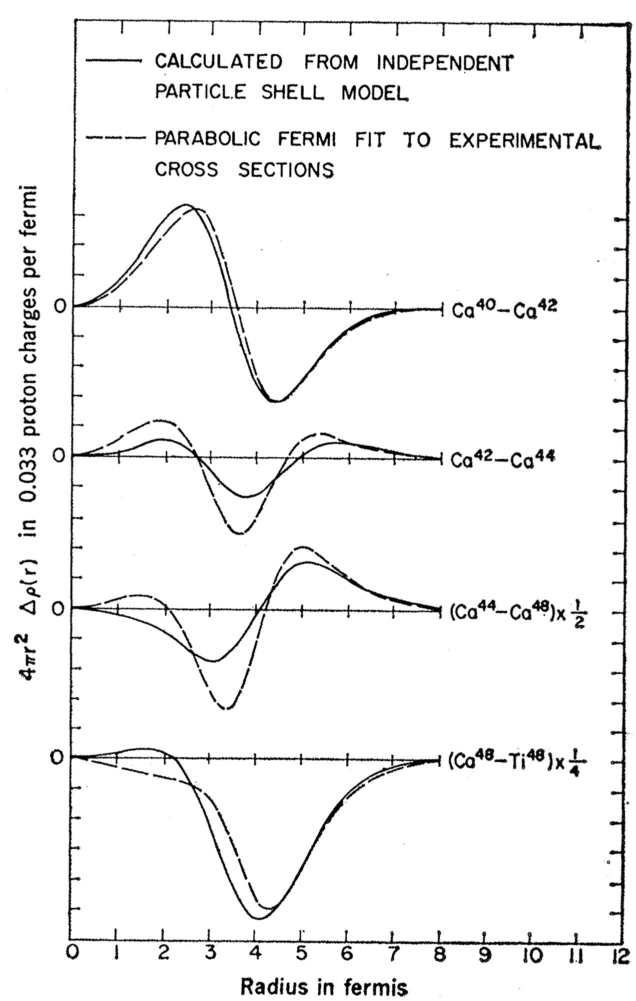
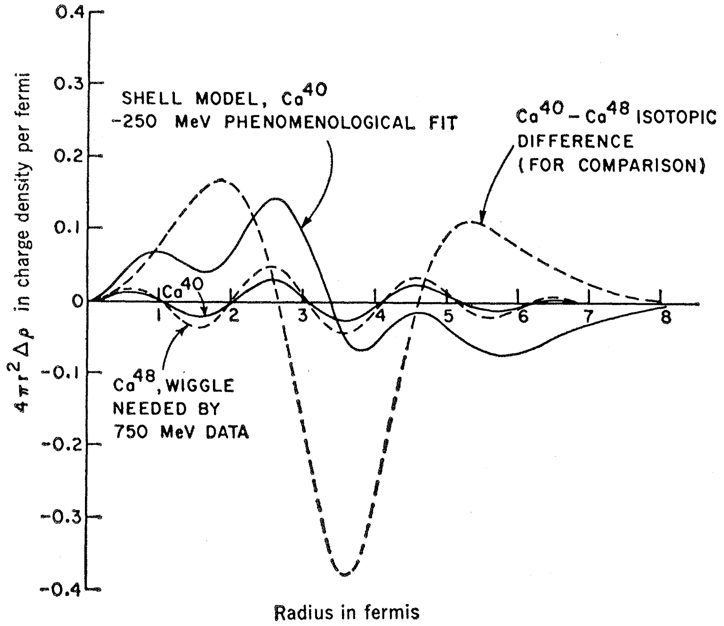


Fig. 1 (above). Elastic scattering cross section for high-energy electrons on He^4. The diffraction minimum occurs at a momentum transfer squared of 10 F^{-2} (*17*). Fig. 2 (right). Differences between the electric charge densities of various isotopic pairs plotted against distance from the center of the nucleus. Dashed curves are obtained from experimental scattering data (*12*) and solid curves are computed on the basis of an independent-particle model (*14*).



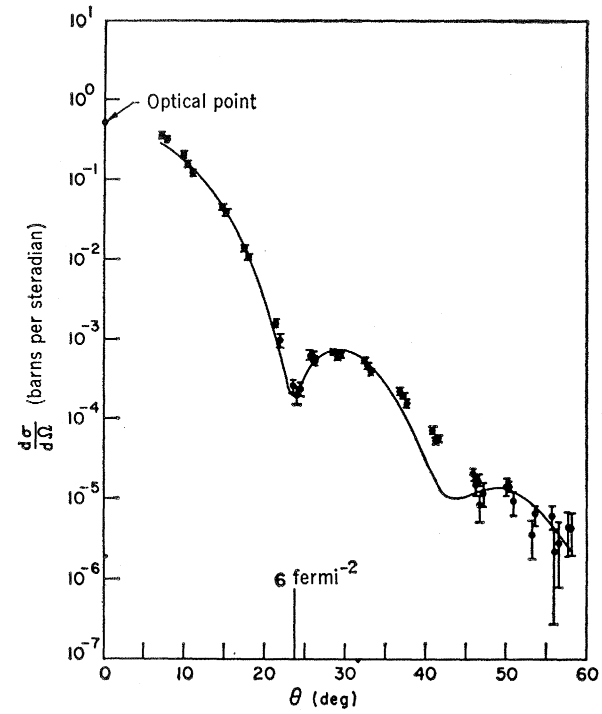


Fig. 3 (above). "Wiggles" in the charge densities of Ca^{40} and Ca^{48} needed to account for the electron scattering data at 750 Mev (*15*). Fig. 4 (right). Elastic scattering cross section for 1000 Mev protons on He^4. The first diffraction minimum occurs at a momentum transfer squared of 6 F^{-2} in units of inverse length squared (*16*).

cleons moving in it, but such self-consistent calculations are extremely arduous and have not yet progressed to the point of comparison with isotopic difference curves such as those in Fig. 2. The fits shown there between the simpler theory (*14*) (solid curves) and experiment (dashed curves) are quite good but still leave room for improvement.

The recent extension of the electron scattering experiments on calcium to 750 Mev (*15*) has revealed a surprising new feature. The smooth charge distributions that fit all of the data at 250 and 500 Mev, that is, somewhat beyond the second diffraction minimum, continue to fit the data at 750 Mev this far. But they predict scattering curves that deviate significantly from the data at 750 Mev in the neighborhood of the third diffraction minimum and beyond. It is possible to fit the new experiments without damaging the agreement with the old by the introduction of a small modulation of the charge density, as shown in Fig. 3. The dashed curve labeled "Ca^{48} wiggle" is the additional charge density needed to fit the data at 750 Mev on Ca^{48}; the similar solid curve is the corresponding modulation for Ca^{40}. It is interesting that they agree very well in wavelength (2 F) and in phase, and they are similar in amplitude. As expected, modulations at such short wavelengths show up only at the large momentum transfers that are feasible with the higher-energy electrons. The difference in charge density between Ca^{40} and Ca^{48} is also shown for comparison in Fig. 3; as with the second and third dashed curves of Fig. 2, this can be interpreted as showing a much sharper edge of the charge distribution in the heavier isotope. The solid curve of Fig. 3 labeled "shell model phenomenological fit" shows the deviations from a smooth charge distribution calculated for Ca^{40} on the basis of the nuclear shell model (*14*). The wavelength and phase of the calculated modulations agree fairly well with the experimental interpretation, but the amplitude is too large. More detailed experimental information, which may help to decide whether or not the "wiggle" provides a unique interpretation, should become available from experiments at 900 and 1000 Mev which are now in progress.

Elastic Electron and Proton Scattering from Helium

Experiments have recently been performed at the Brookhaven cosmotron on the elastic scattering of protons whose energies reach 1000 Mev from several light nuclei (*8, 16*). These nuclei, He^4, C^{12}, and O^{16}, were chosen since they all have first excited states that are more than 3 Mev above the ground state; thus the experimental energy resolution of 3 Mev was adequate to distinguish elastic from inelastic scattering. Diffraction curves are obtained, which superficially resemble those obtained from electron scattering. The data on proton scattering from He^4 is shown as the points with error bars on Fig. 4, which is a plot of the differential cross section against angle. At the time this work was published, it was thought that the data on electron scattering from helium could only be fitted by a gaussian charge distribution; as a consequence, it was considered significant that the proton scattering from a gaussian potential decreased much more rapidly with increasing angle than the experimental points. The solid curve in Fig. 4 is calculated from a potential of the Saxon-Woods or Fermi form, which is nearly constant for small r and falls off exponentially (rather than as a gaussian curve) for large r.

However, at about the same time, new results from Stanford were published on the scattering from He^4 of electrons with energy up to 800 Mev (*17*). The experimental points in Fig. 1 show a sharp diffraction minimum which is inconsistent with the gaussian charge distribution but which can be fitted very well with a distribution that closely resembles the Fermi form used in the analysis of the Brookhaven work.

As is seen from Fig. 1, the diffraction minimum in the electron scattering occurs at a momentum transfer (measured in inverse length units) of about $(10)^{\frac{1}{2}} = 3.2$ F^{-1}. In accordance with the rule cited earlier in this article, this corresponds to $r_{\frac{1}{2}} \cong 4.2/3.2 = 1.3$ F for the electric charge distribution. On the other hand, Fig. 4 shows that the first diffraction minimum in the proton scattering occurs at a momentum transfer of about $(6)^{\frac{1}{2}} = 2.5$ F^{-1}, or $r_{\frac{1}{2}} \cong 4.2/2.5 = 1.7$ F. Electric charge measures the proton distribution as spread out by the finite size of the proton, while proton scattering measures the nuclear matter distribution (protons and neutrons) as spread out by the range of nuclear forces. This effect of spreading out can be estimated by adding squares of radii. Then, since the two protons and two neutrons in He^4 are expected to be distributed in the same way, the two experimental values of $r_{\frac{1}{2}}$ disagree unless the range of effective nuclear force is appreciably larger than the electromagnetic size of the proton.

Thus far the scattering data for protons on helium have been analyzed on the basis of the eikonal approximation, as extended by Glauber to include multiple scattering effects (*18*). These calculations (*19, 20*) are in general agreement with each other. The later calculation (*20*) describes the experiments somewhat better, out to what appears to be a second minimum in Fig. 4, but predicts a dip there which is too marked. In both papers, the authors point out that their calculational method is not expected to work except at small angles, whereas the experiments extend to nearly 60°.

It is known that the eikonal approximation in the form used is valid only for angles θ (measured in radians) such that $kr_{\frac{1}{2}}\theta^2$ is fairly small in comparison with unity; more complicated versions of the eikonal approximation are available (*21*). The quantity k here is the incident particle momentum measured in inverse length units; for a proton whose energy is 1000 Mev, $k = 8.8$ F^{-1}. Thus the calculations just cited should be reliable only for scattering angles somewhat smaller than 15°, that is, not even up to the first minimum. It is perhaps fortunate, therefore, that the solid curve of Fig. 4, calculated on the basis of potential scattering (*16*), agrees so well with experiment. Another calculation based on potential scattering (*22*) shows fairly good agreement with the experiments on elastic scattering of 1000-Mev protons from C^{12} and O^{16}, although it uses the small-angle approximation.

The principal reason for the failure of the simple form of the eikonal approximation at the larger angles is that the longitudinal component of the momentum transfer vector is neglected. When an incident particle is scattered through a small angle, the initial and final momentum vectors are nearly parallel, and the momentum change is nearly perpendicular to the initial direction. At larger angles, however, the component of the final momentum vector along the incident direction becomes appreciably different from the initial momentum, and the component of the momentum transfer vector along the incident direction can no longer be neglected. This gives the integrand of the scattering amplitude an additional oscillatory part, which decreases its magnitude. A calculation of the effect has been made by Ross (*23*), who used a very simple nuclear model. For $kr_{\frac{1}{2}} = 10$, the amplitudes based on the Glauber formalism are decreased by a factor 1.27 at 10°, 3.36 at 20°, and 6.58 at 30° (*23*). While these numbers must be regarded as tentative at this time, the fact that the scattering probability is proportional to the square of the amplitude suggests that the simple eikonal approximation cannot be relied upon for the interpretation of scattering data for protons on helium beyond the first diffraction minimum.

Summary

Principal emphasis in the 1955 review was placed on the use of high-energy electrons for the exploration of nuclear structure. The application of high-energy techniques to the study of the low-energy properties of atomic nuclei has become enormously more extensive in the intervening 13 years, both in variety and in the detail encompassed. Particular attention is paid to the recent work on the elastic scattering of electrons from the calcium isotopes and to the comparison of the elastic scattering of protons and electrons from He^4.

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