Reports

Solar Oblateness and Fluid Spin-Down

Abstract. *An experiment performed on a differentially rotating, density-stratified fluid shows that "spin-down" need not occur under the condition of stratification. Inasmuch as density stratification occurs in the interior of the sun, spin-down probably does not exist in the solar interior, and the observed solar oblateness may indicate that the sun contains a rapidly spinning core.*

Experimental evidence is presented in favor of the contention (*1*) that a fluid with a sufficient density gradient need not undergo "spin-down" (*2*), a rapid damping of rotation due to the convection of angular momentum into a thin shell of viscous shear. This point is crucial to an explanation (*3*) of an observed solar oblateness (*4*).

It has been suggested (*3, 5, 6*) that the sun may have a large gravitational quadrupole moment which is the effect of a rapidly rotating core deep in the interior. Subsequently the sun was observed to be oblate (*4*), and the value for the oblateness was in approximate agreement with that first suggested (see *3*).

Although magnetic coupling with the core could eliminate the rotation, viscous coupling between the core and outer layer is believed to be too weak to seriously affect a rapid spin of the core in the accepted lifetime of the sun (*3, 5*).

Objections to the proposed model of a rapidly rotating solar core have been raised by Howard, Moore, and Spiegel (*2*). They contend that a fluid dynamical phenomenon of much greater consequence than simple viscous diffusion of angular momentum would come into play and quickly (10^5 years) damp out a rapid core rotation. This damping phenomenon is Ekman pumping of angular momentum to a very thin layer of large shear in the velocity distribution at the surface of the core, below the convective layer. The resulting loss of angular momentum to the outer parts of the sun is called "spin-down" and is accounted for as follows:

When a liquid is placed in a rotating vessel and given enough time to adjust to the vessel's angular velocity, surfaces of constant pressure adjust themselves so that centripetal acceleration at every point in the liquid is balanced by a pressure gradient. If the vessel's angular velocity is diminished, a thin layer of liquid next to the container's bottom surface (the Ekman layer) experiences a diminution in centripetal acceleration, while pressure gradients remain practically unchanged. The result is that liquid in the thin Ekman layer is pumped toward the axis of rotation and new liquid is drawn into the layer. The large velocity gradient in this layer implies a rapid loss of angular momentum. The Ekman pumping transports the angular momentum to the layer and brings about a quick damping of the liquid's differential rotation.

If one stirs a cup of tea and watches tea leaves moving toward the center of the cup, one is observing the effect of Ekman pumping. Because the tea leaves have a density greater than that of the tea, one might expect them to move away from, not toward, the center of the cup. The damping time of the tea's rotation is observed to be only a small fraction of that expected from viscous diffusion of angular momentum alone.

The northern and southern hemispheres of the sun (if we assume a rapidly spinning core) may be likened to a fluid rotating in an open hemispherical container whose angular velocity is less than that of the fluid. The analog of the container is the sun's slowly rotating outer layer, whereas that of the rotating liquid is the proposed rapidly spinning core.

It has been argued that the spin-down effect of Howard, Moore, and Spiegel (*2*) requires a constant density fluid and that the density gradient (*7*) in the sun's interior precludes the formation of an Ekman layer and the resulting Ekman pumping (*1*). It is shown that the pressure gradient which induces Ekman pumping in a constant-density fluid is balanced by a buoyancy force in a differentially rotating fluid of variable density.

In a constant-density fluid, centrifugal effects are responsible for spin-down. If one solves the equation of motion, neglecting centrifugal terms, for a liquid in a cylindrical vessel, one should arrive at a solution valid only in the absence of spin-down. It will be shown that such a solution does predict an approximate value for the damping time of a density-stratified liquid, while giving results completely inapplicable to a uniform-density liquid.

If centrifugal and gravitational terms are neglected, the Navier-Stokes equation for an incompressible fluid rotating with respect to its cylindrical container is

$$\Delta \mathbf{v}/\Delta t = \nu \nabla^2 \mathbf{v} \qquad (1)$$

If one assumes purely toroidal flow

$$\mathbf{v} = V_\theta(r, z, t) e_\theta$$

Eq. 1 becomes

$$\frac{\Delta V_\theta}{\Delta t} = \nu \times \left(\frac{\Delta^2}{\Delta z^2} + \frac{\Delta^2}{\Delta r^2} + \frac{1}{r} \cdot \frac{\Delta}{\Delta r} - \frac{1}{r^2} \right) V_\theta \qquad (2)$$

where $\nu = \eta/\rho$ is the liquid's kinematic viscosity, and V_θ is the linear velocity measured in a coordinate system rotating with the container. Boundary conditions to be satisfied are that the fluid's velocity be zero at the surfaces of the containing cylinder, and that there be no shear (and thus no z-component of velocity gradient) across the liquid's free surface.

The lowest normal decay mode solution of Eq. 2 satisfying the required boundary conditions is, for constant ν

$$V_\theta = A J_1 (x_{11} r/R) \sin (\pi z/2H)\, e^{-t/\tau} \qquad (3)$$

with

$$1/\tau = \nu\, [(x_{11}{}^2/R^2) + (\pi^2/4H^2)] \qquad (4)$$

where $x_{11} = 3.832$ is the first root of

J_1 (x), and R and H are the cylinder's radius and depth.

An experiment has been carried out to help illuminate the role of density gradients in fluid spin-down. It consists basically of placing a density-stratified liquid in a rotating tank, inducing a differential rotation between liquid and tank by changing the tank's rotational speed, and then observing the manner in which the liquid adapts itself to the new rotational speed, attempting to compare the surface velocity time dependence with Eqs. 3 and 4. The experiment was quite similar to one cited by Greenspan and Howard (*8*), but a density-stratified liquid was used in place of one of constant density. Similar questions have been discussed by Pedlosky (*9*) and Barcilon and Pedlosky (*10*).

In our density-stratified fluids the density ρ decreased by about 20 percent from the bottom of the container to the top. The kinematic viscosity ν was proportional to the density and $\nu/\rho = 9.3 \times 10^{-3}$ cm^5/g sec. While this variation of ν with z implies that Eq. 3 is not strictly applicable to a stratified fluid, there is another more important reason for not expecting the damping time to be accurately given by Eq. 3. It has been shown (*1*) that in a density-stratified fluid the angular-velocity and density distributions are interconnected. Thus, the angular-velocity distribution cannot generally assume the form of the lowest mode of Eq. 3. As viscous diffusion of angular momentum takes place, the density distribution is modified through fluid circulation in r-z planes, and this circulation also transports angular momentum. Whereas this situation is complex, the damping constant obtained from Eq. 3 should be approximately correct. Equation 1 is a Euler equation obtained from a variational principle which demands an extremum for the fractional rate of viscous dissipation of kinetic energy in the fluid. Because of the stationary character of the dissipation rate, it is little affected by a small change in the velocity distribution. While the decay of the rotation of the surface layer need not accurately indicate the average dissipation in the fluid as a whole, it would not be expected to depart greatly. Although Eqs. 1 to 3 are not strictly applicable, the decay constant obtained from these equations would be expected to be approximately correct.

The cylindrical tank used in the experiment had a radius R equal to 7.32 cm, and was filled with liquid to a depth of 5 cm. Each of the liquids had an average kinematic viscosity very near ν equal to 0.01 cm^2/sec. Thus one expects from Eq. 4 an e-folding time for surface velocities,

$$\tau = 266 \text{ seconds} \tag{5}$$

The cylindrical tank was mounted on a phonograph turntable driven by a variable-frequency oscillator and power amplifier. While the data were being taken, a stroboscopic light, triggered by a mechanical contact on the turntable, "froze" the rotation of the table and tank, and it was possible for a camera to record motions of surface floats on the liquid relative to the container. From the photographs, the angular coordinate θ of a given surface float was tabulated for intervals of 1 minute. From this tabulation, differences $\Delta\theta$ in float position, were extracted for 1-minute intervals. If the fluid adapts itself to the lowest normal mode of decay, $\Delta\theta$ should have the same $e^{-t/\tau}$ time dependence predicted for surface velocities in Eq. 3.

Operating conditions during data taking were as follows. All observations were made by varying the container's angular velocity between the two limits of 18.7 and 18.5 rev/min, the change being made by varying the audio oscillator frequency between the limits of 71 and 70 cy/sec. The driving frequency was varied both upward and downward for the density-stratified liquids to observe any asymmetry that might exist between the processes of spin-up and spin-down. It was found that too large a change in angular velocity in too short a time resulted in the separate mixing of the top and bottom portions of the fluid and led to the formation of a sharp boundary between the two portions. Also, it was necessary to cover and seal the tank to preclude atmospheric drag, and to align carefully (within about 1 minute of arc) the turntable's rotational axis with the vertical. Unless these steps were taken, a lingering differential rotation between liquid and tank would set in and assume a constant nonzero value.

Tests carried out to assess the constancy of the turntable's rotational speed indicated that long-term changes were less than 0.005 rev/min^2. Short-term change (within one revolution of the table) of the table's angular position from constant velocity were within ⅓ °. Tests on the ratio of turntable speed to driving frequency yielded a constant value within ¼ percent for a frequency range 60 to 120 cy/sec.

The liquid used in the observations varied from pure water to density-stratified solutions of $Cu(NO_3)_2$, with the specific gravity varying from 1.00 to 1.20. Cupric nitrate was chosen for

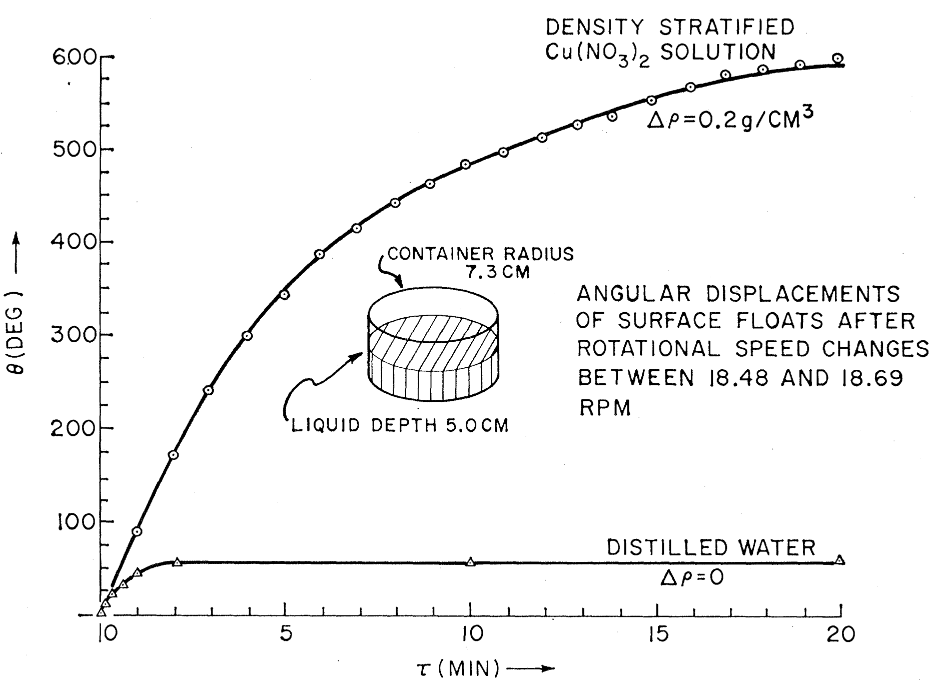

Fig. 1. Angular displacements of surface floats after changes in rotational speed between 18.48 and 18.69 rev/min.

its combination of high solubility and blue color in solution. It was important here to achieve smooth density variation in the fluid, for sharp boundaries could lead to separate layers of the liquid spinning down independently, as mentioned above. To effect a smooth density distribution, cupric nitrate solution of specific gravity 1.20 was placed in the crystallizing dish; water was poured on cautiously, initially forming a sharp interface with the dense blue solution. Careful stirring yielded a smooth color gradient, after which four specifically prepared floats of densities 1.05, 1.10, 1.15, and 1.20 could be placed in the solution to give quantitative information on the density distribution. Typically, the distribution had a linear z-dependence, ranging from 1.20 g/cm^3 at $z = 0$, to 1.00 g/cm^3 at $z =$ 5 cm.

Figure 1 shows angular displacements relative to the container plotted against the time for surface floats on two types of liquids: (i) density stratified $Cu(NO_3)_2$ solution; (ii) distilled water. In both cases the liquid was allowed half an hour to assume the container's rotational speed. At $t = 0$, a change of 0.21 rev/min was made in rotational speed (for the particular observation chosen to represent type i, the speed change was positive, while that for type ii was negative).

Fourteen observations on density-stratified liquids were made; seven each for increased and decreased container rotational speeds. Angular displacements per minute, $\Delta\theta$, as described earlier, were plotted against time on semilog coordinates. It was found that after the liquid had been rotated for about 2 minutes, the expected purely exponential decrease in surface velocities did set in. Exponential decay was seen for approximately 15 minutes, after which surface velocities were too low to be measured accurately. The e-folding times for the linear portions of the semilog curves in the cases of decreased rotational speed of the container averaged 342 seconds, whereas those for increased rotation averaged 394 seconds. The average of these is 368 seconds, which is to be compared with the 266-second decay time found in Eq. 5.

The difference in the first two decay times can be understood qualitatively. As noted above, for stratified fluids the rotational speed and density distributions are interconnected. Thus, after initial transient circulatory currents have ceased, the distribution of rotation will be dependent upon both the initial density distribution and the change in angular velocity. The rotational distribution will generally not be that of a normal decay mode and will be different in the two cases of spin-up and spin-down.

Three liquids of constant density [including a constant-density $Cu(NO_3)_2$ solution] were observed, all giving results of the type presented in Fig. 1 (lower curve). Surface velocities for these liquids depart grossly from decay mode (*3*). Damping was much stronger than for the stratified fluids investigated: final angular displacements of surface floats following equal changes in container rotation were about ten times greater for stratified fluids than for constant-density fluids. One is led to conclude from this great difference in damping rates that the occurrence of Ekman pumping and the ensuing "spin-down" in a fluid can indeed be inhibited or eliminated by a sufficient density gradient in the fluid. When these observations are related to our initial discussion of hydrodynamic processes within the sun, our experiments suggest that the density stratification in the solar interior (*7*) should preclude "spin-down" damping of differential rotation.

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Biochemical Genetics of Oxidative Phosphorylation

Abstract. *The usefulness of mutants in the unraveling of complex, highly organized, membrane-bound processes such as oxidative phosphorylation is illustrated by a study of a single recessive gene mutation in yeast, designated* op_1, *which has abolished the efficiency in vivo and in vitro of oxidative phosphorylation without impairing the electron transfer.*

During the last two decades oxidative phosphorylation has been studied by a variety of methods. These include the kinetic, functional, and chemical analyses of mitochondrial components in both the isolated and *in situ* states, the investigation of the sites of action of specific inhibitors, and the fractionation of individual components followed by their reconstitution into more complex functional units. Despite extensive efforts of many laboratories, the fundamental mechanism underlying oxidative phosphorylation is still far from being completely understood (see *1*).

The usefulness of mutants as a means for the analysis of metabolic pathways no longer needs to be stressed. It is obvious that if this approach could be systematically applied to the study of complex, highly organized, membrane-bound processes such as oxidative phosphorylation new insights into its mechanisms might be gained.

In yeast a total absence of respiration leads only to a conditional lethality, since fermentative energy supply is able to palliate the deficiency. The same should be true for a block in oxidative phosphorylation in this organism. After introduction of the genetic methodology in the study of electron transfer (*2, 3*) it seemed logical to extend this approach to the oxidative phosphorylation research. The purpose of this report is to present first examples of the use of mutants in this field.

Numerous respiratory-deficient mutants in yeast have been isolated which lack one or several components of the respiratory chain. Genetically they result either from a chromosomal gene mutation (abbreviated p) or from an extrachromosomal, mitochondrial DNA mutation (abbreviated ρ^-) (*2–4*). The phenotype used to screen such "petite" mutants is based on the fact that they are unable to grow on nonfermentable substrates (for example, glycerol, lac-