ent. By dropping some of these assumptions, one can find more general models, but the required effort in computation is much greater than is needed for the present model.

The dynamo reverses the fields somewhat more quickly than does Sun, but the field magnitudes and gross structure, and particularly the process of field reversal, agree with the observations. These results support the assumption of large-scale thermally induced Rossby-mode disturbances in the solar convection zone and photosphere.

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## Wake Collapse in Stratified Fluid: Experimental Exploration of Scaling Characteristics

Abstract. *Passage of a submerged self-propelled body or other mixing device produces a region of more or less homogeneous fluid, in a fluid having a stable vertical density gradient (stratified), which initially expands vertically and then falls back (collapses). Maximum expansion* $Z_2$ *at time* $t_2$ *after the start of mixing are dependent variables related to the diameter* $Z_1$ *of propeller or mixer and to the Väisälä-Brunt period* T *by* $T/t_2 = 2.5$ *and* $Z_2 t_2/Z_1 T = 1.3$. *These scaling relations are first-order approximations.*

In general the oceans, the atmosphere, and even a glass of water tend to be stably stratified in the vertical direction. Stable stratification means that lower particles of fluid are denser than higher particles. I shall not consider the recognized important unstable anomalies, usually near the air-Earth and water-air interfaces.

In sea water, in which compressibility can often (but not always) be neglected, particles of heavy cold or salty water (or both) tend to migrate downward, and particles of lighter warm or less-salty water (or both) tend to migrate upward. A measure of the magnitude of the vertical stability is given by the Väisälä-Brunt period:

$$T = 2\pi/[(g/\rho)(d\rho/dz)]^{1/2} \text{ sec/cycle}$$

where $g$ is the acceleration of gravity; $\rho$, the mean density of the fluid; and $d\rho/dz$, the vertical gradient of density. Conceptually, Eckart (*1*) describes $T$ as the period during which a particle of fluid would oscillate about its equilibrium position in a stratified fluid if it could be lifted or depressed from this position and be released. A strongly stratified fluid has a shorter oscillation period (lower value of $T$) than a weakly stratified fluid, for which $T$ is longer.

Some aspects of the novel phenomena of "wake collapse" following the mixing of a region of stratified fluid have been discussed (*2*, *3*). Figure 1 illustrates the mechanism of collapse of vertical wake; the region below the top wavy line represents water that has increasing density with depth; the circle below the surface is an idealized prepresentation of the end view of a "tube" of water that has been turbulently mixed by the passage, of a self-propelled body, perpendicular to the center of the circle. After mixing, the average density of the water inside the circle is more or less uniform, having an average value approximately equivalent to the density of the undisturbed fluid at the level of the center of the circle. Thus the water in the upper part of the circle is heavier than the water just outside, and the water in the lower part is lighter than the water immediately outside. The result is an unstable condition wherein the circle of mixed fluid flattens out (collapses) in the vertical direction and spreads horizontally —as symbolized by the cigar-shaped curve.

Pictures of a submerged self-propelled body leaving a dye-marked wake have been used (*2*); the wake-collapse phenomenon produced significant internal waves in the stratified fluid for some time after passage of the body. Pictures have shown (*3*) how the passage of a self-propelled body in a stratified fluid can be simulated approximately by experiments with two-dimensional models, with use of a centrally located circular generator of turbulence (called a mixer), in a narrow cell of stratified water; several mixers of different diameters were used. Each mixer was a multibladed propeller-like device that did not rotate. Localized turbulent mixing was accomplished by a 2.5-second small-amplitude to-and-fro movement of a mixer, imparted to it by a lever system actuated by a motor outside the cell. The propeller-like mixer had a narrow shroud around it to concentrate turbulent mixing and minimize nonturbulent circulation.

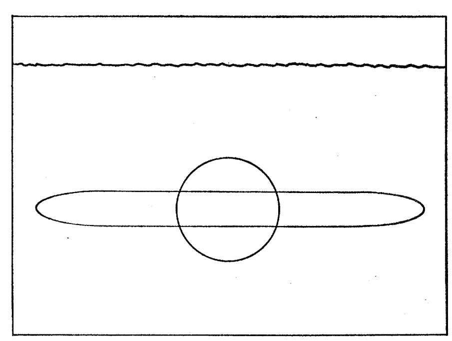

Fig. 1. An idealized "slice" of a volume of stratified fluid through which a self-propelled body has recently passed, perpendicularly through the center of the circle. The mixed wake of the body has turbulently diffused outward, as is represented by the area within the circle. Turbulent diffusion in the vertical direction is eventually limited by the density structure of the stratified fluid. Soon after maximum vertical expansion is reached, the mixed fluid will "collapse" vertically and spread horizontally, as is represented by the cigar-shaped pattern.

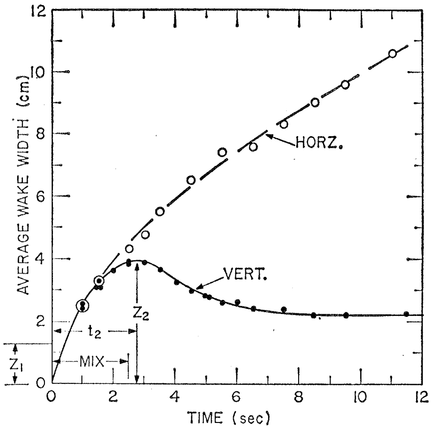


Fig. 2. Representative time history of expansion of vertical wake and "collapse" in stratified water, where the Väisälä-Brunt period $T = 7$ sec/cycle and mixer diameter $Z_1 = 1.27$ cm.

Figure 2 shows an example of the average vertical and horizontal dimensions of a particular wake-collapse event as determined by dye measurements (*2*, *3*). In this instance a mixer having a diameter of $Z_1 = 1.27$ cm was located near the center of a water-filled transparent cell, 30 cm wide, 7.3 cm deep, and 2.5 cm thick. A Väisälä-Brunt period $T = 7$ sec/cycle was maintained in the region of the mixer by electrical cooling of the bottom of the cell and heating of the top. Time-zero was measured from the start of mixing. In Fig. 2 the vertical and horizontal wakes first expanded at about rates; when $t_2 =$ 2.75 seconds, the vertical wake reaches a maximum of $Z_2 = 3.9$ cm before collapsing to a lesser value. The horizontal wake expands continuously during the time scale shown. Wall effects that become important for long values of $t$ have been discussed (*2*).

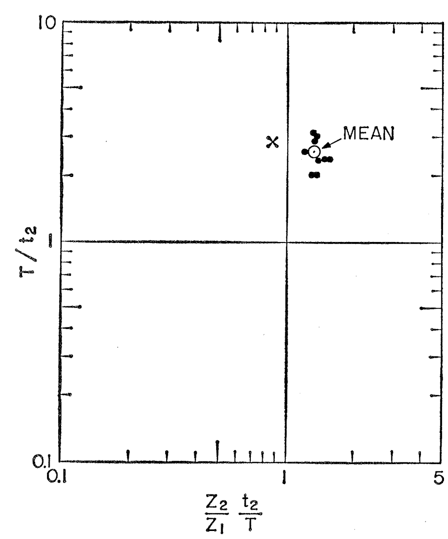


Fig. 3. Dimensionless plot of Väisälä-Brunt period $T$, divided by the time for maximum expansion of vertical wake before collapse, $t_2$, versus the product of the maximum diameter $Z_2$ of vertical wake and the time between the start of turbulent mixing and the time at which maximum vertical expansion occurs ($t_2$), divided by the product of propeller (or mixer) diameter $Z_1$ and the Väisälä-Brunt period $T$. Point $x$ is from a small-scale three-dimensional experiment; the small round points, from experiments using a two-dimensional model in which the independent variables $T$ and $Z_1$ were varied. It is interesting, as a first approximation, that the mean value of $T/t_2 \simeq 2.5 = k_1$ and that $Z_2t_2/Z_1T \simeq 1.3 = k_2$; thus the dependent variables $Z_2$ and $t_2$ are approximately simply related to the independent variables $T$ and $Z_1$.

Experiments used six values of $T$ between 7 and 16 sec/cycle when everything else was held constant. For one value of $T = 7$ sec/cycle, experiments used four values of $Z_1$ ranging from 0.95 to 2.54 cm. Another experiment (*2*) also yielded data for $T = 2.8$ sec/cycle and $Z_1 = 1.8$ cm (propeller diameter). The results of all experiments are displayed in Fig. 3. All points are sufficiently closely grouped around the mean values of the dimensionless quantities $T/t_2 = 2.5 = k_1$ and $Z_2t_2/Z_1T =$ $1.3 = k_2$ to evoke interest. The root-mean-square deviation in both instances is about 0.1.

The dimensionless quantities of Fig. 3 appear to have physical significance. For example, it would be expected that, when $T$ is large, $t_2$ would be large, everything else being held constant. Also it appears reasonable that the product of $Z_2t_2$ would be large when the product of $Z_1T$ was large. Thus to a first approximation the dimensionless quantities of Fig. 3 should be constant when mixing is essentially complete. From the first experimentally determined equation given in the previous paragraph it is evident that $t_2 = (1/k_1)T$ $\cong 0.4T$. The first and second equations of the previous paragraph indicate that $Z_2 = k_1k_2Z_1 \cong 3.4Z_1$. These simple expressions relate the unknowns — of time ($t_2$) between the start of mixing and the point of maximum vertical expansion before collapse, and the maximum vertical expansion amplitude $Z_2$— with the known Väisälä-Brunt period $T$ in the region of the mixer, and with the mixer (or propeller) diameter $Z_1$. The expressions should be helpful for first-order scaling and for theoretical refinements of the wake-collapse phenomenon. There are several reasons for the spread of points around the mean in Fig. 3:

1) The experimental determination of average dimensions of the turbulent wake, with dye as a marker, is not precise; the magnitude of error is not easily assessable.

2) The thoroughness of mixing (initial kinetic energy) is a factor needing further investigation. Various degrees of less-than-complete mixing certainly affect the magnitude of $Z_2$. It is likely that mixing was less complete in one experiment (*2*) than in another (*3*). This difference may be the reason why point $x$, from (*2*), in Fig. 3 has a significantly lower value of $Z_2t_2/Z_1T$ than have the other points for which mixing was quite uniform and probably more complete.

3) The limited experiments in which $Z_1$ was varied indicate that $k_1$ tends to be somewhat smaller for the larger values of $Z_1$.

4) As mentioned in (*3*), $Z_2$ appears to be a weak function of $T$.

5) One could not make a completely uniform vertical-density structure by my earlier method (*3*).

The simple dimensional relation of Fig. 3 should aid understanding of the complex phenomenon of wake collapse. It is presented as a first approximation because there are several second-order effects that are also important, some of which have just been mentioned.

The wake-collapse phenomenon involves additional parameters that require rather arbitrary definitions. The magnitude of the maximum rate of vertical collapse, and its associated time after the start of mixing, are important. The maximum rate of horizontal expansion after collapse and the associated time [$t_H$ in (*3*)] may be important. The minimum width of vertical wake many Väisälä-Brunt periods after mixing and the maximum limit of expansion of horizontal wake are factors sensitive to boundary conditions and cannot be determined satisfactorily in small model tanks. An important quantity that may be amenable to small-scale experiments is the initial rate of expansion of the wake near time-zero, but uncertainties may arise due to differences in the times of the start of mixing and its duration.

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