Mercury Has Two Permanent Thermal Bulges

Abstract. *If Mercury has been rotating with a period exactly two-thirds of the period of orbital revolution for at least 60,000 years, there exist two permanent thermal bulges on opposite sides of Mercury's surface that alternately face Sun at every perihelion. The bulges increase the strength of the resonance lock and tend to prevent drifting out.*

Mercury's rotational period of 58.65 days is gravitationally locked (*1*), and the natural cause of this lock has been of great concern. I have investigated effects of thermal expansion on the positive difference of moment of inertia $B - A$.

The angle of rotation of Mercury is defined by

$$\Psi = f + \Phi(f) \tag{1}$$

where f is the true anomaly, and $\Phi(f)$, angle of diurnal motion of Sun as seen from Mercury, is governed by

$$\Phi = \left[\sin^{-1}\left(\frac{e + \cos f}{1 + e\cos f}\right) - \frac{e(1-e^2)^{\frac{1}{2}} \sin f}{1 + e \cos f} - \frac{\pi}{2}\right]\frac{P_1}{P_2} - f \tag{2}$$

in which e is the orbital eccentricity, and P_1 and P_2 are the orbital and rotational periods of Mercury. For $e = 0.206$ and $P_1/P_2 = 3/2$, it has been found that Mercury's instantaneous orbital angular velocity around Sun equals Mercury's rotational angular velocity at points that it reaches in its orbit 4 days before and after perihelion. Furthermore it has been found that the axis of Mercury that points to Sun at perihelion (the perihelion axis) is also directed at Sun from points that Mercury reaches 7 days before and after perihelion. Another axis of Mercury (the aphelion axis) points to Sun at every aphelion. In each Mercurial day (176 Earth days), Sun remains at the local meridian for points around the perihelion axis for about 60 days near perihelion. For points near the aphelion axis, Sun stays at its meridian position for only 8 days. The surfaces around the perihelion axis, therefore, absorb more solar heat than do those around other axes. This heat, when conducted inward, will cause the planet to bulge slightly along this axis because of thermal expansion, giving rise to a positive difference in the moments of inertia.

Table 1. Values of the ratio $(B - A)/C$ when $\alpha = 2 \times 10^{-5}$/deg, $\gamma = 2420$ km, and $\Delta T = 100°$C.

β	y (km)	$(B - A)/C$
0.0000000	2.420×10^3	1×10^{-4}
.7000000	$3 \times 2.420 \times 10^2$	2×10^{-5}
.8000000	$2 \times 2.420 \times 10^2$	1×10^{-5}
.9000000	$1 \times 2.420 \times 10^2$	3×10^{-6}
.9900000	2.420×10	3×10^{-8}
.9990000	2.420	3×10^{-10}
.9999000	2.420×10^{-1}	3×10^{-12}
.9999900	2.420×10^{-2}	3×10^{-14}
.9999990	2.420×10^{-3}	3×10^{-16}
.9999999	2.420×10^{-4}	3×10^{-18}
1.0000000	0	0

The difference in surface temperature between the regions around the perihelion and aphelion axes can be estimated by solution of a simple differential equation of heat transfer in a semi-infinite medium. The surface of Mercury is imagined as a homogeneous material characterized by thermal conductivity k, density ρ, and specific heat c. In such a material the temperature T at depth y below the surface at time t may be found from the one-dimensional heat-conduction equation

$$\delta T/\delta t = (k/\rho c)(\delta^2 T/\delta y^2) \tag{3}$$

The heat flux outward at any point is

$$F = k(\delta T/\delta y) \tag{4}$$

In the case of Mercury, the surface receives heat I from Sun and eventually reradiates this to space. The boundary condition that must be obeyed is

$$\sigma T_0^4 = I + F_0 \tag{5}$$

where σ is the Stefan-Boltzmann constant and the subscript zero signifies the surface. The insolation on Mercury's surface is

$$I = \frac{G(1 - A)(1 + e\cos f)^2}{a^2(1 - e^2)^2}\cos\Phi \tag{6}$$

where G is the solar constant, a is the major semi-axis of Mercury's orbit in astronomical units, and A is the albedo of the surface; the value of Φ is given by Eq. 2. When

$$2n\pi + (3\pi/2) \geq \Phi \geq 2n\pi + (\pi/2) \quad (n = 0, 1, 2, 3, \ldots)$$

I is zero. In order to transform the true anomaly f to the mean anomaly M, the following equations are adopted:

$$\tan(f/2) = [(1 + e)/(1 - e)]^{\frac{1}{2}} \tan(E/2) \tag{7}$$

and

$$E - e \sin E = M \tag{8}$$

where E is the eccentric anomaly. The equations were solved on an IBM-7094 computer; it was found that the difference in surface temperature, ΔT, between the regions around the perihelion and the aphelion axes is about 100°C.

The result of the greater thermal expansion of the region around the perihelion axis can be estimated by application of Bousinesq-Papkovich potentials (*2*). The thermoelastic problem for a sphere has been the subject of many earlier investigations (*2*, bibliography). If we assume that (i) the extra density change due to solar thermal expansion, along any radius of Mercury, can be represented by a linear drop from the surface value to zero at the same depth y; and that (ii) at longitude θ the surface temperature is

$$\Delta T \cos\theta \cos^{\frac{1}{2}}\lambda$$

(λ is latitude) higher than that at the longitude of the aphelion axis, then the ratio of $(B-A)/A$ is given by

$$\frac{B - A}{A} = \frac{3(2 - 5\beta^3 + 3\beta^5)}{100(1 + \beta + \beta^2)} \times \alpha \cdot \Delta T \tag{9}$$

in which α is the coefficient of linear thermal expansion and $\beta = (\gamma - y)/\gamma$, where γ is the aphelion radius. For $\alpha = 2 \times 10^{-5}$/deg, $\gamma = 2420$ km, and $\Delta T = 100°$C, the values of the ratio $(B-A)/C$ are given in Table 1.

Goldreich and Peale (*3*) have shown that, if $(B - A)/C$ is as great as 10^{-10}, a 3 : 2 resonant lock is possible; Table 1 makes it clear that this condition requires y to exceed 1.4 km. It is well known that solutions of the one-dimensional conduction Eq. 3 lead to the result that the depth of heating is approximately

$$[(k/\rho c)t]/y^2 \approx 1 \tag{10}$$

The value of $k/\rho c$ is approximately 0.01 for most silicate materials (*4*), so that

$$t \approx 2 \times 10^{12} \text{ seconds} \tag{11}$$

or roughly 60,000 years, which is brief compared to the age of the solar system. Thus it appears quite possible that the process of capture of Mercury has been naturally affected by thermal expansion, and that the two thermal bulges on Mercury's surface contribute significantly to the dynamic stabilization of the planet's rotation.

HAN-SHOU LIU

Goddard Space Flight Center, National Aeronautics and Space Administration, Greenbelt, Maryland

References and Notes

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Search for a Frequency Shift of the 21-Centimeter Line from Taurus A near Occultation by Sun

Abstract. *The 21-centimeter absorption line from the direction of Taurus A was used for detection of a shift in frequency when the source passed near Sun. A possible decrease in frequency of 150 cycles per second was detected, which cannot be caused by general relativity or by the plasma around Sun.*

A search for a shift in frequency of the 21-cm neutral hydrogen-absorption line from Taurus A during its near-occultation by Sun was made with the U.S. Naval Research Laboratory's 25.6-m radio telescope at Maryland Point. When, every June, Sun approaches the line Earth–Taurus A one can check the influence of Sun's vicinity on the frequency of the line. A shift in frequency is expected because of three effects:

1) According to general relativity the optical path of an electromagnetic wave depends on the strength of the gravitational potential along its path (*1, 2*). Whenever the optical path changes with time, a shift in frequency results. Thus, as Sun moves toward the line Earth–Taurus A, the optical path gradually increases, giving rise to a red shift when it approaches the Earth–Taurus line and to a blue shift when it recedes (*3, 4*).

The fractional change in frequency is

$$\Delta\nu/\nu = -(1/c)(dL/dt)$$

where the optical path $L = L_0 + c \cdot \Delta Tr$; L_0 is the classical distance, and ΔTr is the additional time delay predicted by the general theory of relativity. According to Shapiro (*1*)

$$\Delta Tr = \frac{2GM}{c^3}\left(\ln\frac{4XeXp}{l^2} - \frac{3Xe + Xp}{2Xe}\right)$$

for one-way propagation, where G is the gravitational constant, M is Sun's mass, Xe is the distance along the line of sight from Earth to the point of closest approach to Sun, Xp is the distance from this point to Taurus A, and l is the distance from this point to the center of Sun. Differentiation of ΔTr with respect to time gives

$$\frac{\Delta\nu}{\nu} = \frac{4GM}{lc^3}\frac{dl}{dt} = \frac{1.93 \times 10^{-5}}{l}\frac{dl}{dt}$$

Thus the frequency shift is independent of the distances Xe and Xp, and for a ray passing at 1.25 deg from Sun (where it was on the day of closest approach, 15 June 1967), and for

$$dl/dt = 20.7 \text{ km/sec } (5)$$

$\Delta\ \nu/\nu$ is equal to 1.16×10^{-10}. The maximum expected shift, due to general relativity, in the 21-cm line from Taurus A is 0.16 hertz; the shift predicted as a function of time is plotted in Fig. 1.

2) The plasma around Sun also causes a shift in frequency. The time delay due to plasma, Tp, is related to the distance from the center of Sun by (*1*)

$$Tp = [(6.5 \times 10^{24})/\nu^2 l] \times [\tan^{-1}(Xp/l) + \tan^{-1}(Xe/l)]$$

and the shift in frequency is then

$$(\Delta\nu/\nu)\ p \cong -(dTp/dl) = (6.5 \times 10^{24})/l^2\nu^2 \times (dl/dt)$$

The ratio between the relativistic and the plasma shifts is

$$\Delta\nu r/\Delta\nu p \cong [(1.93 \times 10^{-5})/(6.5 \times 10^{24})]l\nu^2 = 1.45$$

for the 21-cm line when Taurus A is 1.25 deg away from Sun (*6*).

3) The third effect is highly speculative but worth examining nevertheless. Assuming an analogy between electrons and photons, Fürth (*7*) hypothesized that, whenever a deflection of light occurs due to gravitational effect of a mass, a decrease in frequency follows; thus he can explain the cosmological red shift and the excessive red shift found for some of the spectral lines from Sun. A wave passing at a distance l from the center of Sun will be red-shifted by

$$\Delta\nu/\nu = K\,(4GM/lc^2)$$

where K is of the order of 1. At the closest approach of Taurus A

$$\Delta\nu/\nu = 1.64 \times 10^{-6}$$

which fact implies a shift of 2300 hertz for the 21-cm line.

The general relativistic and the plasma effects are too small to be detected with our present instrumentation. The resolution of frequency is limited by

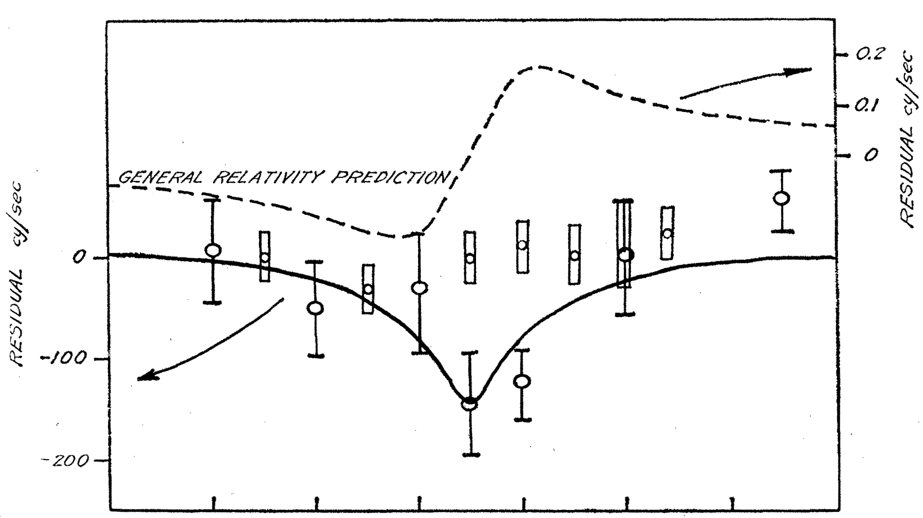

Fig. 1. Circles represent the experimentally determined residual frequency (measured minus calculated) of the 21-cm line during the days before and after the closest approach of Taurus A to Sun. The error flags represent the actual spreads of the 20 measurements of frequency taken each day. The solid curve is a tentative $1/r^2$ line that seems to fit the data. The dashed line is the general relativistic effect of the change of frequncy as Sun approaches the line of sight Earth–Taurus, as predicted for the situation in June 1967. The rectangles represent Taurus-A data received during 7 days of testing between 7 and 15 March 1967, when Sun was far away from Taurus A. The results exclude a theory by Fürth; the small decrease in frequency found by us has yet to be confirmed with better instrumentation.