data presented here are consistent with the pH-dependent equilibrium between enzyme conformations shown in Fig. 1, and indicate that the complex ionization constants K^*_E and K^*_{ES} can be determined by application of the approach described in this paper.

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Space-Filling Polyhedron: Its Relation to Aggregates of Soap Bubbles, Plant Cells, and Metal Crystallites

Abstract. *A fourteen-faced space-filling polyhedron which closely approximates the actual distribution of four-, five- and six-sided polygons found in packings of soap bubbles and biological cells is proposed as an alternative to the Kelvin tetrakaidecahedron as the ideal polyhedron for these packings. This polyhedron may also have relevance to crystallite morphologies and crystal structures.*

In nature, partitions of space into polyhedral cells by "close-packing" of bodies, such as aggregates of soap bubbles (*1*), plant cell tissue (*2*), and metal crystallites (*3*) tend to conform to at least three rules: (i) the average number of faces on aggregated bodies approaches 14 faces per body; (ii) the average number of sides per face is 5.143; and (iii) the vertices are generally tetrahedral, formed by four cells whose juncture angles are close to 109°28′ (*4*). The latter restriction is a consequence of the minimization of surface energy, which causes each body to enclose the greatest volume with the least amount of surface area (*5*), given the special circumstances that determine the form of each body.

Thus far, the only polyhedron which satisfied these conditions and packed with other identical units to fill space was the "tetrakaidecahedron" (Fig. 1a) of Lord Kelvin (*6*). This is closely related to the truncated octahedron, one of the 13 Archimedean semiregular polyhedra; it has eight doubly curved hexagonal faces and six quadrilateral faces with bowed edges. The curved surfaces are a requirement of the minimization of surface energy (*7*).

Matzke and Nestler (*8*), however, have demonstrated that the statistical distribution of polygon faces on packed soap bubbles differs markedly from Kelvin's tetrakaidecahedron in that the bubbles showed a predominance of pentagonal faces. Studies of metal crystallites (*7*) and vegetable cells (*1*) showed similar distributions (Table 1 and Fig. 2).

To date, there has apparently been no report of a polyhedron with the appropriate distribution of kinds of faces and with the ability to pack to fill space. However, such a polyhedron, the β-tetrakaidecahedron, is now proposed (Fig. 1c). It can be mechanically derived from the Kelvin polyhedron (α-tetrakaidecahedron) by taking any edge common to two hexagons plus the edges that meet at each end of this edge (Fig. 1a), rotating them 90° and reconnecting them. The resultant polyhedron (Fig. 1b) with four quadrilateral, four pentagonal, and six hexagonal faces will also pack to fill space. The same operation is then performed with the same group of edges on the opposite side of the polyhedron (Fig. 1c). This transformation retains the same number of faces (14), vertices (24), and edges (36) as the α-tetrakaidecahedron, and the vertex juncture angles remain at 109°28′. The β-tetrakaidecahedron has two quadrilateral, eight pentagonal, and four hexagonal faces, which give an average of 5.143 sides per face.

The percentage distribution of the kinds of faces on the α-tetrakaidecahedron and the β-tetrakaidecahedron and their relationship to the distribution of faces in natural packings are shown in Table 1 and Fig. 2.

A packing of a group of β-tetrakaidecahedra is shown in Fig. 3. The packing arrangement belongs to the space group $P4_2/mnm\text{-}D_{4h}^{14}$ (*9*). The centers of the polyhedra correspond to special position 2*a*, and the nodal points of the interstitial network correspond to positions 4*d* and 8*j* of that space group (*9*).

The centers of β-tetrakaidecahedra cells form a body-centered tetragonal lattice which, if the c/a axial ratio is set at unity, is equivalent to the body-centered cubic lattice. Therefore, it is

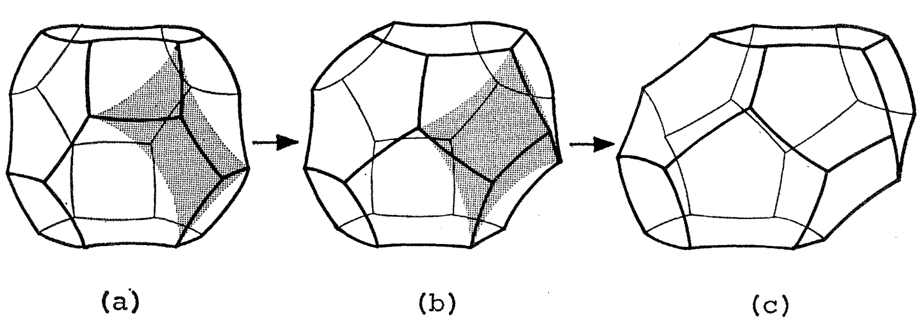

Fig. 1. The transition from the α-tetrakaidecahedron (a) through a polyhedron with four quadrilateral, four pentagonal, and six hexagonal faces (b) to the β-tetrakaidecahedron (c).

Table 1. Summary of distribution of polygon faces in bubbles, vegetable cells, metal grains, α-tetrakaidecahedron (Kelvin), and β-tetrakaidecahedron.

Edges per face (1) (No.)	600 Uniform bubbles 0.1 or 0.2 cm³ (1) (%)	100 Small bubbles (0.05 cm³) in mixture (8) (%)	50 Large bubbles (0.4 cm³) in mixture (8) (%)	Mixture of 50 large and 100 small bubbles (8) (%)	450 Vegetable cells (1) (%)	30 Beta brass grains (3) (%)	Kelvin and alternative forms of the tetrakaidecahedron (%)	
							α	β
3					5.1	2.5		
4	10.5	32.9	11.3	22.9	27.3	20.2	42.9	14.3
5	67.0	58.1	48.1	56.1	39.7	43.6		57.1
6	22.1	8.9	28.3	19.8	25.4	28.7	57.1	28.6
7	0.4		11.2	6.0	6.3	4.6		
8			5.9	0.3	0.8	0.7		
9			1.1	0.05	0.1			

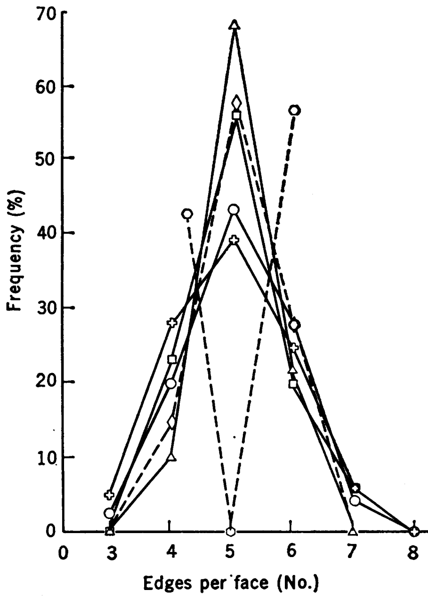

Fig. 2. Percentage distribution of polygon faces in vegetable cells (crosses), uniform bubbles (triangles), β brass grains (circles), mixed bubbles (squares), α-tetrakaidecahedron (hexagons), and β-tetrakaidecahedron (diamonds).

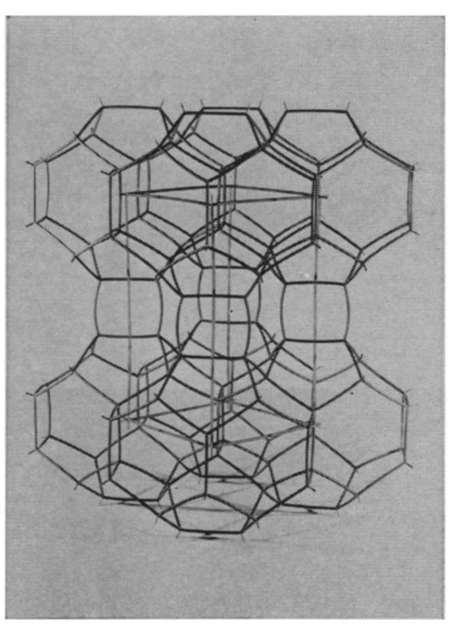

Fig. 3. A packing of β-tetrakaidecahedra with the tetragonal lattice included.

no surprise that the β-tetrakaidecahedron is related to the α-tetrakaidecahedron, which is a basic polyhedron packing in the body-centered cubic configuration.

Continuous topological transformations of points and bonds permit a number of other space-filling polyhedra to be derived from this polyhedron. For example, as the parameters x and z approach zero (that is, as position $8j$ approaches position $2b$), the β-tetrakaidecahedron packing transforms to the rhombic dodecahedron packing (basic polyhedron packing in a face-centered cubic configuration). If z goes to zero ($8j \rightarrow 4f$), the β-tetrakaidecahedron transforms to a space-filling polyhedron with eight curved pentagonal faces and four rhombic faces. Then continuing from this polyhedron, if y ceases to equal x ($4f \rightarrow 8i$), a distorted form of α-tetrakaidecahedron is defined. This is the minimum symmetry version of the α-tetrakaidecahedron packing in this space group.

The properties of this polyhedron and its packing must be more thoroughly examined; namely, the dihedral angles must be calculated, the reciprocal polyhedron and net determined, relationship of this polyhedron to ellipsoid packing systems and its hierarchical placement in the family of unitary space-filling polyhedra found, and relationship of the surface area to volume calculated. The relevance of this polyhedra packing to the structure of liquids (*10*) and to clathrate compounds such as the gas hydrates must also be considered.

In preliminary calculations of the ratio of surface area to volume, the β-tetrakaidecahedron was found to require roughly 4 percent more surface area to enclose the same volume as the α-tetrakaidecahedron. Nonetheless, for reasons as yet unknown, natural packings of bodies seem to prefer the composition of faces exemplified on the β-tetrakaidecahedron to the more regular composition of faces on the α-tetrakaidecahedron.

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Distilled-Deionized Water: A System for Preparing and Distributing Large Volumes

Abstract. *A system for preparing and distributing 100 liters of distilled-deionized water per day is described. Novel features are an overflow regulator, a "barometer" tube (permitting secondary reservoirs), and a pressure-controlled shutoff valve.*

A simply operated, inexpensive, and self-regulated system for preparing and distributing large volumes of distilled-deionized water is described (Fig. 1).