satisfactory in view of the simplicity of the model. Although this model is certainly not unique, it suggests that models of this type should be explored in attempting to interpret pulsars. If this model is correct, it shows that the diameters of the emitting region and the central object are very much less than the distance given by ct_s, and therefore much less than 3300 km. This is consistent with the limit suggested by the width of the final subpulse of Pulsar 4. This model is most consistent with those concepts of pulsars in which the central object is a pulsating star.

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Parameters of the Plasma Affecting the Radiation of Pulsar 1

Abstract. *Analysis of the relation between time delay and frequency for pulses from Pulsar 1 shows that the dispersive region of the ray path must exceed 300 astronomical units and have an average electron number density less than 8000 per cubic centimeter and average magnetic field strength less than 2×10^{-3} gauss. These requirements almost guarantee that the observed dispersion takes place in the interstellar medium.*

One of the remarkable features in the recent observations of pulsars is the high degree of accuracy with which the arrival time t_1 and t_2 of pulses at different radio frequencies ν_1 and ν_2 satisfy the condition

$$t_1 - t_2 = B\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right)$$

where B is a constant proportional to the total electron content along the path from source to receiver. In fact, data taken at Arecibo between 40 and 430 Mhz for the source located in the constellation Vulpecula shows that this relation is satisfied to an accuracy of about 1 part in 3000 at 40 Mhz (*1*). However, as the relation is an approximation based upon the assumption that the radio frequency greatly exceeds both the plasma and cyclotron frequencies in the ionized interstellar gas, we shall show that the absence of any sizable deviations from this law can be used to establish upper limits on the average electron density and magnetic field along the path of propagation and, from this, lower limits on the extent of the ionized region.

When one retains first-order corrections to the expressions for the travel time of a pulse through a uniform (collisionless) plasma with electron number density n_e and magnetic field strength B_o, one finds that

$$t_1 - t_2 = B\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right)(1 + T_1 + T_2)$$

where

$$T_1 = 3\nu_p^2\,(\nu_1^2 + \nu_2^2)\,/\,4\nu_1^2\,\nu_2^2$$

and

$$T_2 = \pm 2 f_c \cos\gamma\,(\nu_2^3 - \nu_1^3)\,/\,\nu_2\,\nu_1\,(\nu_2^2 - \nu_1^2),$$

with ν_p, f_c, and γ the plasma frequency, cyclotron frequency, and angle between the magnetic field and the direction of wave propagation, respectively. From the data at Arecibo over frequencies between 40 and 430 Mhz, we conclude that T_1 and T_2 must be less than 3×10^{-4}. Hence, from T_1 we find

$$n_e < 8000 \text{ electron/cm}^3$$

while (assuming cos γ is of order unity), we have from T_2 that

$$B_o < 2 \times 10^{-3} \text{ gauss}$$

Although these estimates assume a uniform plasma, one can easily show that for a nonuniform plasma where n_e equals $n_{eo} + \delta n_e$, the condition on the average electron number density, n_{eo}, becomes even more restrictive since one obtains $n_{eo} + \langle(\delta n_e)^2\rangle / n_{eo} < 8000$ cm^{-3}, while the magnetic field condition is modified somewhat and becomes simply $\langle n_e B_o \cos\gamma\rangle / n_{eo} < 2 \times 10^{-3}$ gauss.

The upper limit on average electron number density together with the value of 3.84×10^{19} cm^{-2} obtained for $n_{eo} L$ shows that L must exceed 5×10^{15} cm $\cong$ 300 astronomical units. This result eliminates the possibility of the dispersion taking place in a stellar corona. Although an H II region has been observed in the source area at Arecibo, it appears far too small to provide the needed electron content. Hence, it seems almost certain that the dispersion is taking place in the general interstellar medium.

One final point that arises from the expression for time delay is that the time difference at a single frequency between the ordinary and extraordinary waves is given, to lowest order in B_o, by

$$\Delta t = \frac{4\,B\,f_c \cos\gamma}{\nu^3}$$

From the above results one should expect $\Delta t < 15$ μsec (at 430 Mhz), which is considerably less than the observed pulse width of 37 msec. This, plus the fact that the observed pulse shape is qualitatively the same at all frequencies (*2*), suggests that the shape cannot be caused by a difference in arrival time of the two magnetoionic waves.

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10 April 1968

Thermally Driven Rossby-Mode Dynamo for Solar Magnetic-Field Reversals

Abstract. *There is increasing interest in the possible existence of large eddies or "Rossby waves" in Sun's convection zone and photosphere. It is shown that many flows of this type, driven by an equator-pole temperature difference, act as hydromagnetic dynamos to produce magnetic fields that periodically reverse. The periods and field amplitudes agree with solar phenomena within an order of magnitude.*

Ward (*1*) postulates the existence of a "Rossby type" general circulation of the solar photosphere, in which large horizontally flowing waves or eddies (larger in dimensions than sunspot groups) carry momentum from pole to equator and maintain the equatorial acceleration. He supports this view with extensive statistical analyses of the proper motions of sunspots. Citing the studies of large-scale bipolar and unipolar magnetic regions (*2*), Starr and Gilman (*3*) consider the energetics of such waves and argue that they would be hydromagnetic in character. From the field data Bumba *et al.* (*4*) suggest the possibility of very-large-scale cellular disturbances. In addition, Plaskett (*5*) deduces the existence of Rossby waves from measurements of Doppler shift, but, as Ward (*6*) observes, his data sample is too small for conclusive results.

These Rossby waves or large-scale solar eddies do not originate spontaneously; they must somehow be excited. In Earth's atmosphere similar waves originate in response to latitudinally nonuniform solar heating. The idea of a solar equator-to-pole temperature difference to drive axisymmetric circulations, including the differential rotation, is old, dating to Eddington and Bjerknes. However, the idea that such a temperature difference can create large-scale asymmetric eddy or wave-like motions is much more recent, dating from Ward (*1*).

The observational evidence of a temperature difference at photospheric levels is very conflicting. Perhaps stronger gradients occur inside the convection zone where they cannot be seen; they will exist if local rotation influences significantly the vertical heat flux in granulation and supergranulation. We know that the onset of convective instability is inhibited by rotation (*7*); Weiss (*8*) has estimated its inhibitive effect on the heat transport in fully developed cells, and Roxburgh (*9*) has applied his results to the solar convection zone.

For years Plaskett (*10*) has espoused the idea that the differential rotation is a "heliostrophic" wind: that is, a flow for which horizontal pressure gradient forces, arising from horizontal temperature gradients, are to the lowest order balanced by Coriolis forces. However, he considers it a purely photospheric phenomenon; he argues that the underlying convection zone is in solid rotation as a result of turbulent mixing.

I feel that the heliostrophic wind may pervade a large fraction of the solar convection zone, even to its inside boundary. Also, the dynamics of this wind surely is a hydromagnetic problem. In particular, strong subsurface zonal or toroidal magnetic fields are important. With this in mind, I made a linear, normal-mode analysis of the stability to Rossby-wave disturbances (*11*, *12*) of heliostrophic-type flows in a toroidal field. This work is a direct hydromagnetic generalization of the classic "baroclinic stability" problem of dynamic meteorology (*13*). The model is necessarily oversimplified in several respects as far as Sun is concerned. For example, the fluid is nearly incompressible and confined to a channel between two latitude circles within a single hemisphere (Fig. 1). Nevertheless, several of the results warrant further investigation.

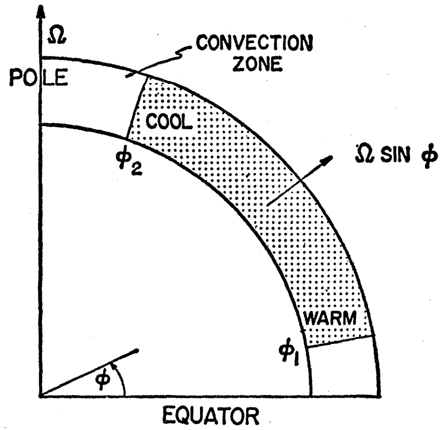

Fig. 1. Schematic cross section of Sun, with region for dynamo stippled.

The linear solutions show growing disturbances transporting heat horizontally from warm to cold latitudes, just as in the nonmagnetic problem. Small vertical motions in the disturbances, whose principal function is to sustain the instability by converting potential into kinetic energy, also drag up horizontal fields and produce weak vertical flux loops. The horizontal disturbance motions then separate the ends of these flux loops, carrying fields of predominantly one sign toward the pole, and fields of the opposite sign toward the equator. In this way an axisymmetric meridional or poloidal field is produced. In other words, the linear solutions show the first half of a hydromagnetic dynamo cycle. For Sun, Babcock (*14*) postulates a process of this type, forming unipolar fields near the poles. Bumba and Howard (*2*) observed the systematic poleward movement of magnetic flux, but could not assess the role of large-scale meridional flow.

When these disturbances occur in a flow which in basic profile resembles the differential rotation, the vertical field patterns tilt upstream like the observed bipolar magnetic regions. A similar tilt, occurring in the velocity field, implies horizontal momentum transport toward the equator, as Ward deduced from the spot statistics (*1*). The "*e*-folding" time for the growth and spreading of the disturbances is only a few solar rotations, a time comparable to that for growth and spreading of observed bipolar regions.

These results encourage me to undertake a study based on a somewhat more general form of the equations including nonlinear effects. The major question to be answered is: Will the dynamo cycle be completed and field reversals occur? As a prelude to the linear work described above (*12*) I expanded the complete nonlinear scaled equations in powers of the Rossby number (the ratio of fluid-particle acceleration to Coriolis force: about 10^{-1} for the solar differential rotation) and retained only terms of the zeroth and first orders. In so doing this preliminary nonlinear system was rendered incapable of acting as a hydromagnetic dynamo. Vertical flux loops and an axisymmetric poloidal field were produced, but they could not stretch out under the differential rotation to produce a new toroidal field. In the present nonlinear analysis, terms of second order in Rossby number which involve this stretching process are included; they produce a thermally forced hydromagnetic dynamo (*15*) having field reversals.

The nonlinear dynamo model is developed as follows: First I simplify the equations to apply to two superposed layers of fluid, and then expand each variable in Fourier harmonics. The particular harmonics allowed for each variable depend on its boundary conditions. The channel is bounded by rigid, perfectly conducting side walls, and rigid, nonconducting top and bottom. (I would get essentially the same results if the bottom were perfectly conducting, which condition is perhaps more realistic for Sun.) Fluid slip occurs at the sides and for some cases also at the top, but not at the bottom. Fields are completely confined within this region. The resulting infinite set of differential equations in time for the Fourier amplitudes are truncated; six harmonics for each variable (four containing longitude and latitude dependence, and two having only latitude dependence) are retained. These equations are then integrated numerically. The approach stems directly from that used successfully by Lorenz (*16*) for study of nonlinear, thermally induced Rossby modes without hydromagnetic effects. Lorenz studied particularly the well-known "rotating annulus" experiments (*17*) including the phenomenon of "vascillation."

The results of these integrations show several of the large-scale elements of a solar cycle. Large classes of solutions found so far are of Rossby-mode type in the velocity field and periodic in the magnetic fields. (I find also some nonreversing dynamos.) This period depends primarily on the strength of the velocities: the larger the velocities, the shorter the periods. The periods do not depend very much on the electrical resistance of the fluid. For horizontal velocities comparable to the solar differential rotation (for example, 100 m/sec), the period for the fields (that is, two successive field reversals) is about 4 years, compared to 20 years for the true solar cycle. Because of the simplifications inherent in the model one should not expect better agreement.

Because the equations describe explicitly only very-large-scale motions and fields (that is, greater than 10^5 km in horizontal scale on Sun), the effects of all the smaller scales, such as granulation and supergranulation, must be lumped into diffusion coefficients for heat, momentum, and magnetic-field transfer. These coefficients for Sun are rather uncertain, but in any case are larger by many orders of magnitude than the corresponding molecular values. For the cases studied, the "effective" Reynolds number UD/ν_e (U being a differential-rotation scale velocity; D, the depth of the fluid; and ν_e, the eddy viscosity) is of magnitude $\sim 10^2$ (*18*). Furthermore the thermal conductivity (*19*) is of the same magnitude as the eddy viscosity. Sustained dynamo action occurs when the effective magnetic Reynolds number $UD/\lambda_e \geq 10^3$, λ_e being an "eddy" resistivity. Increased thermal forcing gives dynamo action at smaller ordinary and magnetic Reynolds numbers.

A typical dynamo solution evolves in the following way. First the finite-amplitude nonmagnetic solution is allowed to establish itself. Then a small "seed" toroidal field is added; its magnitude is chosen so small that it will have no initial effect on the motion. The flow induces poloidal fields from this initial input in the manner described for the linear solutions. These poloidal fields stretch out into new toroidal fields opposite in sign from the original fields. The whole process then repeats, producing new poloidal fields also of the opposite sign, and so on.

In the early stages, the peak amplitudes of the fields increase with each successive cycle. After many cycles, the toroidal fields grow so strong that the associated Maxwell stresses affect the motion which then adjusts so as to reduce the induction of new fields, and successive peaks in field amplitude level off. Thereafter the fields remain of finite amplitude, reversing with a period only slightly altered. Dissipation then balances induction of fields, on the average. (The leveling-off process is less smooth for magnetic Reynolds numbers significantly above the dynamo threshold.) The fields generally stop growing when the energy stored in them (principally in the toroidal component) is 10 or 20 percent of the kinetic energy in the flow. Thus equipartition of magnetic with kinetic energy is not quite reached. This bound on the field strengths allows one to compare the magnitudes with solar observations. If one chooses the mean density for the layer in the convection zone to be about 10^{-4} g/cm^3, the toroidal fields are about 10^2 gauss—comparable to most prior estimates. The vertical fields of 1 to 10 gauss in magnitude compare well with the observed unipolar and bipolar magnetic regions; their values depend on the vertical and horizontal length scales.

How large a temperature difference ΔT between equator and pole is needed to drive the dynamo? Because of difficulties in carrying over the results of my incompressible model to Sun, whose convection zone is many scale heights thick, I can give only an estimate. The latitudinal temperature gradient relates to the vertical shear of heliostrophic flow by the "thermal wind" relation (*20*). If I use as vertical coordinate the number of pressure scale heights N over which the flow changes by its own magnitude, the thermal wind is approximately

$$(U/N) \sim (R/m\Omega)\,[\Delta T/(\pi a/2)]$$

where Ω is the mean solar rotation, m is the mean molecular weight, R is the universal gas constant, and a is the solar radius. Therefore

$$\Delta T \sim (m\Omega/R)\,(\pi a/2)\,(U/N)$$

For Sun, $a = 7 \times 10^{10}$ cm, $\Omega = 3 \times 10^{-6}$ sec^{-1}, $m = 1/2$ (ionized hydrogen), and $U \sim 2 \times 10^4$ cm/sec, so that $\Delta T \sim 40°K/N$. If, for example, the appropriate depth is 10 scale heights, $\Delta T \sim 4°K$. Roxburgh estimates that the influence of rotation upon convection can produce temperature differences of about 300°K near the top of the solar convection zone. If he is right, there is even more thermal forcing than needed for the dynamo model. On the other hand, eddy viscous stresses may be larger (effective Reynolds number, lower) than in my present model. The correct lowest-order force balance would then include viscous stresses as well as pressure and Coriolis forces. If this were the case, greater thermal forcing would be needed to drive the dynamo.

The model without magnetic fields can be contrasted with the differential-rotation models of Kippenhahn (*21*) and Cocke (*22*) who ignore thermal effects totally and equate viscous and inertial (Coriolis and centrifugal) forces. In these models the convection layer has an exactly adiabatic vertical temperature gradient as well as no horizontal gradient. My model, in addition to having horizontal temperature gradients, also assumes that the vertical temperature gradient is slightly subadiabatic. Support for this latter assumption comes from recent studies, both theoretical and experimental, of finite-amplitude nonlinear convection (*23*).

Because of the artificial sidewalls, the differential rotation in my nonlinear model is a maximum in the middle of the channel (that is, in middle latitudes) rather than at the equator. It is expected that more general models, for a complete spherical shell, will produce an equatorial acceleration as well as dynamo behavior; this possibility is being explored by others. Obviously, future models also should take better account of compressibility, for which more work is first required on the flow problem without magnetic fields.

A final difficulty with the dynamo should be mentioned. In hydromagnetic turbulence, magnetic energy may cascade by nonlinear interactions into the higher wave numbers and dissipate at a faster rate than that of the resupply of kinetic energy at lower wave numbers (*24*). If this were to happen, it would destroy the dynamo; in my present model it is prevented by the scaling restrictions and truncated harmonic representation. With more harmonics and more layers in the vertical, one can test this effect, but only partially; the same scaling assumptions are still present. By dropping some of these assumptions, one can find more general models, but the required effort in computation is much greater than is needed for the present model.

The dynamo reverses the fields somewhat more quickly than does Sun, but the field magnitudes and gross structure, and particularly the process of field reversal, agree with the observations. These results support the assumption of large-scale thermally induced Rossby-mode disturbances in the solar convection zone and photosphere.

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23 February 1968

Wake Collapse in Stratified Fluid: Experimental Exploration of Scaling Characteristics

Abstract. *Passage of a submerged self-propelled body or other mixing device produces a region of more or less homogeneous fluid, in a fluid having a stable vertical density gradient (stratified), which initially expands vertically and then falls back (collapses). Maximum expansion* Z_2 *at time* t_2 *after the start of mixing are dependent variables related to the diameter* Z_1 *of propeller or mixer and to the Väisälä-Brunt period* T *by* $T/t_2 = 2.5$ *and* $Z_2t_2/Z_1T = 1.3$*. These scaling relations are first-order approximations.*

In general the oceans, the atmosphere, and even a glass of water tend to be stably stratified in the vertical direction. Stable stratification means that lower particles of fluid are denser than higher particles. I shall not consider the recognized important unstable anomalies, usually near the air-Earth and water-air interfaces.

In sea water, in which compressibility can often (but not always) be neglected, particles of heavy cold or salty water (or both) tend to migrate downward, and particles of lighter warm or less-salty water (or both) tend to migrate upward. A measure of the magnitude of the vertical stability is given by the Väisälä-Brunt period:

$$T = 2\pi/[(g/\rho)(d\rho/dz)]^{1/2} \text{ sec/cycle}$$

where g is the acceleration of gravity; ρ, the mean density of the fluid; and $d\rho/dz$, the vertical gradient of density. Conceptually, Eckart (*1*) describes T as the period during which a particle of fluid would oscillate about its equilibrium position in a stratified fluid if it could be lifted or depressed from this position and be released. A strongly stratified fluid has a shorter oscillation period (lower value of T) than a weakly stratified fluid, for which T is longer.

Some aspects of the novel phenomena of "wake collapse" following the mixing of a region of stratified fluid have been discussed (*2, 3*). Figure 1 illustrates the mechanism of collapse of vertical wake; the region below the top wavy line represents water that has increasing density with depth; the circle below the surface is an idealized prepresentation of the end view of a "tube" of water that has been turbulently mixed by the passage, of a self-propelled body, perpendicular to the center of the circle. After mixing, the average density of the water inside the circle is more or less uniform, having an average value approximately equivalent to the density of the undisturbed fluid at the level of the center of the circle. Thus the water in the upper part of the circle is heavier than the water just outside, and the water in the lower part is lighter than the water immediately outside. The result is an unstable condition wherein the circle of mixed fluid flattens out (collapses) in the vertical direction and spreads horizontally —as symbolized by the cigar-shaped curve.

Pictures of a submerged self-propelled body leaving a dye-marked wake have been used (*2*); the wake-collapse phenomenon produced significant internal waves in the stratified fluid for some time after passage of the body. Pictures have shown (*3*) how the passage of a self-propelled body in a stratified fluid can be simulated approximately by experiments with two-dimensional models, with use of a centrally located circular generator of turbulence (called a mixer), in a narrow cell of stratified water; several mixers of different diameters were used. Each mixer was a multibladed propeller-like device that did not rotate. Localized turbulent mixing was accomplished by a 2.5-second small-amplitude to-and-fro movement of a mixer, imparted to it by a lever system actuated by a motor outside the cell. The propeller-like mixer had a narrow shroud around it to concentrate turbulent mixing and minimize nonturbulent circulation.

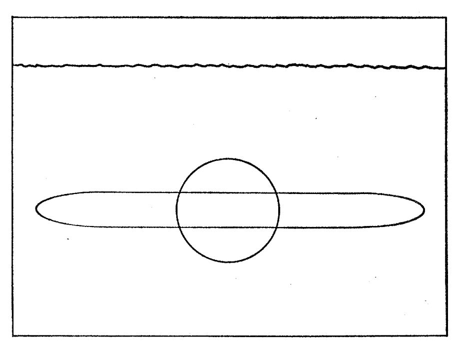

Fig. 1. An idealized "slice" of a volume of stratified fluid through which a self-propelled body has recently passed, perpendicularly through the center of the circle. The mixed wake of the body has turbulently diffused outward, as is represented by the area within the circle. Turbulent diffusion in the vertical direction is eventually limited by the density structure of the stratified fluid. Soon after maximum vertical expansion is reached, the mixed fluid will "collapse" vertically and spread horizontally, as is represented by the cigar-shaped pattern.