Reports

Gravitational Inconsistency in the Lunar Theory: Numerical Determination

Abstract. *Preliminary numerical integrations of the lunar motion indicate that defects in the lunar ephemeris, due to omissions in the revised Brown lunar theory, produce errors of the order of several hundred meters in the coordinates at certain times. Such errors are large enough to affect adversely analyses of data from spacecraft, as well as determination of ephemeris time. Distinct planetary periodicities seem to appear in the residuals.*

The tabulated positions on which both astronomers and space-flight engineers rely for knowledge of the motion of Moon are based on revised versions of the Brown lunar theory (*1*, *2*). Thus the question of errors in the theory is one of great concern to us, for they will be reflected in the JPL (Jet Propulsion Laboratory) Ephemeris Tape System, the ephemeris used for virtually all computations of spacecraft trajectories in the U.S. space program.

An earlier report (*3*) showed range residuals from Lunar Orbiter spacecraft to be greatly reduced by application of known corrections to the lunar ephemeris. These results were in fact interpreted as experimental confirmation of the validity of the corrections. More-recent tracking data from lunar spacecraft, however, have exhibited much larger residuals—frequently several hundred meters in range. Velocity anomalies were observed in the transponder data from Surveyor I sitting on the lunar surface.

As a result of the JPL seminar (*4*) we have integrated the equations of Moon numerically over the period between 25 April 1966 and 26 April 1968. Comparison of the integration with the coordinates and velocities of Moon, tabulated in the theoretical ephemeris, indicates considerable departure of the theory from strict gravitational consistency. It appears that the smallness of the final residuals (*3*) was at least partly due to anomalously small values of the gravitational defect in the theory during the time period involved. The results (*3*) are in no way compromised, but they are not indicative of the overall accuracy of the current JPL ephemeris.

The lunar theory consists of algebraic expressions for the geocentric spherical coordinates of Moon. These expressions, Fourier series whose parameters are explicit functions of time, were obtained by formal integration of the equations of motion, suitably expanded in series form. The author of such a theory must exercise judgment in determining the precision to which the expressions are carried. If the theory contains omissions of terms that are significantly large, the theory does not predict positions that strictly accord with the law of gravitation.

The Brown lunar theory suffers from such omissions; such an admission is no slur on Brown, for he would have had to have been a remarkable visionary to foresee the demands that technology would make on his theory half a century later. Clemence (*5*) has estimated that the errors due to neglected terms of planetary perturbation may exceed 0.1 second of arc in longitude and latitude.

The current JPL lunar ephemeris LE 4 (*2*) consists of an evaluation of the "improved lunar theory" (*1*) modified in four respects: (i) LE 4 contains corrections making it consistent with the currently adopted International Astronomical Union (I.A.U.) constants (*6*), the value of J_2 excepted; (ii) the Eckert transformation corrections (*7*) have been added; (iii) a one-term error found by Eckert has been corrected; and (iv) the coordinates are transformed to give geometric rather than apparent positions. Velocities were obtained by numerical differentiation. The integration was compared to this ephemeris, and the position residuals were used to correct the scale factor and the initial conditions of the integration by Gaussian least squares. The integration and fitting sequence was repeated until convergence was obtained.

The resulting ephemeris is designated JPL Lunar Ephemeris No. 5 (LE 5). The residuals for the 2-year span of the fit (Fig. 1) provide an empirical verification of Clemence's estimate. The maximum residuals are 0.16 second of arc in longitude and 0.12 second in latitude, corresponding roughly to 300 and 225 m, respectively. The standard deviations for these coordinates are 0.057 and 0.053 second of arc; these are perhaps half the size of the corrections given by

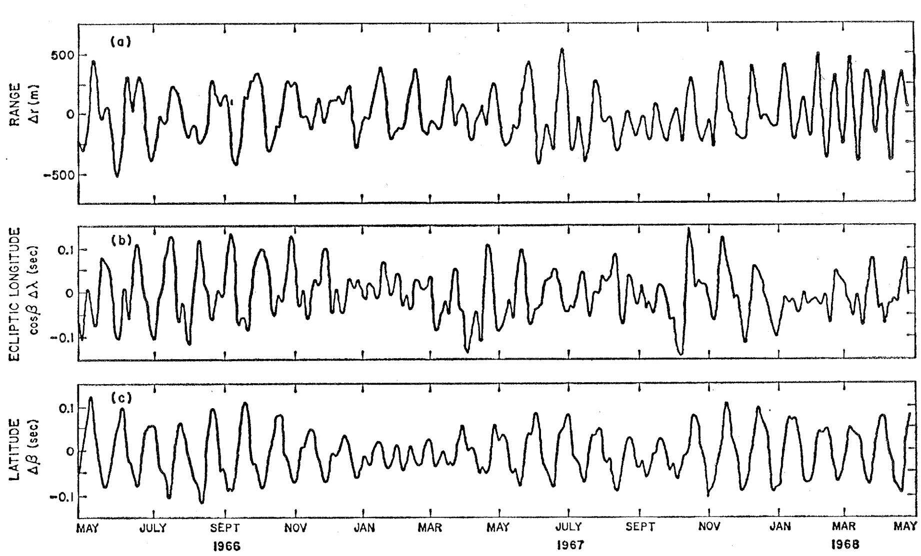

Fig. 1. Differences (LE 4 − LE 5) between the current JPL lunar ephemeris and the numerical integration, in geocentric ecliptic spherical coordinates.

Eckert (*7*) for the solar terms. The range residuals are somewhat worse than those in longitude and latitude. In terms of sine parallax, the maximum is 0.0047 second of arc and the standard deviation is 0.0018 second (roughly 525 and 200 m in range). The residuals in range rate are about 1 to 2 mm/sec.

As must be expected, the residuals show several very distinct periodicities, some of which are more pronounced in the rectangular coordinates and velocities than in the spherical coordinates of Fig. 1. We have not yet performed a spectral analysis of the residuals, but visual examination of the data leads us to conclude that periodicities appear closely resembling several planetary arguments, such as an Earth + Venus period (140 days) and the synodic periods of Venus (600 days) and Jupiter (400 days). In addition to these, of course, there are periodicities of about 1 month. If a spectral analysis reveals that most of the residuals can be correlated with physically significant periods, one may artificially construct terms causing the theory to represent gravitationally consistent motion more adequately.

Our result implies a problem in timekeeping. Ephemeris time, which was until recently the closest determinable approximation to Newtonian time, is determined in practice by comparison of observations with the theoretical longitude of Moon; it is assumed that the theoretical longitude is not contaminated by any error. If such errors exist, the practical foundation of ephemeris time is compromised. An error of 0.16 second of arc in the theoretical longitude produces an error of about 0.30 second of time in the determination of ephemeris time. It appears that, if ephemeris time is to have any validity on time scales below the 1-second level of precision, the planetary portion of the lunar theory must be recalculated.

If our work indicates defects of hundreds of meters in the ephemeris, one may ask why the residuals (*3*) were so small. By a fortunate coincidence the gravitational defects in the ephemeris were smaller than usual during the two periods of spacecraft data, especially that of November 1966, during which the range residuals (Fig. 1) never exceed 150 m and have a mean value near zero.

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References and Notes

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5. G. M. Clemence, in *Proc. JPL Seminar on Uncertainties in the Lunar Ephemeris*, J. D. Mulholland, Ed. (Jet Propulsion Laboratory Tech. Rept. 32-1247, in press).
6. *Proc. General Assembly Intern. Astron. Union 12th Hamburg 1964* (Academic Press, New York, 1966), vol. **12B**, p. 595.
7. W. J. Eckert, M. J. Walker, D. Eckert, *Astron. J.* **71**, 314 (1966).
8. The constants used in the integration were those of the I.A.U. System of Astronomical Constants, except for the following: astronomical unit, 149,597,900 km; Earth's gravitational constant, 398,601.3 km^3/sec^2; Earth-Moon mass ratio, 81.302; Earth's oblateness factor (J_2), 0.00111157; scale factor, 6378.1495 km. Apart from J_2, these are consistent with radar and radio tracking results.

9. Supported by NASA contract NAS 7-100. We thank G. M. Clemence for drawing our attention to this problem. Working independently, F. M. Sturms has found residuals similar in nature and magnitude. We have benefited from the cooperation of W. L. Sjogren and C. Cary who are studying the spacecraft data from Lunar Orbiter and Surveyor, respectively.

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Gravitational Inconsistency in the Lunar Theory: Confirmation by Radio Tracking

Abstract. *When range and Doppler observations of space probes near or on Moon are reduced by use of a lunar ephemeris calculated from the Brown lunar theory, residuals as large as 440 meters in position and 1.5 millimeters per second in velocity are observed. When the calculations are repeated with use of LE 5, the integrated lunar ephemeris described* (1), *the residuals are greatly reduced.*

The numerically integrated lunar ephemeris (LE 5) described (*1*) has been tested by reduction of spacecraft data. Range data from five Lunar Orbiters, as well as Doppler (range-rate) data from a Surveyor resting on the lunar surface, confirm the presence of systematic errors in the currently adopted Jet Propulsion Laboratory (JPL) lunar ephemeris (LE 4) and demonstrate the greater accuracy of LE 5.

Observations of a Lunar Orbiter are used for detection of ephemeris errors in the following way: The spacecraft moves in an elliptical orbit about Moon. Range and Doppler measurements, which are independent of each other, are recorded at the tracking station. The Doppler data alone are used for determination of the spacecraft's orbit, from which its selenocentric position vector **r** can be calculated within 100 m. This computation is insensitive even to very large errors in the lunar ephemeris. From a priori information, **R**, the position vector of the tracking station, is known within 30 m. The geocentric position vector of Moon, $\mathbf{A}_{☾}$, is obtained from the ephemeris. The range to the spacecraft is calculated at

$$\rho = |\boldsymbol{\rho}| = |\mathbf{A}_{☾} + \mathbf{r} - \mathbf{R}|$$

with an additional error of ± 4 m being introduced at this point by the finite size of the computer word. A

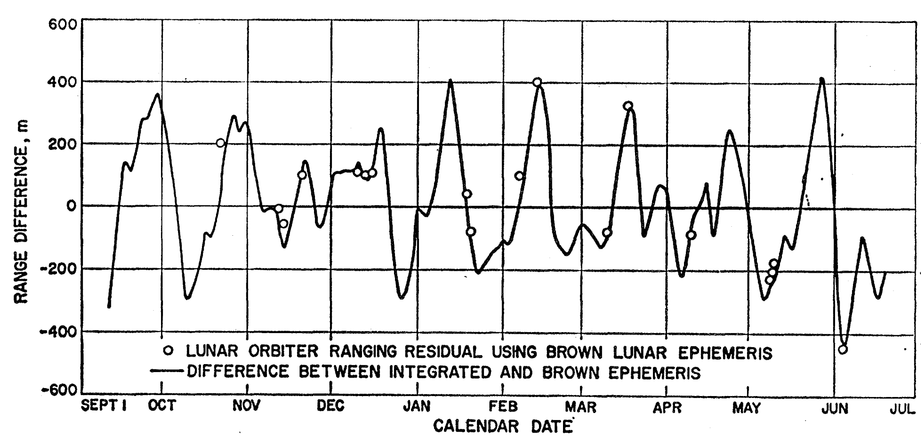

Fig. 1. Range residuals on the geocentric radial coordinate of Moon, 1966–67.