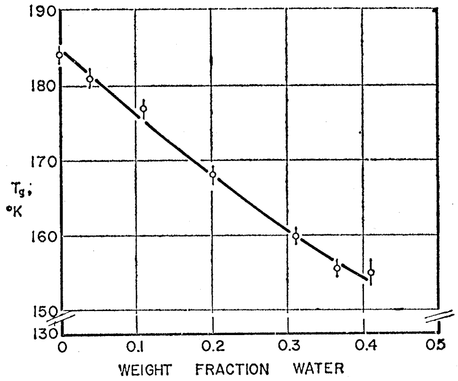

Fig. 1. Depression of the vitrification (glass-transition) temperature, T_g, of glycerol (184° ± 1°K) by addition of water. A smooth line has been drawn through the experimental points in order to reduce the error in the graphic procedure for the estimation of T_g for pure water.

into account earlier observations (*10*) of the vitrification temperature of two aqueous solutions of sulfuric acid, was that T_g for water lies between 123° and 148°K.

Various empirical rules have been proposed for prediction of T_g for a particular substance on the basis of the known magnitudes of other transition temperatures or other quantities characteristic of the substance. Thus it has been observed that T_g/T_m lies (*1*) between 0.50 and 0.67, T_g/T_b lies (*2*) between 0.25 and 0.33, and $\Delta H_v/kT_g$ (where ΔH_v is the molecular heat of vaporization of water at T_b, and k is Boltzmann's constant) lies (*2*) between 38 and 53. Table 1 lists three experimentally derived estimates of T_g for water, and estimates of the predicted range within which T_g should occur, as the estimates are computed by use of these three empirical rules.

Meteorological studies of formation of cloud and snow have been hampered by inability to supercool water by more than about 40°C. This circumstance is universally attributed to the ubiquity of foreign particles (motes) that induce nucleation prematurely and heterogeneously. In order to eliminate motes from water samples and subsequently to supercool these samples by about 35° to 40°C several techniques have been devised, including (i) copious distillation and deionization (*11*); (ii) suspension of water droplets at the interface between two liquids (*12*), or coating of such droplets with an inert skin (*13*); and (iii) production of very fine water droplets in cloud chambers (*14*). My success in supercooling water by more than 115°C (Fig. 1) was gained at the expense of gross contamination (more than 60 percent by weight) with glycerol. Nevertheless I suggest that the search for a suitable solute, that in trace amounts could suppress the crystallization of water, might prove considerably more rewarding than any of the sample purification and size-reduction procedures that I have just mentioned.

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1 February 1968

Mass of Vesta

Abstract. *The asteroid 197 (Arete) approaches asteroid 4 (Vesta) within 0.04 astronomical unit once every 18 years. It may therefore be possible to determine the mass of Vesta from observations of Arete. From 72 observations a value of (1.20 ± 0.08) × 10^{-10} solar mass was derived, the indicated uncertainty being a probable error.*

Asteroid 197 (Arete) approaches asteroid 4 (Vesta) within 0.04 astronomical unit once every 18 years. Since its discovery in 1879 there have been five such approaches—in 1885, 1903, 1921, 1939, and 1957.

It may therefore be possible to determine the mass of Vesta from observations of Arete. A discussion of 59 observations led to the value (1.17 ± 0.10) × 10^{-10} solar mass, the indicated uncertainty being a probable error (*1*).

A final discussion of the observations will be possible as soon as the search for older observations has been completed. In the meantime, a new analysis of 72 observations in 28 oppositions was made by comparison with two orbits. Orbit 1 was a numerical integration which did not take account of any Vesta perturbations. Orbit 2 was a numerical integration which included the effect of Vesta. The value 1.17343 × 10^{-10} solar mass, used for the mass of Vesta, was obtained in the aforementioned preliminary solution. The extra figures have no physical significance and were dropped in the quoted results, but they were retained in the construction of orbit 2 for computational convenience. The residuals before the differential corrections were obtained from P. Herget and C. Bardwell of the Cincinnati Observatory.

The whole period from 1879 to 1966 was divided into five basic differential correction intervals, each having the time equivalent to one of the five approaches near the middle. Fifteen combinations of basic differential correction intervals were considered, each consisting of one, two, three, four, or five basic differential correction intervals. For each of the 15 combinations three solutions were made. One was a correction of orbit 1 for an adjustment of the position-velocity vector at the epoch; the second was a correction of orbit 1 with a determination of the mass of Vesta included. The third solution was a correction of orbit 2 consisting of an adjustment of the position-velocity vector and the determination of a correction to the value used for the mass of Vesta in constructing orbit 2.

No meaningful results were obtained where only one or two basic differential correction intervals were considered. Table 1 shows the results for the cases where at least three basic differential correction intervals have been used. The corrections of orbits 1 and 2 in each case lead to the same value for the mass of Vesta.

For the cases where three or four differential correction intervals were used, the results indicate some, although not complete, consistency among themselves and with the solution covering the whole period. The reason for the incomplete consistency may be the paucity of observations. The final analysis, when additional observations are available, may show whether the increase in the number of the observations would increase consistency and whether the true uncertainties of the values obtained for the mass of Vesta are larger than those indicated by the formal probable errors. The formal probable error for the overall solution (about 10 percent of the value found for the mass of Vesta) indicates that this value has some substance.

O'Keefe (*2*) has pointed out that the value for the mass of Vesta in the overall solution and Barnard's value for the diameter (*3*) lead to a value of 8 g cm^{-3} for the density of Vesta. This makes it probable that Vesta consists of iron.

The selection of 7 seconds as a rejection limit may have been too conservative. A few trials with different rejection limits suggest that in the final analysis great care must be exercised with respect to which observations are to be rejected and in assigning weights to the observations.

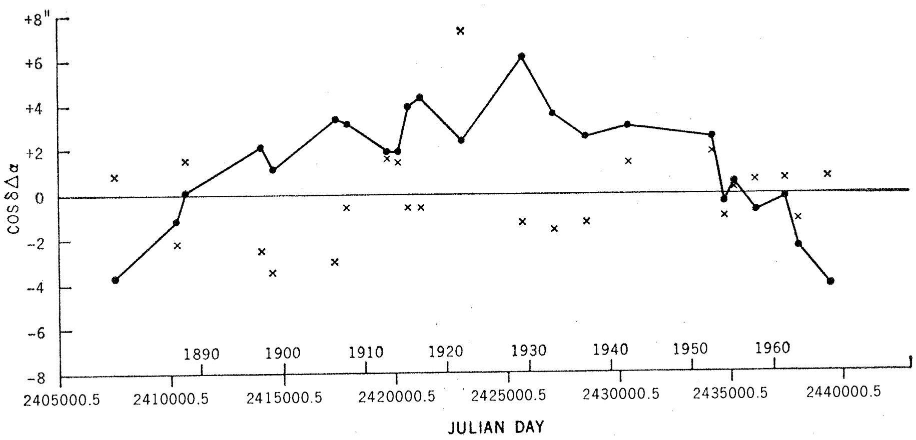

Fig. 1. Residuals in right ascensions for opposition normals; $\cos\delta\Delta\alpha$ indicates the adjusted difference between the observed and computed right ascension.

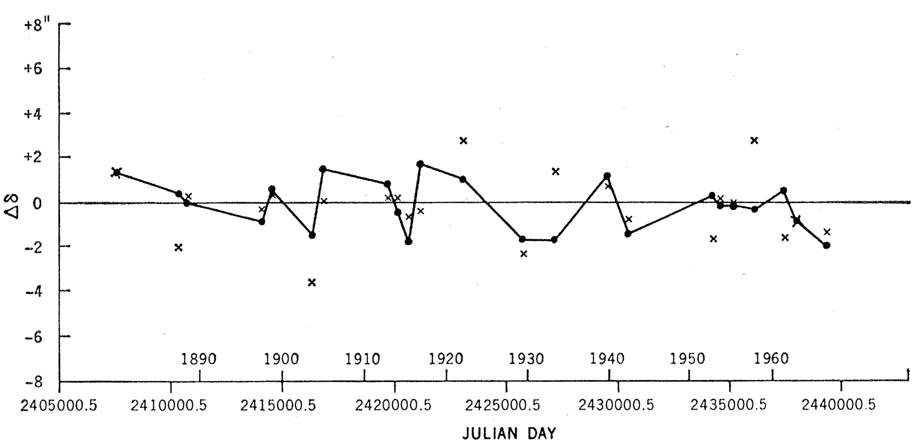

Fig. 2. Residuals in declination for opposition normals; $\Delta\delta$ indicates the difference between observed and computed declination.

Table 1. Corrections of orbits 1 and 2 and solutions for the mass of Vesta. Basic differential correction intervals (BDC) are: interval 1, from 1879 to 1894; interval 2, from 1894 to 1912; interval 3, from 1912 to 1930; interval 4, from 1930 to 1948; interval 5, from 1948 to 1966. The probable errors (p, p', and p'') of an observation of weight 1 are: p, after correcting orbit 1 without allowing for mass of Vesta; p', after correcting orbit 1 with effect of mass of Vesta included; p'', after correcting orbit 2 including correction to mass of Vesta. The masses of Vesta (m' and m'') ($\times 10^{-10}$ solar mass) and corresponding probable errors ($p'm$ and $p''m$) are derived from corrections of orbits 1 and 2. Residuals >7 seconds were rejected.

BDC intervals	Number of observations	Probable error (sec)			Mass of Vesta and corresponding probable error			
		p	p'	p''	m'	m''	$p'm$	$p''m$
1, 2, 3, 4, 5	59	2.24	1.58	1.58	1.19	1.20	0.08	0.08
2, 3, 4, 5	42	1.47	1.41	1.41	0.65	0.67	.16	.16
1, 2, 3, 4	40	1.88	1.66	1.66	1.57	1.58	.23	.23
3, 4, 5	35	1.50	1.44	1.45	1.20	1.21	.31	.31
2, 3, 4	23	1.23	1.24	1.24	−0.59	−0.57	.45	.45
1, 2, 3	37	1.90	1.67	1.67	2.26	2.26	.33	.33

Figures 1 and 2 show, for right ascensions and declinations, respectively, the mean of the residuals in each observed opposition from the corrected orbit 2 corresponding to the first line of Table 1. These data are indicated by crosses. Also shown as dots connected by a line are the computed positions of orbit 1 corrected without the effect of Vesta included. The data again correspond to the first line of Table 1. The distances of the crosses from the dots with the same abscissa are the residuals from corrected orbit 1 without the mass of Vesta being included. The relative closeness of the crosses to the line of zero ordinates or to the dots indicates whether the value derived for the mass of Vesta represents the observation better than if no allowance is made for the mass of Vesta. While the data for the right ascensions indicate a concentration toward the orbit with the mass of Vesta included, no preference is strongly indicated in the case of the declinations.

The coefficients of the mass of Vesta or of the correction to the mass of Vesta in the differential corrections were obtained by comparing two numerical integrations, one constructed which includes the effect of Vesta and one which does not. It would be interesting to compute these coefficients from the results of an evaluation of Vesta perturbations of Arete. Carpenter (*4*) has attempted to determine such perturbations but was unable to obtain meaningful results because of the close approach of the orbits of Vesta and Arete. Perhaps a regularization procedure would be helpful. In the final analysis, an attempt will be made to allow as much as possible for the fact that better star positions are available now than at the times of some older observations.

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15 February 1968