phate (Sigma), 0.15 mg per ml; fast blue RR salt (Sigma), 0.50 mg per ml; incubation time, 2 hours at 37°C. The monolayers were rinsed twice with 95-percent ethanol and were extracted with 2 ml of a 1/100 dilution of freshly prepared solution of absolute ethanol saturated with KOH in absolute ethanol for 30 minutes at 60°C. The extracts were transferred to conical centrifuge tubes to which were added 2 ml of the following mixture (in volumes): conentrated HCl, 3; absolute ethanol, 47; $CHCl_3$, 50. The precipitates that formed were removed by centrifugation, and the optical densities at 570 mμ of the supernatants were determined.

12. Supported by grants GMO8217 and GMO6983 from NIH. This is paper No. 1128 from the Genetics Laboratory, University of Wisconsin, Madison. We thank Drs. A. C. Crocker, D. W. Smith, and J. M. Opitz for helping us to study their patients.
* Research fellow of the Helen Hay Whitney Foundation.

24 May 1967

New Method for the Detection of Light Deflection by Solar Gravity

Abstract. *The prediction of Einstein's theory of general relativity that light will be deflected by the sun may be tested by sending radio waves from the earth to Venus or Mercury when either passes behind the sun and detecting the echoes with a radar interferometer.*

One of the three classical tests of general relativity originally proposed by Einstein concerns the prediction that the path of starlight will be deflected by the sun (*1*). Although attempts to detect this frequency-independent deflection have been made during almost every total solar eclipse since 1919, the results have been quite discordant and have verified the prediction only to within about ± 20 percent (*2*). The purpose of this report is to point out and discuss a new method with the potential for measuring the gravitational bending of light to substantially higher accuracy. The essential element of this method is the use of a radar interferometer. Since the maximum predicted bending is 1.75 arc sec for light just grazing the limb of the sun, conventional radar systems would be unable to detect such a deflection: the finest angular resolution at present obtainable with an interplanetary radar system is about 4 arc min—over 100 times greater than the largest predicted general relativistic effect. By employing two radar receivers in an interferometer arrangement, however, the error in determining the angle of arrival of a signal can be decreased approximately in inverse proportion to the receiver separation. Such interferometric techniques cannot be used effectively at optical wavelengths for this purpose because the phase ambiguities introduced by the atmosphere are practically impossible to unravel (*3*). By contrast, at centimeter wavelengths, the atmosphere introduces relatively few rotations of the phase vector which can probably be determined to a small fraction of a rotation.

A specific realization of this method could involve illuminating Venus or Mercury with a radiation of 8×10^3 Mhz (X-band) when either passes behind the sun and receiving the radio waves over a base line of approximately 10 km (*4*). The deflection predicted by general relativity would then be measurable to within 10 to 15 percent. Longer base lines could yield proportionately higher accuracies.

The theoretical expression for the bending can be obtained, for example, by calculating in isotropic coordinates (*5*) the difference η in angle of arrival between a ray propagating rectilinearly and according to general relativity. To first order in r_0, we find

$$\eta = \frac{2r_0}{r_e} \tan\left(\frac{\theta}{2}\right) \qquad (1)$$

where $r_0 \approx 1.5$ km denotes the so-called gravitational radius of the sun (*6*), r_e the heliocentric coordinate distance of the earth observer at the time of detection, and θ the planet-sun-observer angle with the planet's position being at the time of reflection. This apparently new result is remarkable in showing that the gravitational deflection undergone by an electromagnetic ray is independent of the position on a given radial line from which the ray originates. The farther out the source is on the radial line, the farther the ray's closest approach is from the sun, but the longer its path to the receiver; these two opposing influences on the total deflection just cancel as indicated by Eq. 1.

Near superior conjunction when the radar signal to a planet passes closest to the sun, Eq. 1 may be written in the following way

$$\eta \approx \frac{4r_0}{d}\left(\frac{r_p}{r}\right); \quad d \ll r_e, r_p \qquad (2)$$

where r_p is the heliocentric distance of the planet at reflection, r the rectilinear distance between the earth observer and planet, and $d = (r_e\ r_p/r) \sin\theta$ the approximate distance of closest approach of the signal to the sun. Because of the finite planet-observer separation, the maximum deflection of $4r_0/d(\approx 1.75$ arc sec) is not attained. Thus, with Venus the target and with the signal grazing the limb of the sun, $\eta \approx 0.73$ arc sec.

For a two-element interferometer operating at X-band with a 10-km base line, a change of one fringe would correspond to a difference in angle-of-arrival of at least 0.7 arc sec. But, depending on the signal-to-noise ratio and other factors, the phase within a fringe may be measurable to within about 10°, implying an error in the determination of the angle of arrival of only about 0.02 arc sec for this situation. Larger separations would lead to proportionately smaller errors.

To determine the feasibility of such an experiment, we must consider the possible limitations on d, the refractive effects of the earth's atmosphere and the solar corona, the angular size of the target planet, the uncertainty in the orbital positions of the target and antennas, the possible ambiguities in the comparison of the echo phases received at the two sites, the absolute phase stability required between receiver elements, and the necessary radar-system sensitivity. We discuss each in turn.

The minimum usable value of d is set mainly by three constraints: the noncoplanarity of the orbits of earth and target planet, the radio noise emitted by the sun, and the turbulence of the inner corona. The resultant limitation, although depending on the radar system and varying with the solar cycle and from conjunction to conjunction, lies at about $d = 3r_s$ (r_s is the solar radius) and implies that the largest detectable general relativistic deflection will be about 0.25 arc sec for planetary echoes.

Variations in the earth's atmosphere set limits on the accuracy with which ground receivers can be used to infer the exo-atmospheric direction of propagation of radio signals. At the very low elevation angle of 7°, the 2-day root-mean-square fluctuations σ in the estimate of azimuth angle for X-band signals are about 2 arc sec for a 1-km base line (*7, 8*). These fluctuations decrease with the cosecant of the elevation angle [thus, σ (45°) $\approx$ 0.3 arc sec], with a decrease in the observing interval, and with an increase in base line (*7*). For example, we might anticipate (*9*) that σ (45°) $\approx$ 0.02 arc sec for the root-mean-square fluctuations over a several-hour interval with a 100-km base line. By making use of simultaneous measurements of atmospheric properties near both sites, it may well be possible to reduce σ further (*9*). Such measurements would also make possible precise estimation of the phase biases introduced by the atmosphere. Compensating for these effects probably represents the most difficult obstacle to be overcome in improving the accuracy achievable with interferometers.

The solar corona appears not to be a serious impediment at X-band. During a "quiet-sun" period, an electron density $N(r)$ given by

$$N(r) = 5 \times 10^5\,(r_s/r)^2 \text{ electron/cm}^3 \quad (3)$$

characterizes the data (*10*) reasonably well from about $r = 3r_s$ to $r = 30\ r_s$. The actual N increases more rapidly for $r \lesssim 3r_s$ and decreases more rapidly than in (*3*) for $r \gtrsim 30\ r_s$; at times of maximum solar activity, N may be as much as a factor of 5 higher at any given r (*10*). Using geometric optics and neglecting the minute magnetic field and plasma collision effects, we easily find that near superior conjunction the total deflection ϵ undergone by a ray propagating between a planet and the earth is

$$\epsilon \approx -\pi\left(\frac{r_s}{d}\right)^2 \Delta n;\ \Delta n \ll 1;$$
$$3\,r_s < d \ll r_e, r_p \quad (4)$$

where $\Delta n \approx (2 \times 10^{13}/f^2)$, and f in hertz is the frequency of the signal. For $f = 8 \times 10^3$ Mhz and $d = 3r_s$, Eq. 4 yields $\epsilon \approx -0.02$ arc sec. The corresponding plasma-related difference $\delta\varphi$ in the phases of the waves arriving at the two receivers is given by

$$\delta\varphi \approx \frac{2\pi^2 r_s^2 r_p\,\Delta n}{\lambda\, r\, d}\left\{\frac{\delta r}{r_e} + \frac{r_e\,\delta\theta}{d}\right\} \quad (5)$$

where λ is the signal wavelength, and $(\delta r, \delta\theta)$ are the differences in the heliocentric polar coordinates of the two receivers. For a 10-km separation and values of f and d as given above, $\delta\varphi < 10°$ with Venus the observed planet. The contributions of the earth's ionosphere to ϵ and $\delta\varphi$ are far smaller. All the effects of plasma between target and receiver can therefore be neglected at X-band for almost all configurations of interest. In any event, two high frequencies can be used simultaneously to determine the plasma effects (*11*).

At superior conjunction, the angular diameters of Venus and Mercury are approximately 9.7 and 4.8 arc sec, respectively. Since these planets reflect radar signals quasi-specularly, only about the central 10 percent of the illuminated disk contributes appreciably to the backscattered signal. Moreover, by utilizing the principles of delay-Doppler mapping (*12*) and with the planetary rotation rate and scattering law obtained from high signal-to-noise radar observations at inferior conjunctions, it may be feasible to locate the center of the disk to within about 0.03 arc sec (*13*).

From the accumulated optical observations and radar measurements made during the past few years, the orbital positions of the three innermost planets have already been determined in isotropic coordinates with an error of the order of only 0.03 second of heliocentric arc (*14*), more than an order of magnitude higher accuracy than was achievable previously with optical observations alone. Further substantial improvements can be expected from radar observations currently being made. The positions of the receiver elements can be determined from interferometric observations of either planets or (point) radio sources (*15*). The effects of radomes and of any variations in focal positions caused, for example, by thermal expansion, steering of the antenna, or tidal motions, are expected to be negligible (small compared to a wavelength) or measureable from series of observations (*16*).

Ignoring any possible error in the general relativistic theory of light deflection, we see that the a priori uncertainty in the angle of arrival of the echo will correspond to less than one fringe at X-band for base lines under about 200 km. For an uncertainty of the order of the maximum predicted deflection, a 10-km base line would be sufficient to remove the fringe ambiguity. More generally, intercomparison of an extended series of data would suffice to remove all ambiguity even for base lines in excess of several hundred kilometers. In an actual experiment, a series is mandatory if only to check the consistency of the planetary positions predicted from other observations with the interferometric results (*17*).

Maintaining absolute phase stabilities at X-band frequencies between separated sites apparently poses no difficulty. Stability to within 3° has already been demonstrated over a propagation path length of 15 km (*18*); if necessary, an improvement of one order of magnitude could easily be achieved (*9*).

What facilities are available to perform this experiment? Lincoln Laboratory's Haystack radar is the most sensitive system currently in operation and can be used to transmit signals at 7840 Mhz and detect echoes from Venus or Mercury when either is at superior conjunction (*19*). The other arm of the interferometer receiver could be provided by Lincoln Laboratory's 60-foot (18-m) diameter Westford antenna, located slightly more than 1 km from Haystack (*20*). A far longer base line perhaps useful only for point sources (*13*) could involve either the 140-ft diameter antenna at Green Bank, West Virginia (base line $\approx$ 850 km) or Lincoln Laboratory's 60-ft diameter antenna at Camp Parks, California (base line $\approx$ 5000 km). Another possibility, provided that the X-band antenna efficiency requirements could be met, would be to illuminate the planet with the Haystack transmitter and use as the interferometer receiver the Jet Propulsion Laboratory's 210-ft and 85-ft diameter antennas, separated by 20 km. It may even prove feasible to use Haystack alone as an interferometer with the earth's motion providing the base line (*21*). (During the time required to produce a useful base line, the diffraction pattern of the target reflection must not change so greatly as to prevent proper correlations from being made.) If the regional facility proposed by the Cambridge Radio Observatory Committee (*22*) comes into being, it, combined with Haystack, would likely yield the most powerful interferometer for this experiment. Although some of these possibilities are obviously long-range in several senses, plans for using the Haystack-Westford system as a radar interferometer for planetary experiments are already being implemented (*21*).

Other variations of this deflection-detection experiment may well prove feasible. A suitably instrumented space probe in orbit about the sun, a transponder on a planet, or a strong extrasolar point source of X-band radiation (perhaps from still unobserved maser action in this frequency range) might be used, provided that occultation by the sun occurs.

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References and Notes

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2. See, for example, H. Klüber, in *Vistas in Astronomy* (Pergamon Press, London, 1960), vol. 3, p. 47.
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4. This experiment is complementary to, but philosophically distinct from the radar time-delay test of general relativity proposed by I. Shapiro [*Phys. Rev. Lett.* **13**, 789 (1964)].
5. See, for example, A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge University Press, 1960), p. 93.
6. ———, *ibid.*, p. 87.
7. M. C. Thompson, Jr., H. B. Janes, W. B. Grant, in preparation.
8. The (uncorrected) elevation-angle fluctuations are considerably greater (*7*), but the azimuth angle is the relevant quantity for detecting a solar gravitational deflection.
9. M. C. Thompson, Jr., personal communication.
10. See, for example, W. C. Erickson, *Astrophys. J.* **139**, 1290 (1964).
11. At 2×10^3 Mhz and below, effects of turbulence in the corona become discernible at solar distances up to about 10 r_s (R. B. Dyce and R. M. Goldstein, personal communications) and may make the deduction of plasma densities difficult.
12. P. E. Green, *J. Geophys. Res.* **65**, 1108 (1960).
13. A report on the problems involved in the actual processing of the planetary echoes to achieve the required resolution, as well as the role of atomic clocks in long base line interferometry, is in preparation.
14. M. E. Ash, I. I. Shapiro, W. B. Smith, *Astron. J.* **72**, 338 (1967).
15. If intercontinental base lines become feasible, perhaps at somewhat lower frequencies, then detection of continental drifts of the order of 1 cm per year might be possible. An island such as Puerto Rico, with its Arecibo radar facility, might be an ideal first choice for one arm of such an interferometer. Radio beacons or phase-coherent transponders on the moon or on synchronous earth satellites might be more appropriate than planetary targets for this purpose. With a source on the moon, high-accuracy studies of lunar libration and orbital motion would also be possible. The "geodesic precession" might be detectable as well.
16. M. C. Thompson, Jr., and H. B. Janes, in preparation.
17. Such interferometric measurements can, of course, also be used to improve orbit determinations.
18. M. C. Thompson, Jr., D. Smith, D. M. Waters, R. O. Gilmer, *National Bureau of Standards Report No. 9144* (1966, unpublished).
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20. Unfortunately, the Haystack-Westford base line is nearly north to south and has its largest component along the most useful direction for detecting gravitational light deflection only near dawn and dusk when atmospheric effects are most severe.
21. T. Hagfors, personal communication.
22. "Objectives and study program for a regional radio and radar astronomy research facility" (Cambridge Radio Observatory Committee, 1965, unpublished).

23. Lincoln Laboratory is operated with support from the U.S. Air Force.

22 May 1967

Clear-Air Turbulence: Simultaneous Observations by Radar and Aircraft

Abstract. *Ultrasensitive radars and uninstrumented jet aircraft in concert have probed regions of the clear atmosphere in search of clear-air turbulence. All sources of clear-air radar echoes above 6 kilometers that were probed simultaneously by the aircraft were found to be turbulent.*

At Wallops Island, Virginia, ultrasensitive radars of 3.2-, 10.7-, and 71.5-cm wavelength are being used to investigate the nature of all radar echoes from a clear atmosphere. One object of the program is to determine the feasibility of using radar for detecting clear-air turbulence (CAT); such turbulence at high altitude has recently attracted attention by the government and the airlines. Aircraft accidents have been attributed to unexpected encounters with turbulence in cloudless skies. For increased safety and because of the high costs of damage to aircraft and of flying diversionary routes to avoid turbulence, a system for detecting turbulence from a distance is obviously desirable.

The Wallops Island radar facility has been described (*1*). Its more important characteristics for this discussion are the beamwidths of 0.21, 0.48, and 2.9 deg, and the minimum detectable signals of −110, −115, and −112 dbm (decibels referred to milliwatts) for the radars of 3.2-, 10.7-, and 71.5-cm wavelength, respectively. Using this multiwavelength facility, one can distinguish between clear-air radar echoes and echoes caused by clouds, precipitation, or any other types of particulate matter (*1, 2*). The clear-air echoes arise because of scattering caused by variations in refractive index; many have studied theoretically the mechanism responsible for the scattering (for example, *3*). Although clear-air echoes are often observed in the lower troposphere, clear-air radar layers from the tropopause also have been described (*4*); the tropopause marks the upper boundary of the troposphere and the lower limit of the stratosphere.

It is known that no unique relation exists between the intensity of clear-air radar echoes and the severity of turbulence (*4*). Instead, the intensity of the echoes depends on both the magnitude of the mean vertical gradient of refractive index and, in a complex manner, the severity of turbulence. The large vertical gradients of refractive index that frequently exist in the lower troposphere are detectable even when the turbulence within the region is much too little to affect aircraft. In the upper troposphere, however, the vertical gradient of refractive index is limited because of the almost negligible presence of water vapor in the atmosphere at high altitude. Under these circumstances it was inferred on theoretical grounds (*4*) that turbulence sufficiently intense to affect aircraft appeared to be necessary before the region of turbulence at the tropopause became detectable. This theory is supported by the results of simultaneous probing of CAT regions with narrow-beam microwave radars and by aircraft; this is a preliminary report of the results.

The experimental procedure with radar consisted primarily in taking slow-scan (0.1 deg/sec) photographs of the range-height indicators of the three radars. This type of scan provides a picture of the atmosphere in a vertical plane, from which one can determine the location and extent of any high-altitude clear-air radar layer; this type of layer is revealed only by the 10.7- or 71.5-cm radar (*4*). Airplanes were used as direct probes to determine whether turbulence was associated with the source of the radar echoes. Aircraft (*5*) made spiral ascents and descents, level flights, or porpoise runs (a sawtooth pattern about a mean height) in predetermined flight zones. About every 15 seconds the pilots reported their altitudes and qualitatively estimated the severity of turbulence encountered; their estimates were recorded on tape.

The results of four aircraft flights and observations by the 10.7-cm radar for regions above 6 km are summarized in Fig. 1. The heights of the sources of radar echo and their vertical extent were obtained from the range-height photographs. The altitudes of the radar layers and of the turbulences reported by aircraft are accurate within about 200 m. All high-altitude clear-air radar echoes are weak and are usually detected over a horizontal range of only 10 to 20 km. (The radar range of detectability is indicated qualitatively by the horizontal extent of the layers shown in Fig. 1.) Three of the radar layers occur near a height of 11.5 km, which corresponds to the level