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Precession of the Earth as the Cause of Geomagnetism

Experiments lend support to the proposal that precessional torques drive the earth's dynamo.

W. V. R. Malkus

The geomagnetic field is presumed to be due to motions in the earth's fluid core. Doubts (*1*) concerning the possibility of dynamo action in a homogeneous electrically conducting fluid have been removed by the theoretical construction (*2*) of velocity fields which exhibit magnetic instability. However, the origin and character (*3*) of the motions responsible for the earth's dynamo have not been determined. That is to say, the elementary kinematic problem has been resolved while the numerous dynamic problems have barely been touched. It is generally believed that heat sources in the core produce convective motions which in turn drive the geodynamo (*4*). Although there is little progress to report (*5*) on the determination of interdependent velocity and magnetic fields due to buoyancy forces, convection is a theoretically attractive mechanism because of its possible applicability to stellar dynamos as well as to the earth. However, it has been difficult to find a heat source (*6*) sufficient to meet even the minimum energy needs of a geodynamo and compatible with surface observations of heat flux. This difficulty is compounded 25-fold by the dynamic requirement (*7*) that any convective "engine" in the core is at most 4 percent efficient in transforming heat flux, through fluid motion, into ohmic dissipation. Unfortunately, then, the heat source needed to drive a realistic dynamo places severe strains on current theories of the thermal history of the earth.

The author is a professor of geophysics and mathematics at the University of California, Los Angeles.

An alternative process which might drive the geodynamo is the precession of the earth. Recent experiments suggest that the motions in the core can be due entirely to the precessional torques. To acquaint the reader with the nature of the forces involved, an estimate is made here of the magnetic fields which could result from the earth's precession. This estimate is based on the assumption that the flow in the core is turbulent. Laboratory experiments on precessing fluids are then described, which establish the conditions for steady and turbulent flow.

The Poincaré Force

Of the several mechanisms for inducing motion in the core that have been proposed, the precession of the earth is the only one whose magnitude and character are well known. The earth precesses with a period of 25,800 years, due to the gravity fields of the moon and the sun acting on the earth's equatorial bulge. Figure 1 illustrates the aspects of earth structure and average motion that are relevant to this study. The precession vector $\Omega = 7.71 \times 10^{-12}$ radian per second and is normal to the plane of the ecliptic. The earth's angular rotation vector $\omega = 7.29 \times 10^{-5}$ radian per second and is inclined at 23.5 degrees relative to Ω. The lightly shaded zone in Fig. 1 is the earth's molten core, of mean radius $R = 3.47 \times 10^8$ centimeters. The mean radius of the earth as a whole, R_1, is roughly $2R$, while the radius, R_2, of the presumably solid inner core is roughly $0.4R$. The rotation of the earth causes an equatorial bulge, resulting in a difference between the moments of inertia about the polar and the equatorial axes. The ratio of this difference to the moment of inertia about the polar axis is known as the *dynamic ellipticity*. The dynamic ellipticity of the earth as a whole is 3.28×10^{-3}, and that of the core alone is $2.45 \times 10^{-3} \pm 2$ percent (*8*). The dynamic ellipticity of the core is three-quarters that of the mantle, due to the core's greater density (approximately 10 grams per cubic centimeter).

The precession rate of a planetary body is directly proportional to its dynamic ellipticity, but independent of its radius or mass (*9*). Hence, if the core and mantle were not coupled, the core would precess at only three-quarters the rate of the mantle. Thirty thousand years of such uncoupled precession would lead to relative velocities of 10^4 centimeters per second at the core-mantle boundary. Of course, core and mantle are coupled by torques resulting from their relative motion. These torques will increase until core and mantle precess at the same average rate. The local forces needed to produce this common precession are most easily described in terms of a coordinate system fixed in the earth, sharing both the earth's rotation and its precession. The transformation of coordinates from inertial space to a uniformly rotating system leads to the usual centrifugal and Coriolis forces. The additional transformation of the coordinates to a uniformly precessing system adds the unfamiliar precessional force per unit mass

$$[(\omega \times \Omega) \times \mathbf{r}]$$

where **r** is the position vector measured from the center of the earth and where ω and Ω are precession and angular rotation vectors, respectively, as defined for Fig. 1. Here I refer to this force as the Poincaré force, in honor of H. Poincaré, who made early studies of precession (*10–13*). The Poincaré force is akin to the usual centrifugal force, but since it produces a net torque it cannot be balanced by pressure alone. Due to the difference in core and mantle ellipticity, one-fourth of the Poincaré force remains unbalanced in the core and produces fluid motion.

A fluid velocity **V** in the core will give rise to the usual Coriolis force per unit mass, $2\omega \times \mathbf{V}$. Such electric currents as may be associated with this flow will produce the Lorentz force per unit mass (μ/ρ) $(\mathbf{j} \times \mathbf{H})$, where ρ is the density of the fluid, μ is the magnetic permeability, **H** is the magnetic field, and the curl of **H**, or $(\nabla \times \mathbf{H})$, is 4π times the electric current density **j**. Gaussian electromagnetic units are used in which $\mu = 1$ for any fluid.

I assume, tentatively, that the Poincaré force is large enough to cause a fully magnetoturbulent flow in the core. I define *fully* as meaning that a gross balance has been struck between the fluctuating Coriolis and Lorentz forces in the interior of the flow and that each of these fluctuating response forces is of the same magnitude as the steady Poincaré driving force. Forces due to pressure gradients, and even viscous forces, can be as important as the Coriolis and Lorentz forces, but I will show that it is sufficient to consider only the two major inertial forces and the electromagnetic force in order to obtain an estimate of the velocities and magnetic fields which would be associated with such flow.

This gross balance is written

$$\frac{1}{4} \mid (\omega \times \Omega) \times \mathbf{r} \mid \cong \mid 2\omega \times \mathbf{V} \mid \cong \mid \frac{1}{4\pi\rho} (\nabla \times \mathbf{H}) \times \mathbf{H} \mid. \qquad (1)$$

Hence the estimate for the magnitude of a characteristic velocity V is

$$V \cong \frac{\mid \omega \times \Omega \mid}{8 \mid \omega \mid} R \cong \frac{4}{3} \times 10^{-4} \text{ cm/sec.} \quad (2)$$

For scales of motion comparable to the radius of the core, ∇ in Eq. 1 is replaced by $1/R$, and the estimate for the magnitude, H, of a characteristic magnetic field is

$$H \cong (\pi\rho \mid \omega \times \Omega \mid R^2)^{1/2} \cong 29 \text{ gauss.} \quad (3)$$

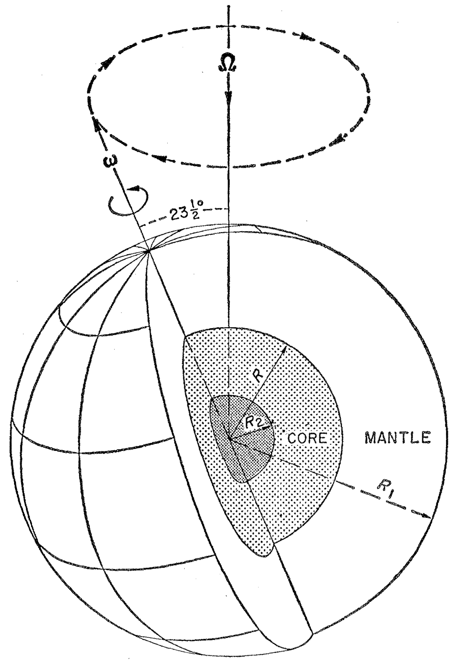


Fig. 1. The precessing earth The angular rotation vector ω is 10^7 times as large as the precession vector Ω. The dynamic ellipticity of the core is three-fourths as large as that of the mantle.

The vector character of the force balance (Eq. 1), requires that the estimated magnetic field H^2 be interpreted as the product H_pH_T, where H_p is the magnitude of the poloidal components of the field and H_T is the magnitude of the toroidal component of the field. The poloidal components lie in meridian planes containing the axis of rotation. The toroidal component is at right angles to these planes. Similarly, the characteristic velocity V should be interpreted as representative of the geometric mean of poloidal and toroidal components. A balance of the average radial Coriolis force and the average radial Lorentz force is not included in the estimates given above, but is used in the following section. Neither V nor H are directly observable quantities, since they are estimates for fields within the molten core material.

Estimate of the Dipole Field

The magnitude of the earth's dipole field at the core-mantle boundary, as inferred from surface observations, is approximately 5 gauss. In order to determine the external dipole associated with the internal field H, one must establish how much of H can "leak out" of the electrically conducting mantle. A gross balance of the terms in the magnetic-diffusion equation provides such an estimate, but this estimate depends explicitly upon the value of core conductivity.

This balance between the loss of magnetic field by ohmic dissipation and its regeneration by the fluid motion is

$$\mid \frac{1}{4\pi\sigma} \nabla \times (\nabla \times \mathbf{H}) \mid \cong \mid \nabla \times (\mathbf{V} \times \mathbf{H}) \mid \qquad (4)$$

where σ is the electrical conductivity of the fluid. For scales of motion comparable to the radius of the core, the balance of Eq. 4 may be interpreted as the estimate

$$H_0 \cong \frac{1}{4\pi\sigma R} \frac{H}{V} = \frac{2 \mid \omega \mid}{\sigma R} \left(\frac{\rho}{\pi \mid \omega \times \Omega \mid} \right)^{1/2} \qquad (5)$$

where H_0 is defined as the magnitude of the magnetic field at right angles to the velocity **V** and is presumed to be small compared to H, and where the ratio $H{:}V$ is expressed in terms of the precession rate through the use of Eqs. 2 and 3. Since the average fluid velocity must be parallel to the core-mantle boundary, H_0 is also an estimate for the magnetic field external to the core. A mean value for the electrical conductivity of core material, determined from the extrapolation of laboratory measurements, is 7×10^{-6} (abohm centimeter)$^{-1}$. However, an uncertainty of at least a factor of 3, in either direction, is given in the literature (*14*). With this choice for σ, the estimate (Eq. 5) for the external magnetic field at the core-mantle boundary is

$$H_0 \cong 7 \text{ gauss} \pm 300\%.$$

If the estimate for H_0 had led to a value equal to or larger than H, the appropriate interpretation of the dissipation-regeneration balance equation would have been that the magnetic field could not be maintained by the velocity field and would vanish. This would occur if the precession $|\Omega|$ were $1/16$ its present value, or if the conductivity σ were $1/4$ the mean value used in arriving at the estimate given above.

Having found an approximate value for H_0, we can make a secondary estimate of the internal fields. Since the estimated value for H_0 is also an estimate for one component of the poloidal field just inside the core-mantle boundary, it sets a lower bound on the magnitude of the internal poloidal field H_p. Then $H^2 \equiv H_pH_T > H_0H_T$, so an upper bound for the internal toroidal field H_T

can be found from the estimates for H^2 and H_0. From Eqs. 3 and 5, we obtain the relation

$$H_T < \frac{H^2}{H_0} = \frac{\sigma}{2\,|\,\omega\,|}\,\rho^{\frac{1}{2}}\,\pi\,|\,\omega \times \Omega\,|^{\frac{3}{2}}\,R^3$$

$$\cong 120 \text{ gauss} \pm 300\%. \quad (6)$$

Finally, a secondary estimate for a mean value of the toroidal velocity field, V_T, follows from the balance, in Eq. 1, of the radial Coriolis force and the radial Lorentz force. This relation is written:

$$V_T \cong \frac{H_T{}^2}{8\pi\rho\,|\,\omega\,|\,R} < \frac{(\sigma\pi)^2 R^5}{32}\left(\frac{|\,\omega \times \Omega\,|}{|\,\omega\,|}\right)^3$$

$$\cong 2.3 \times 10^{-3} \text{ cm/sec} \pm 1000\%, \quad (7)$$

where Eq. 6 is used to express $H_T{}^2$ in terms of the precession rate and the conductivity. Order-of-magnitude accuracy is the most one might expect from a secondary estimate such as this. If V_T is interpreted as representative of the westward-drift component of core surface velocity, then the value in Eq. 7 is one-sixth the mean drift velocity inferred (*3*) from recent geomagnetic data.

Energy Source of the Dynamo

The foregoing estimates drawn from the Poincaré force led me to be reasonably confident that precession does indeed sustain the geodynamo. Oddly enough, it was Poincaré's early work which was quoted in the literature (*4*) to suggest the implausibility of precession as a cause of geomagnetism. Poincaré (*10*) had concluded that an inviscid fluid precessing in an oblate spheroidal cavity would achieve equilibrium by a tilt of its axis of rotation with respect to the axis of the cavity. In the case of the earth, the tilt would be approximately 10^{-5} radian, and would result in a smooth diurnal oscillation of the core fluid, with a velocity of several millimeters per second. Neither the magnitude nor the structure of the motion was thought adequate to produce dynamo action. This belief was reinforced by the recent work of Stewartson and Roberts (*12*), in which they considered how the viscosity of the core fluid would modify Poincaré's results. They concluded that the addition of a thin boundary-layer flow met the viscosity requirements, and left Poincaré's solution for the fluid motion essentially unaltered.

The stability of a Poincaré-like fluid motion for precessional flow in the earth's core remained undetermined. Large-scale flows are notoriously unstable. Roughness of the core-mantle boundary or unanticipated secondary flows can trigger the disturbances which grow into a dominating turbulence. Magnetic instabilities, like the ordinary hydrodynamic instabilities, draw their energy from the shearing flow. Both types of instability transport momentum through the fluid. One can anticipate that magnetic instabilities, like the ubiquitous shear-flow instabilities, also occur whenever the relative velocities and conductivity of the fluid are large enough. To test the hypothesis that precessional turbulence sustains the geodynamo, one must initially determine whether sufficient energy is available to drive the geodynamo through the eons.

The energy consumption of the precession-driven geodynamo can be found directly from the earlier estimates. The characteristic stress acting on the core-mantle boundary must equal the momentum transport just inside the boundary. From Eq. 3 we find this momentum transport to be

$$\frac{H_p H_T}{4\pi} \cong \frac{\rho}{4}\,(\,|\,\omega \times \Omega\,|\,R^2) \cong 68 \text{ dyne/cm}^2. \quad (8)$$

An estimate of the total work, W, done by the mantle on the core is then, from Eqs. 7 and 8,

$$W \cong \left(\frac{H_p H_T}{4\pi}\right) V_T\,(4\pi R^2)$$

$$< \frac{\rho\sigma^2\pi^3 R^9\,|\,\omega \times \Omega\,|^4}{32\,|\,\omega\,|^3}$$

$$\cong 2.3 \times 10^{17} \text{ erg/sec} \pm 1000\%, \quad (9)$$

and this would be consumed in ohmic heating primarily. For comparison, note that the estimated rate of dissipation due to tidal interaction of earth, moon, and sun is 3×10^{19} ergs per second. Like the tidal dissipation process, the energy for the precession-driven geodynamo must come from the kinetic energy stored in the earth's rotation. Unlike the tidal process, the response of the core fluid to precessional forces does not produce a reaction torque on the moon or sun. Hence, in order for the total angular momentum of the earth, moon, sun system to be conserved, only the rotational energy in the nonconserved component of the earth's rotation can supply the dynamo. This is the component of rotation in the plane of the ecliptic, the component at right angles to Ω in Fig. 1; it is sufficient at present to maintain the geomagnetic field for many earth lifetimes. However, in an earlier eon when the moon was considerably closer to the earth than it is now, the dynamo dissipation might well have exceeded the total tidal dissipation and would have contributed significantly to internal heating in the earth.

It should be noted that any other source of internal heating in the earth would reduce or remove the constraints on precessional turbulence which would exist if the core were stably stratified. The radial heat flux, which Verhoogen (*6*) proposes would result from growth of the solid inner core, could produce an adiabatic lapse rate in the core fluid. If a weak convection resulted from this heat flux, it could contribute to the precessional turbulence and to the dynamo energetics.

However, the foregoing arguments and estimates are based on the assumption that the Poincaré-like flow in the earth becomes turbulent without the assistance of a secondary mechanism. In the next section I report on experimental and theoretical tests of this assumption.

Turbulence in Laboratory Flows

The first experiments performed with a precessing oblate spheroidal cavity exhibited dramatic departures from laminar Poincaré flow in the contained fluid. Figure 2 illustrates these experiments. A plastic spheroid, of major axis 25 centimeters and minor axis 24 centimeters, was filled with water and rotated at 60 revolutions per minute around its minor axis. The spheroid, its supports, and its motor were mounted on a horizontal, rotatable table, with the minor axis of the spheroid inclined at 30 degrees to the vertical. A pinch of aluminum powder dispersed in the water was illuminated by a beam of light normal to the rotation axis, permitting visualization of the process. Rotation of the table at ¾ revolution per minute caused the steady two-dimensional flow seen in Fig. 2a. Rotation of the table at 1 revolution per minute led to the wavelike instability of the steady flow seen in Fig. 2b, while rotation at ⁴⁄₃ revolution per minute caused the turbulent flows seen in Fig. 2, c and d.

A program of measurement was undertaken to determine the flow field prior to instability, the critical param-

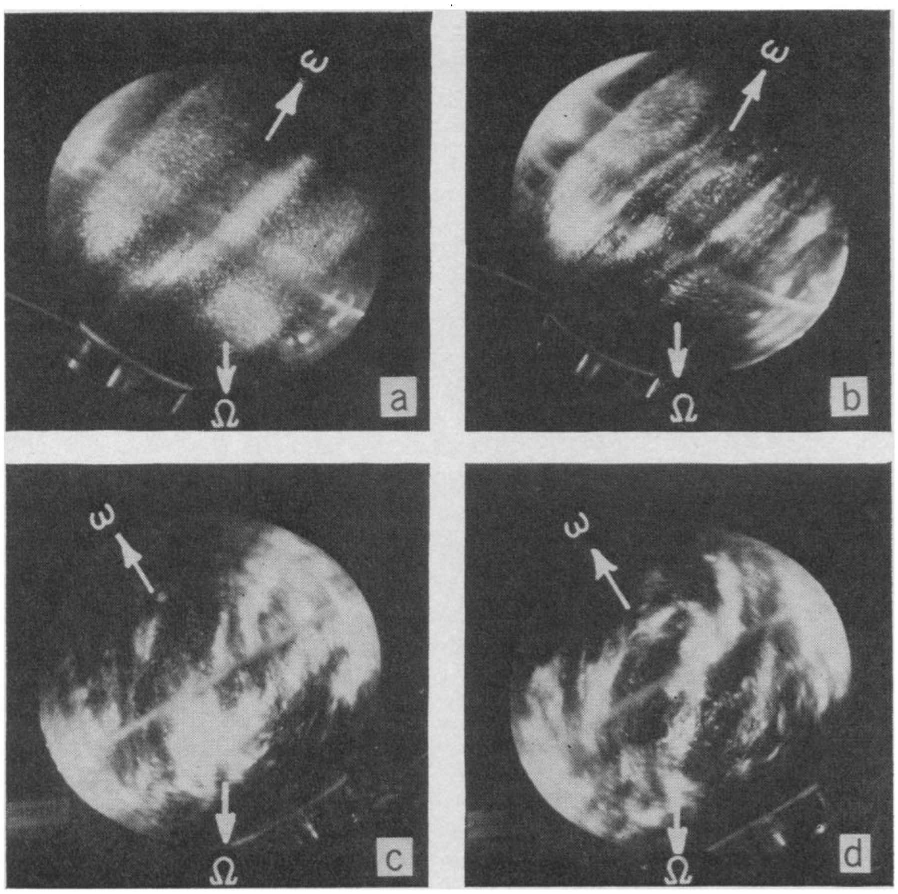


Fig. 2. Precessional flow in a laboratory spheroid. (a) Laminar nonlinear flow; (b) initial wavelike instability; (c and d) turbulent flow.

eters determining instability, and the motor torques needed to sustain the flow. Figure 3 is a tracing from the photographic records of a dye streak introduced into a laminar flow such as that of Fig. 2a. The dye was produced electrically in a dilute solution of thymol blue by means of a straight wire probe extending from the equator of the spheroid to the center of the fluid. The probe was withdrawn, and a very slow precession was started. Photographs of the developing flow were taken at 1-minute intervals. A steady toroidal velocity relative to the container was reached after several minutes. The parameter which determines the shape of these flows is called the Ekman number, E; $E = \nu/\omega R^2$, where ν is the kinematic viscosity of the fluid, ω is the angular velocity of the container, and R is the mean radius of the container. The square root of the Ekman number measures the ratio of the typical thickness of the viscous boundary layer to the average radius. The Ekman number of the flow in Fig. 3 is 3.6×10^{-6}.

In the Poincaré-like flows predicted by Stewartson and Roberts (*12*) the dye line of Fig. 3 would not have moved at all. The interior flow in Poincaré core fluid is a smooth, zero-average, diurnal oscillation. I traced the origin of the axisymmetric toroidal flow seen in Fig. 3 to the nonlinear momentum advection in the boundary layers, neglected in the Stewartson and Roberts study. The most obvious features of the toroidal flow are the general retrograde motion (westward drift) in the interior and the retrograde and prograde "jets" on either side of radius 0.86. These "jets" become more and more intense as the Ekman number of the flow is decreased. It was clear that the precessional flow in the limit of vanishing viscosity (Ekman number approaching zero) would be very different from the classical flow found by assuming zero viscosity initially. These qualitative conclusions might have remained little more than speculations had I not been joined in this study by Friedrich Busse. He has prepared a detailed theoretical treatment (*15*) of laminar precessional flow, valid in the limit of vanishingly small Ekman number and small, but finite, amplitude. The observed mean retrograde flow is in agreement with the theoretical prediction, and the observed "jets" approach the theoretical "jets" as the Ekman number is reduced.

The first manifestations of the instability of the jetlike flows are quasi-two-dimensional waves (poorly seen in Fig. 2b), appearing at precession rates greater than a critical value discussed below. The phase velocity of these waves is toward the "west"—that is, retrograde relative to the rotation of the spheroid. They are kinematically similar to the hydromagnetic waves proposed in recent theories (*16*) of the geomagnetic secular variations. No satisfactory theory governing the critical amplitude for instability has yet emerged. However, instability is observed for relative jet amplitudes, A, greater than $(5.0 \pm 0.3)E^{1/2}$, where A is the ratio of the maximum toroidal speed to the peripheral speed of the container and E is the Ekman number. experiments confirm the theoretical amplitude

$$A = \frac{|\omega \times \Omega|^2}{\omega^2 \eta^2} f(E),$$

where η is the ellipticity of the spheroid and $f(E)$ is 0.4 for $E > 10^{-5}$. There are a few laboratory data to support the theoretical prediction that $f(E)$ approaches $E^{-1/6}$ at values of $E < 10^{-7}$. In order to apply these results to the earth, we must know the kinematic viscosity, ν, of the core fluid. Estimates in the literature (*4*) range from $\nu = 0.001$ to $\nu = 0.1$ square centimeter per second; therefore, in the earth, $10^{-16} \leqq E \leqq 10^{-14}$. Hence a laminar precessional flow in the core would be stable if $E = 10^{-14}$, marginally unstable if $E = 10^{-15}$, and quite unstable if $E = 10^{-16}$. One can also conclude that the core was very unstable in the past when the moon was closer to the earth than it is today. However, the instability that produces waves does not assure the fully turbulent flow assumed in the first sections of this article.

The turbulent flow seen in Fig. 2, c and d, can be explored in many ways. Measurement of the mean toroidal velocity as a function of radius and determination of the relative magnitudes of the various terms which enter into the local force balance will be of considerable interest. However, the measurements which I have undertaken to date have been made to determine the torque on the minor axis of the spheroids relative to the precession frequency. The example plotted in Fig. 4 shows dramatically the several features of these flows. A spheroid of major axis 7.5 centimeters and minor axis 6.75

centimeters was rotated at 900 revolutions per minute. The angle between the rotation and the precession axes was 96 degrees. In Fig. 4 the laminar region of flow extends from the origin to precession frequency *A*. For a small range of precession frequencies beyond *A*, wave disturbances occur, becoming more and more disordered with increasing distance from *A*. The flow is highly disordered even before precession frequency *C* is reached; however, the constraints of rotation and ellipticity still dominate. Perhaps this lower branch of the torque curve could be called "two-dimensional" turbulence. At precession frequency *B*, the character of the motion abruptly changes. The torque jumps to a "saturation" value, which is independent of further increases in precession frequency. In this range the fluid rotates about an axis almost parallel to the precession axis, and the turbulence fluctuations are comparable in magnitude to the rotation velocity of the container. As the precession frequency is reduced, the torque remains at its "saturation" value to well below point *B*. Finally, at precession frequency *C* the torque "hysteresis loop" is closed in a sudden collapse to the rotation-dominated wave turbulence. This transition at *C* is called a *finite-amplitude instability*. For a range of precession frequencies greater than *C* there are two realizable states for the precessing flow, both of which are "stable" in the presence of small disturbances. However, for arbitrarily large disturbances, only the fully turbulent flow is "stable" beyond *C*.

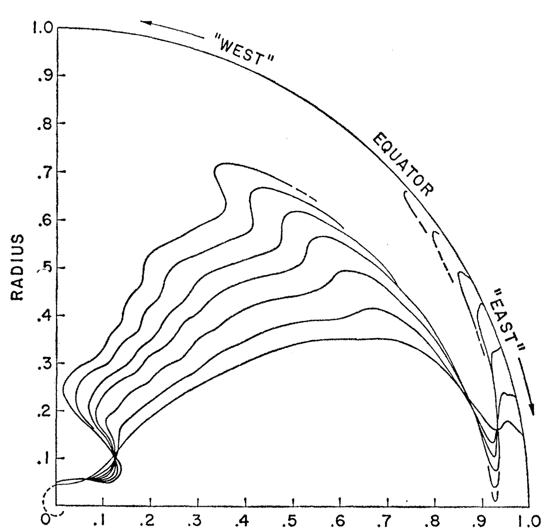
Unfortunately, conventional theory does not provide a basis for investigating problems of finite-amplitude transitions to or from turbulent flows. Yet the transition at *C* may be the most relevant to the geomagnetism problem of any that I have found (*17*). As in the magnetoturbulent flow, the fully turbulent flow at precession frequencies greater than *C* has reduced the constraining effect of the Coriolis force by a momentum-transfer process which opposes this force. In the magnetoturbulence, the electromagnetic Lorentz force balanced the Coriolis force. In the laboratory turbulence, the nonlinear force term $(\nabla \times \mathbf{V}) \times \mathbf{V}$ must play this role. At present, experiments are the only available guide for scaling up the observed transition at *C* to earth size and for interpreting it in terms of an electrically conducting fluid.

Over the limited range of parameters accessible in the laboratory, the experiments suggest that the flow remains fully turbulent if the fluid's frictional response time is smaller than its inertial response time. The inertial response time is determined entirely by the angular velocity of the fluid and the ellipticity of the spheroid. In the earth's core the frictional response time depends almost entirely on the magnetic coupling of core and mantle. This experimental criterion for the persistence of the turbulence state is met if approximately 50 percent of the torque between core and mantle is due to magnetic forces. But my arguments have come full circle. In the first sections of this article I assumed the occurrence of precessional turbulence in order to establish that it could support the geomagnetic field. Now it is concluded from the experiments that full turbulence in the core can exist only if it is magnetoturbulence. Such is the dilemma with finite-amplitude instabilities—and it is faithfully reflected in the mathematical statement of the problem.

Progress Toward an Experimental Dynamo

No proof that the geodynamo is due to precessional torques has emerged from the theoretical or experimental work reported here. In fact, it appears that the laminar precession-induced flows in the earth would support only small hydrodynamical disturbances. However, the conclusion that a sufficiently large magnetoturbulent disturbance in the earth could be sustained as a statistically stable flow by the precessional torques is also consistent with these observations.

To stimulate the continuing theoretical attack on the finite-amplitude magnetic instability of shearing flows (*5*), an experimental program has been started in which liquid metal is used as the fluid in a precessing spheroid. In the first of these experiments my associates and I will undertake to reproduce those com-



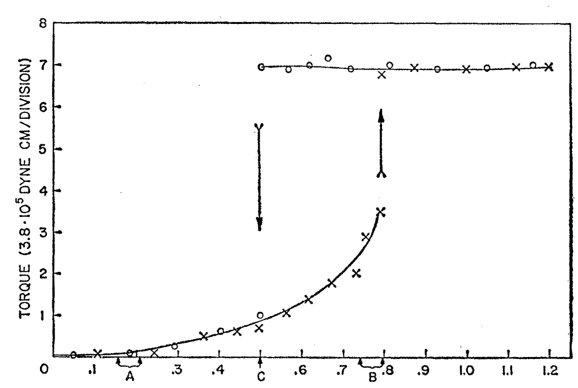


Fig. 3 (left). Tracing from photographic records showing the motion of a dye streak in laminar precessional flow. A sequence of photographs determines the toroidal velocities in the equatorial plane for an Ekman number of 3.6×10^{-6}. Fig. 4 (right). Dissipative torques in precessional flow: the torque on the minor axis of a spheroid plotted against its precession frequency. *A*, Transition to wavy disturbances; *B*, transition to "saturated" turbulence; *C*, hysteresis transition back to waves; *X*, torque data point associated with increasing precession frequency; *O*, torque data point associated with decreasing precession frequency.

binations of hydromagnetic and geometric parameters believed appropriate for the earth's core. A steel spheroid, 25 centimeters in diameter, containing liquid sodium will be rotated about a horizontal axis at 4000 revolutions per minute and caused to precess about a vertical axis. We expect, from the experiments already performed with ordinary fluids, that precession rates of 20 revolutions per minute will cause turbulence velocities comparable to the peripheral velocity of the container, and of a scale comparable to its radius. Magnetic instability would be indicated by a jump in torque similar to the transitions in Fig. 4, and would produce internal magnetic fields of several hundred gauss. Failure to achieve a working dynamo in this first experiment also can serve a purpose. From measurement of the decay rates of magnetic fields imposed upon the fluid, the minimum conditions needed to maintain a dynamo, in a second experiment, can be established.

Summary

I have proposed that the precessional torques acting on the earth can sustain a turbulent hydromagnetic flow in the molten core. A gross balance of the Coriolis force, the Lorentz force, and the precessional force in the core fluid provided estimates of the fluid velocity and the interior magnetic field characteristic of such flow. Then these numbers and a balance of the processes responsible for the decay and regeneration of the magnetic field provided an estimate of the magnetic field external to the core. This external field is in keeping with the observations, but its value is dependent upon the speculative value for the electrical conductivity of core material. The proposal that turbulent flow due to precession can occur in the core was tested in a study of nonmagnetic laboratory flows induced by the steady precession of fluid-filled rotating spheroids. It was found that these flows exhibit both small wavelike instabilities and violent finite-amplitude instability to turbulent motion above critical values of the precession rate. The observed critical parameters indicate that a laminar flow in the core, due to the earth's precession, would have weak hydrodynamic instabilities at most, but that finite-amplitude hydromagnetic instability could lead to fully turbulent flow.

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17. Relaxation transitions back and forth between the low-torque wave-turbulence regime and the high-torque "saturation"-turbulence regime can be achieved in the laboratory flow by using a motor with a constant torque intermediate between those of the two regimes. The average period of these transitions depends upon the torque and the angular momentum of the fluid and its spheroidal container. A phenomenon akin to this may be responsible for the observed reversals of the geodynamo.
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