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25. The most recent available sources were used for the sequence data. In most instances, earlier references will be found in the literature cited. I am particularly grateful to Drs. L. Hood, W. R. Gray, W. J. Dreyer, and F. W. Putnam for allowing me to use extensive amounts of sequence data from manuscripts in preparation. The designations of the proteins are those of the investigators as follows.
   Ag: K. Titani, M. Wikler, F. W. Putnam, (*24*). At positions 7 and 8 the sequence is Ser•Pro, K. Titani and F. W. Putnam, personal communication.
   Roy, Cum: N. Hilschmann and L. C. Craig, *Proc. Nat. Acad. Sci. U.S.* **53**, 1403 (1965).
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26. The abbreviations for the amino acid residues are as follows: Ala, Alanine; Arg, arginine; Asn, asparagine; Asp, aspartic acid; Cys, cysteine; Glu, glutamic acid; Gln, glutamine; Gly, glycine; His, histidine; Ile, isoleucine; Leu, leucine; Lys, lysine; Met, methionine, Phe, phenylalanine; Pro, proline; Ser, serine; Thr, threonine; Try, tryptophan; Tyr, tyrosine; Val, valine; Asx and Glx, aspartic and glutamic acids of unknown amide status.
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29. I have adopted the descriptive word "master" from a related but not identical usage by H. G. Callan and L. Lloyd, *Phil. Trans. Roy. Soc. London Ser. B* **243**, 135 (1960) and H. L. K. Whitehouse (*21*).
30. This argument was suggested to me by Dr. J. F. Crow.

31. This work (paper No. 1134 from the Laboratory of Genetics, University of Wisconsin) was supported by grants NIH (GM08217) and NSF (GB4362). I think my students and colleagues for patient discussions of this work.

Mathematical Theory of Automobile Traffic

Improved understanding and control of traffic flow has become a fast-growing area of scientific research.

Denos C. Gazis

Much has been written about the influence, good and bad, which the automobile has had on our lives. In particular, the subject of traffic congestion has appealed to popular writers, cartoonists, crusaders for saner living, and other serious workers. Some of them have recognized that traffic congestion is not a characteristic of our time only, but may be traced back to Roman times. It certainly existed in major cities near the turn of the century (see Fig. 1), without much contribution from automobiles. However, the emergence and popularity of the automobile has multiplied the problems of congestion and exported them to the suburbs. The centers of the major cities, of course, continue to extract the heaviest toll of delays and frayed nerves. This is illustrated in Fig. 2, which I like particularly because of the unintended caption in the top left-hand corner. The "ecstasy" is, of course, the emotion planned for the users of automobiles by the automobile manufacturers. The "agony" is the true emotion of drivers in New York City, and every other major city.

The investment of money and effort in automobile transportation is nothing less than staggering. In this country alone, the yearly expenditure is of the order of $80 billion, or one tenth of the gross national product. Perhaps it is not surprising that the rapid growth in the use of automobiles has caught us somewhat unprepared to handle the traffic. Compounding the difficulty of understanding and managing so vast an operation has been the fact that it involves not only inanimate objects but also the often unpredictable human being. Under the circumstances, it is not surprising that the management of automobile traffic developed largely as an art, with tools ranging from ingenious to hit-or-miss. Empiricism was of necessity the first approach in management of automobile traffic, and still is the mainstay of traffic engineering. However, in the last 10 years or so, a growing effort has been made to develop a science of vehicular traffic flow and control. The contributions to traffic science have been made by scientists with very diverse backgrounds, each of whom has left the imprint of his discipline on the literature of traffic theory. The growing club of traffic scientists has already convened three times in international symposia (*1–3*) and has contributed to a rapidly growing literature (*4*). In this article I attempt to present some highlights of this work, with emphasis on the more mathematical areas, some

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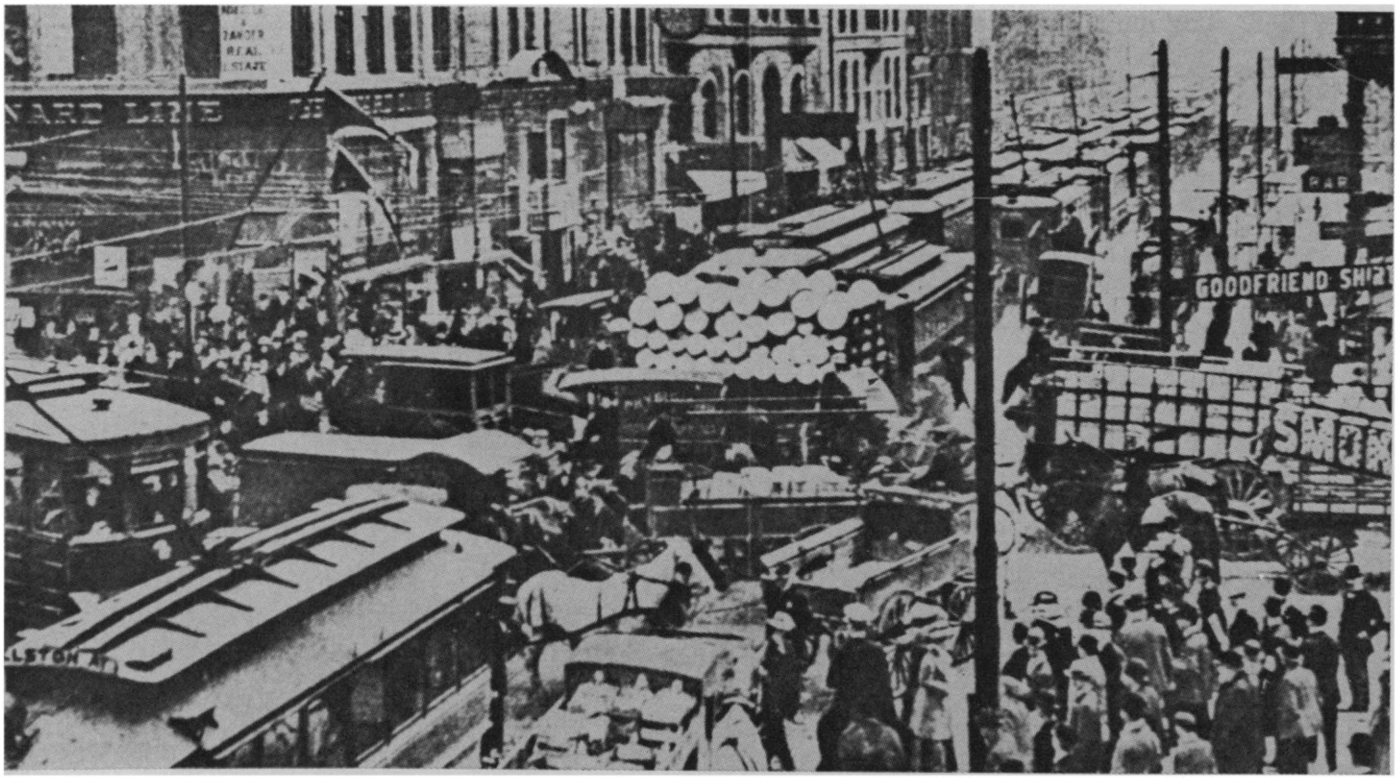

Fig. 1. Corner of Dearborn and Randolph Streets, Chicago, in 1910. [Bureau of Public Roads]

of which have been tested experimentally and have led to direct or indirect applications.

Generally speaking, traffic theorists have addressed themselves to the following problems: flow of traffic on a single-lane or a multi-lane highway; conflicts of traffic streams, as in passing, crossing, or merging; the optimum control, by traffic lights, of a single intersection or a system of intersections; and planning of networks of traffic arteries to accommodate traffic demands. Let us first consider traffic flow.

Traffic Flow Models

In the early 1950's, Reuschel (*5*) and Pipes (*6*) discussed a traffic flow model which takes into account the detailed motion of cars proceeding close together in a single lane. A little later, Lighthill and Whitham (*7*) proposed a model of traffic flow in which traffic was treated as a continuum. These two models are representative of the two different approaches taken over the years in describing traffic flow. The first is a "microscopic" approach in which an attempt is made to describe the behavior of the average individual driver when he must follow the cars ahead in a single lane. The second is "macroscopic," modeled on theories of fluid motion, and may be useful in describing the variation of aggregate measures of either single-lane or multi-lane traffic.

The microscopic, car-following approach has been extensively developed in recent years. Ten years ago, Herman and Montroll planned and carried out the first car-following experiments at the General Motors Research test track. Shortly afterward, Potts, Rothery, and I joined in exploring this area, in collaboration with other workers (*8–14*). The basic notion of the car-following model is that a driver, in a single-lane traffic situation, reacts largely to a stimulus from his environment according to the relationship

$$(\text{Reaction})_{t+T} = \lambda\,(\text{stimulus})_t \qquad (1)$$

where λ is a sensitivity coefficient, t is the time, and T is a reaction time-lag.

Here, the reaction is the acceleration, which the driver controls with the brake and gas pedal. The stimulus may reasonably be expected to be a functional of the differences of various time derivatives of the position of the driver's car and the positions of his neighbors' cars. In the car-following experiments, special gear was used to measure and record the acceleration of a car as well as its speed relative to that of the car ahead (the "leader") and the spacing between the two cars. A high correlation was found between a driver's acceleration and his speed relative to that of the leader, after a time lag of the order of 1 second. This led to the "linear car-following model,"

$$\frac{d^2 x_n(t+T)}{dt^2} = \lambda\left[\frac{dx_{n-1}(t)}{dt} - \frac{dx_n(t)}{dt}\right] \qquad (2)$$

in which n denotes the position of a car in a line of cars (a platoon), λ is a constant, and x_n is the position of the nth car on the highway.

This model was used for investigating the *stability* ("local" or "asymptotic") of traffic, after introduction of a perturbation into the stream. *Local stability* refers to the relative positions of a fixed pair of cars; *asymptotic stability* refers to the attenuation or increase in the amplitude of a perturbation as it propagates down a line of cars. Local stability was investigated by solving Eq. 2 by means of Laplace transform techniques (*9, 13*). It was found that local stability depends on the product λT as follows: If $\lambda T \leqslant 1/e$, a perturbation is damped monotonically (e is the base of natural logarithms, approximately equal to 2.72); if $1/e < \lambda T \leqslant \pi/2$, a perturbation produces oscillations of decreasing amplitude in the distance separating the two cars; and if $\lambda T > \pi/2$, a perturbation produces oscillations of amplitude which increases with time. As might be expected, overcompensation, with a large time lag, is a source of trouble. The asymptotic stability was investigated by considering a sinusoidal perturbation in the speed of an infinite platoon of cars. In the most critical range, the range of low frequencies, it was found that the amplitude would increase or decrease depending on whether λT was greater or smaller than 1/2. From this discussion it may be seen that a line of cars may be locally stable but asymptotically unstable. That is, consecutive drivers may be able to avoid collision with their leaders but may amplify the "signal" as they pass it on to their followers.

The linear car-following model may be satisfactory in describing small fluctuations near a steady-state, constant-speed situation. However, it does not describe well those transitions from one steady state to another which involve appreciable changes of speed and spacing. We therefore considered (*12*) various nonlinear (*15*) models in which the sensitivity is not constant but depends on the speed of the follower and the distance from his leader, according to the relationship

$$\lambda = c\,\frac{\left[\dfrac{dx_n\,(t+T)}{dt}\right]^l}{[x_{n-1}\,(t) - x_n\,(t)]^m} \qquad (3)$$

where c is a constant and (l, m) are integer exponents, particular values of which I use, in parentheses, for identifying particular models. Under the assumption of Eq. 3, Eq. 2 can be integrated over time. This integration yields a relationship between changes of speed and of spacing which may be considered a description of a transition from one steady state to another. Together with an appropriate "boundary condition"—for example, the requirement of zero speed in a bumper-to-bumper situation ("jam concentration")—it yields a relationship between speed, v, and spacing S (or concentration, $k = 1/S$).

The first nonlinear model to be considered (*10*) was the (0, 1) model, from which we find

$$v = c \ln (k_j/k) \qquad (4)$$

where k_j is the jam concentration. The flow q is then given by

$$q = vk = ck \ln (k_j/k). \qquad (5)$$

The parameter c has dimensions of speed and is the characteristic speed of the system, corresponding to maximum q.

It is interesting to note that Eq. 5 was first derived, by Greenberg (*16*), by means of a postulated hydrodynamic equation.

The above nonlinear car-following model was tested in many ways. First, it was found that Eq. 5 agreed quite well with average flow data obtained at the Lincoln Tunnel in New York. Then, Herman and Rothery, together with Edie and Foote of the Port of New York Authority, carried out two studies, a series of measurements of car-following speed and spacing through the vehicular tunnels of New York (*11*) and more accurate measurements of flow and speed at the Lincoln Tunnel (*17*). One of the most interesting results of this series of experiments and observations was the striking agreement between the characteristic speed of cars in the tunnels as determined from gross flow measurements and the values obtained from analysis of the car-following data.

Regarding the more general model of Eq. 3, I should first mention that Edie was responsible for our suggesting this model, by proposing (*18*) the model (1, 2) as a better fit for low density data. Extensive tests of many (l, m) models against car-following data and flow data have been rather inconclusive (*12, 14*). Some bad choices for l, m are easily eliminated, but available data are insufficient to establish absolute superiority of a given model. I should also mention that Newell (*19*) suggested a somewhat different model, in which he lumped the sensitivity and stimulus into a single functional. Recently, Lee (*20*) treated a model proposed by Herman, in which a driver weighs the past stimuli with a memory function instead of reacting to a stimulus received a fixed time interval earlier.

One distinct disadvantage of all car-following models, particularly the non-

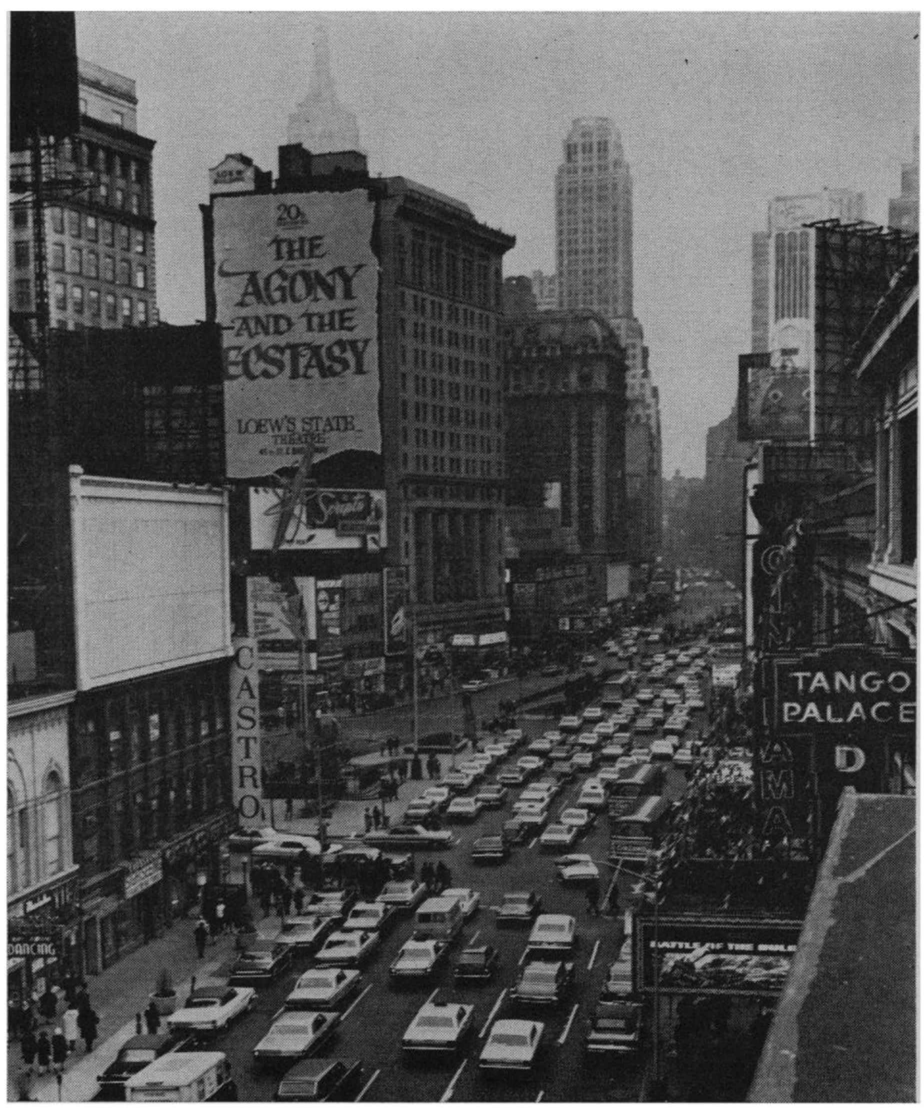

Fig. 2. Looking down Broadway from 47th Street in the Times Square area of New York City. [Philip Gendreau]

linear ones, is the fact that they do not explicitly consider certain physical constraints, such as the limited acceleration capability of cars. Such constraints have been considered only in a simulation study of bottlenecks, by Helly (*21*). Recently I suggested (*22*) a formulation of car following which uses tools of control theory. It is based on the statement that the driver attempts to minimize an "objective function," while the "control parameters" at his disposal are constrained within a given domain. The disadvantage of such a model is the fact that it is not easily amenable to analytical investigation and may be used only in a numerical computation.

Let us now turn to the continuum flow models.

Lighthill and Whitham's theory is based on two postulates (*7*): the constancy of the total number of cars which is expressed mathematically as

$$\frac{\partial k}{\partial t}+\frac{\partial q}{\partial x}=0, \qquad (6)$$

and the existence of an "equation of state" relating the flow $q = vk$ and the concentration k—namely,

$$q = q(k). \qquad (7)$$

From these assumptions, it follows that

$$\frac{\partial k}{\partial t}+V\frac{\partial k}{\partial x}=0, \qquad V=\frac{\partial q}{\partial k} \qquad (8)$$

(V is the speed of propagation of perturbations), which has the solution

$$k = F(x - Vt). \qquad (9)$$

where F is an arbitrary function.

Equation 9 implies that inhomogeneities such as changes in concentration of cars propagate along a stream of traffic at constant speed V. For example, a small change of speed propagates with speed $\partial q/\partial k$, which is positive or negative with respect to a stationary observer, depending on whether the concentration is below or above the optimum concentration corresponding to maximum q (Fig. 3). Furthermore, the theory predicts the existence of shock waves when the density of cars is higher in the direction of traffic movement. That is, a shock front propagates along a stream of traffic, and cars change speeds abruptly as they pass through this front. This somewhat startling feature of theory stems from the fact that the theory neglects the detailed maneuvers of cars in changing speeds. Thus the usefulness of the theory is restricted to cases where disturbances are relatively infrequent. The assumption of an equation of state like

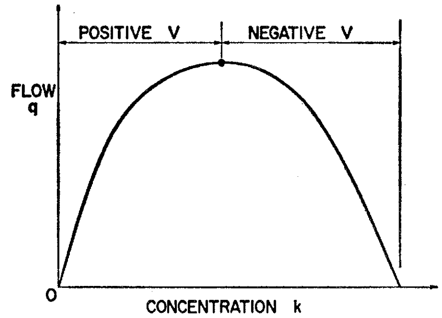

Fig. 3. Diagram showing the general character of the flow-concentration relationship assumed by Lighthill and Whitham (*7*) and the regions of positive and negative speed of propagation of perturbations, v.

that of Eq. 7 is also only approximately verified by observation. This fact may account for the less-than-perfect validation of the theory through the recent experiments of Leutzbach (*23*). Nevertheless, this theory is still the best available description of "kinematic waves" in traffic. It was used by Lighthill and Whitham for describing bottlenecks, and the periodic disturbances caused by traffic lights, by means of streamlines in the time-space diagram. Bick and Newell (*24*) gave a generalization for an undivided highway. Newell (*25*) also has suggested another generalization of the theory, which takes into account the difference in the attitude of drivers during acceleration and deceleration, which involves two distinct equations of state and some mode of transition from one to another. With this model, Newell gave a possible explanation of instabilities in dense traffic which result in amplification of disturbances and congestion.

Another interesting approach to the description of traffic is that of Prigogine and his associates (*26*). This theory is based on a description of traffic in terms of a probability density for the speed of an individual car, $f(x, v, t)$. This density may vary as a function of time, t, and a coordinate x along the highway. The basic equation for the distribution is assumed to be

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}= \left(\frac{\partial f}{\partial t}\right)_{\text{relaxation}}+\left(\frac{\partial f}{\partial t}\right)_{\text{interaction}} \qquad (10)$$

The first term of the right-hand element of Eq. 10 is a consequence of the fact that $f(x, v, t)$ differs from some desired speed distribution $f^0(v)$. The second term of the right-hand element corresponds to the slowing down of a fast vehicle by a slow one. True to his traditions as a leading expert on statistical mechanics, Prigogine frequently refers to this second term as the "collision" term—a rather unsettling choice of words in this context!

The form of these terms has been chosen for mathematical convenience and plausibility. Thus, Eq. 11 has been assumed to have the specific form

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}=\frac{f-f^0}{\tau}+(1-p)\,k(\bar{v}-v)f \qquad (11)$$

where τ is a characteristic relaxation time, p is the probability of a car's passing another car, and $\bar{v}$ is the average speed of the stream of traffic. The second term of the right-hand element of Eq. 11 corresponds to the interaction term of Eq. 10 and tends to approach zero when the concentration of cars is light and the probability of passing is close to unity. In that case, the relaxation term is dominant. If, in addition, we assume a highway with a homogeneous distribution of cars, then $\partial f/\partial x = 0$ and the solution of Eq. 11 is

$$f(v, t) = f^0(v) + [f(v, 0) - f^0(v)]\,e^{-t/\tau}. \qquad (12)$$

If we are interested only in solutions of Eq. 11 that are independent of time and space, then the left-hand element of Eq. 11 is zero. Equation 11 may then be solved (*27, 28*), to yield an equation of state whose general form, for small values of the concentration, is an approximately linear increase of flow with concentration; that is,

$$q \approx \bar{v}^0 k \qquad (13)$$

where $\bar{v}^0$ is the average of the desired speed. As k increases, the flow q falls below the straight line (Eq. 13) due to the increasing influence of interactions. In the range of high concentrations, q is independent of f^0 and depends only on τ and p, according to the equation

$$q=\frac{1}{\tau(1-p)}. \qquad (14)$$

The curve (Eq. 14) may be viewed as a universal curve of "collective flow," which is characterized by high densities and very little passing. For any given f^0, the flow q rises with k along a curve characteristic of the specific f^0 until it intersects the curve (Eq. 14), then decreases monotonically with k according to Eq. 14 (Fig. 4). One very realistic feature of this theory is the fact that it predicts probable stoppage of some vehicles in the case of collective flow. This is certainly in agree-

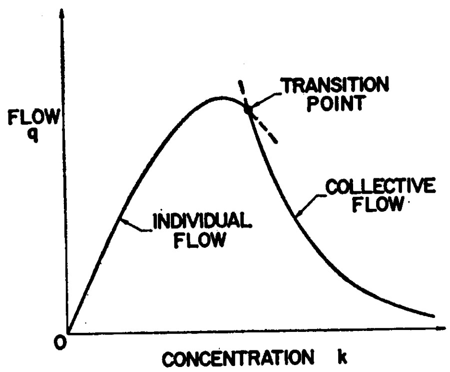

Fig. 4. Flow-versus-concentration relationship obtained by means of the Prigogine theory (*26*).

ment with the common experience of stop-and-go traffic at high concentrations.

Figure 5 shows two traffic situations which illustrate the main points of Prigogine's theory. On the right-hand side we see relatively light traffic in which passing is easily accomplished, and the slowing down of fast cars by slow ones is not too great. The left-hand side shows heavier traffic which has probably reached the state of "collective flow."

I should also mention here the work of Miller (*29, 30*), in which the flow of traffic is described as a process of formation of bunches, or queues, which follows certain rules governing the rates at which faster-moving queues catch up with slower-moving ones, and the rates of passing.

Conflicts in Traffic and Applications of Statistics

In view of the obvious stochastic nature of virtually all the parameters involved in the flow of traffic, it is not surprising that experts in the area of probability and statistics have found this a fruitful area of research.

One class of problems treated by statistical techniques deals with the delay of pedestrians or vehicles attempting to cross a stream of traffic at an uncontrolled intersection. The problem of pedestrian crossing was first considered by Adams (*31*) and, more recently, has been studied by Raff (*32*), Tanner (*33*), and Mayne (*34*). Garwood (*35*) discussed the operation of traffic-actuated signals on the basis of probability theory. Weiss and Maradudin (*36*) used the techniques of renewal theory (*37*) in formulating and solving the problem of a vehicle crossing a single stream of traffic. This method has formed the basis for a series of solutions to related problems, by Weiss and his collaborators (*38–42*). I discuss here a simple case which illustrates the main features of this approach.

Consider a car arriving at an uncontrolled intersection at time $t = 0$. A single line of traffic is passing in front of him, and the time gaps, Δ_i, between cars are assumed to be independent random variables. The probability density for these gaps is some given function $\phi(\Delta)$. I assume, for this discussion,

$$\phi(\Delta) = \sigma\, e^{-\sigma\Delta}, \tag{15}$$

which has been found to be a reasonably close approximation to actual measurements (*31*). We now postulate $\alpha(\Delta)$ as the probability that the driver "accepts" a gap Δ and crosses the street. One such gap-acceptance-probability function, for example, may be the step function

$$\alpha(\Delta) = H(\Delta - G) = \begin{cases} 0 & \Delta < G \\ 1 & \Delta \geqq G \end{cases}, \tag{16}$$

where G is the "minimum acceptable gap" and $H(x)$ is a step function of x. Equation 16 means that the driver always crosses if, and only if, $\Delta \geqq G$. I now need only mention the point that the probability density for the first gap the driver sees upon arrival, which is a fractional "unfinished" gap, is also given by Eq. 15, a convenient feature of the exponential distribution. We may then ascertain that the probability density function $\Omega(t)$ of the waiting time t satisfies an integral equation which is derived by the following argument: either the driver accepts the first gap he sees on arrival, with probability

$$\int_0^\infty \sigma e^{-\sigma x}\alpha(x)\,dx = e^{-\sigma G}; \tag{17}$$

or he finds the first gap of duration x unacceptable, with probability

$$\rho(x)dx = \sigma e^{-\sigma x}\,[1 - \alpha(x)]\,dx. \tag{18}$$

In the latter case, the time x is a "regeneration point" of the "renewal process": the search for an acceptable gap starts again and is successful after an additional time $t-x$. This argument leads to the equation

$$\Omega(t) = e^{-\sigma G}\delta(t) + \int_0^t \rho(x)\,\Omega(t-x)\,dx \tag{19}$$

where $\delta(x)$ is a delta function. If we take the Laplace transform of Eq. 19, we obtain an algebraic expression of the transform of $\Omega(t)$ in terms of known functions. The transform of $\Omega(t)$ may then be used as a moment-generating function. By means of successive differentiations it yields the various moments of the probability density $\Omega(t)$. For example, the first moment—that is, the mean waiting time—is

$$\bar{t} = \frac{1}{\sigma}[e^{\sigma G} - \sigma G - 1]. \tag{20}$$

Several generalizations of the preceding discussion have been given—for example, the possibility that the driver accepts the first gap with probability which differs from $\alpha(\Delta)$, because he may, legally or illegally, keep rolling

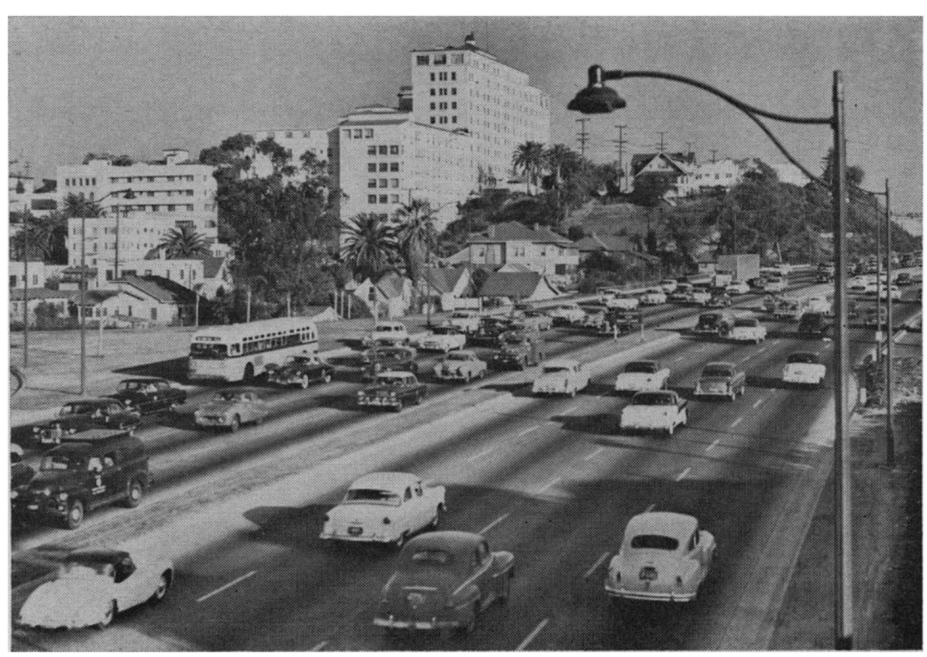

Fig. 5. Freeway traffic, Hollywood, California. [Philip Gendreau]

without coming to a complete stop (*36*); or the possibility that the driver checks not only the immediately available gap but also the gap following it and waits for the latter if it is larger, even though the immediate gap is adequate (*42*). Recently, Newell, Warren, Weiss, and I considered another generalization, the crossing of an N-lane highway (*41*). What makes this problem somewhat more complicated is the possibility that the driver may require a larger time gap in the farther lanes than in the lane nearest to him. A theoretical solution based on "renewal theory" again gave a means for estimating moments of the probability density of waiting time, $\Omega(t)$, such as the mean and the standard deviation. The analytical results were tested by a computer simulation which also yielded histograms of $\Omega(t)$.

Another interesting description of what a driver faces while trying to cross a stream of traffic has been given by Oliver (*43*). He computed the statistics for "blocks" and "gaps"—intervals of time during which crossing is, respectively, impossible and possible.

The preceding discussion describes the delay imposed on a single car, or pedestrian, in crossing a busy highway. When a succession of vehicles or pedestrians arrives and demands service (in the form of a gap or a traffic signal) at the same point, queueing takes place if the service is not instantaneous. The problem of pedestrian queueing is relatively simple because pedestrians can cross the street almost simultaneously as soon as a gap or traffic signal is provided. This problem was first treated by Tanner (*33*), and has been treated in greater generality by Weiss (*39*). Vehicular queueing is a much more involved process, particularly if one tries to consider the fact that different drivers behave differently when trying to cross a stream of traffic. Nevertheless, some useful results have been obtained in this area by Beckmann *et al.* (*44*), Tanner (*45*), Gaver (*46*), and others (*47*). Evans, Herman, and Weiss (*39*) have reported an analytical treatment of a queueing model and more extensive results of a simulation study. A related problem of queueing at a fixed traffic light has been treated by Haight (*48*) and by Newell (*49*).

Another problem is that of passing on a two-lane highway. As in the case of crossing, a driver must find an acceptable gap before he can pass a slow vehicle. Tanner (*50*), Miller (*29*), Kometani (*51*), and Newell (*52*) have considered this problem. More recently, Erlander (*53*) and Gustavson (*54*) have treated the problem of passing complicated by limitations in the distance the driver can see ahead. Essentially, they treated the problem as one in which the driver is confronted with two strings of gaps which move with respect to each other, one being the string of gaps between slow-moving vehicles and the other the string of gaps between obstructions such as hills and curves.

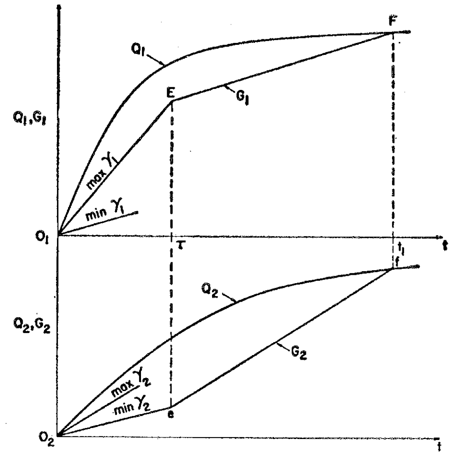

Fig. 6. Optimum control of an over-saturated intersection.

Other applications of statistics and queueing theory are reviewed by Cleveland and Capelle (*55*); a more mathematical survey is given by Weiss (*56*).

Control Problems

Traffic control is the battlefront of the traffic engineer and the heart of modern traffic systems. While some control measures date back to ancient times, traffic control as we know it is barely half a century old, having started with the installation of a primitive traffic light in St. Louis, in 1914 (*57*). Traffic control devices of increasing sophistication were developed in the post-World War II period, from the mid-1940's to the late 1950's, which heralded the age of computers in traffic control. Today there are three functioning digital-computer installations for the control of traffic, one in Toronto, Canada, one in San Jose, California, and one in Wichita Falls, Texas. Many other cities are about to install their own digital computer systems, and many others use special-purpose analog computers of varying flexibility and sophistication.

Before discussing the possible role of the mathematical theory of traffic in the design of control systems for city networks controlled by traffic lights, I should mention some general principles common to such systems. All modern systems involve two distinct time scales, and corresponding control processes. The first is a time scale measured in units of many minutes, and the corresponding control is called "major-loop control." The second is a time scale measured in units of seconds, and the corresponding control is called "minor-loop control." Major-loop control is intended to provide a synchronization of groups of lights which is particularly suited for traffic demands that remain reasonably stable over periods of many minutes. Minor-loop control is intended to provide special second-by-second control of critical points in a traffic system. These two types of control are the two aspects of the problem that have recently been the subject of mathematical research—the development of methods of synchronizing groups of lights and the study of the single intersection, or of small systems involving a few control points.

In the area of synchronization, the old workhorse "progression" has received new attention, and computer techniques for its solution have been proposed by Morgan and Little (*57*), Brooks (*58*), and Yardeni (*59*). The standard progression design aims at providing maximum opportunity for drivers to drive through a system of lights without stopping, at some designated speed, provided the street is not too crowded. It accomplishes this by providing the maximum "through band" —that is, the maximum percentage of green-light phase ("green phase") which such continuously moving traffic can use. Little (*60*) has proposed a mixed-integer linear programming technique for solving a slightly more general problem, in which allowance is made for small changes in speed between successive pairs of intersections. He has also suggested an extension of his method for synchronizing the lights of city networks. Progression design for networks is complicated by the existence of loops which make it virtually impossible to keep every driver moving and happy. The standard arterial progression design can be used only for a "tree" of arteries which contain no such loops. Little's approach yields a design for networks which provides the maximum aggregate "through band" along all possible routes. Another approach, which has been used by Irwin (*61*), is a program which establishes some ideal "relative offsets" between lights and then reduces to a minimum a weighted sum of powers (for example, squares) of the deviation of the design offsets from the ideal set.

Many of us have recognized (*62*) that a design based on the tacit assumption of a relatively empty highway may not be satisfactory in the case of crowded streets where there is a queue of vehicles at the beginning of the green phase. Chang (*63*) has recently proposed a technique which takes these queues explicitly into account in evaluating delay experienced by the users of a city network. Chang's method and computer program are based on use of a search scheme which reduces this delay through small adjustments of the offsets of the traffic lights, in several stages. The method was tried in San Jose and showed substantial advantages over older methods. Hillier (*64*) has also attempted to introduce more realism in the synchronization of lights; he measures delays of platoons between neighboring intersections for different relative offsets and designs a system which matches, insofar as possible, the relative offsets which give minimum delay.

The study of the single intersection is, or should be, the starting point in the design of minor-loop control. An intersection is usually characterized by a set of inputs, q_i, along various competing directions, and by corresponding "saturation flows," s_i, which are the maximum rates of flow when the light is green (rates assumed to be in effect after some time has been lost in startup). A study of the optimum *fixed* setting of a single traffic light has been made by Webster (*65*) and by Miller (*66*). Webster supplemented an analytical investigation by a computer simulation and derived a formula for the delay as a function of (random) inputs. He used this formula to evaluate the optimum cycle and optimum "split" (that is, allocation of green-light time). Even more important in the design of control systems is knowledge of the properties of traffic-actuated signals. Although many such signals have been manufactured and used, very little analytical work has been done in evaluating their effectiveness for various traffic inputs. Recently, Dunn and Potts (*67*) considered a control algorithm which calls for changing the phase of a two-phase light when the queues in the two competing directions satisfy one of two linear relationships. More recently, Grafton and Newell (*68*) used a dynamic programming approach to prove that the "saturation flow algorithm," which calls for switching the light as soon as a stopped queue is completely served, minimizes the delay in many, but not all, cases.

All this work pertains to the intersection where the density of cars is below the saturation point and there is sufficient green-phase time to serve all incoming traffic. Four years ago Potts and I recognized that there are many cases of "oversaturated" intersections in which residual queues build up during a rush period and are served completely only at the end of this period. We found (*69*) that in such cases the control strategy should not depend on instantaneous measurements of queue sizes and inputs but should take into account the expected nature of the demand during the entire rush period. This situation is shown in Fig. 6 where Q_i and G_i are the cumulative-demand curve and cumulative-service curve, respectively, for the ith stream, defined by

$$Q_i(t) = \int_0^t q(\tau)d\tau,$$

$$G_i(t) = \int_0^t \gamma_i(\tau)d\tau,\ (i = 1, 2) \qquad (21)$$

where zero time corresponds to the onset of oversaturation and γ_i are the service rates—namely,

$$\gamma_i = Cg_i/s_i \qquad (22)$$

where C denotes the cycle and g_i denotes the effective green phases. The cumulative-service curves are drawn as smooth lines in Fig. 6, but it should be borne in mind that they have a fine sawtooth form, due to the succession of red and green phases. The fact that the cumulative-service curves are below the cumulative-demand curves is a consequence of oversaturation which sets in when

$$\frac{q_1}{s_1} + \frac{q_2}{s_2} > 1 - \frac{L}{C} \qquad (23)$$

where L is the total lost time during the switching of the light. Finally, the service rates cannot be arbitrarily large or small; very short green phases are impractical or useless, and very long ones are unacceptable to the stopped drivers of the cross traffic, who often assume that the light has failed and must, therefore, be ignored. If $g_{\min}$ and $g_{\max}$ are the bounds of the green phases, then the service rates γ_i satisfy the constraints

$$g_{\min} \leq g_i = \frac{C\gamma_i}{s_i} \leq g_{\max}. \qquad (24)$$

The optimum control policy is that which minimizes the sum of the areas between the cumulative-service and the cumulative-demand curves, and is established as follows (*70*): The sizes of the queues, x_i, satisfy the relationships

$$\begin{aligned} \frac{dx_i}{dt} &= f_i\ (i = 1, 2) \\ f_1 &= q_1 - u \qquad (25) \\ f_2 &= q_2 - s_2(1 - L/C) + (s_2/s_1)u \end{aligned}$$

where $u = s_1 g_1/C$ is the control variable constrained to lie within the control region given by Eq. 24. The objective is to minimize the integral

$$x_0 = \int_0^{t_1} (x_1 + x_2)dt \qquad (26)$$

by selecting an appropriate control function $u(t)$. The t_1 is the end of the rush period, and the sizes of the queues, x_i, satisfy the end conditions

$$x_i(0) = x_i(t_1) = 0, \quad (i = 1, 2). \quad (27)$$

The solution of the problem, which may be obtained by techniques of control theory, is shown in Fig. 6. During the early part of the rush period the direction corresponding to greater flow receives the maximum amount of green-phase time and the other direction receives the minimum amount. Then, from some appropriate switchover time to the end of the rush period, the allocation of green-phase time is reversed. I have been told by an Englishman that this is exactly the opposite of the pattern used by bobbies in London. It appears that in England, as in this country, there is great sympathy for the underdog, which, in this case, is the minor-street traffic.

I have used the same technique in discussing the optimum control of other oversaturated systems, such as two neighboring intersections (*71*), an exit ramp and the highways it connects (*70*), and the reversible lane of an oversaturated tunnel (*72*).

Networks

An excellent demonstration of the often unexpected dividends of mathematical research is the case of "traffic assignment," which is today a standard tool in traffic planning. In 1957 Dantzig (*73*) and Moore (*74*) gave algorithms for computing the shortest routes between various nodes of an idealized network. These algorithms were soon used by traffic engineers in evaluating traffic patterns between zones in a city, or plotting trees of shortest routes from one city to many others. Later developments have led to the work presented at the third international symposium on traffic theory—the treatment of flow-dependent travel times by Almond (*75*), investment payoff for various improvements in a network by Ridley (*76*), and so on. A computer program has also been developed, by Roy and Boulanger (*77*), for evaluating the detailed motion of cars through a city network in which each intersection is a subnetwork whose arcs correspond to the various permissible movements. An interesting study of networks in towns has been made over the last few years by Smeed (*78*). By assuming some special forms of city networks, such as a circular polar grid for a circular city, he derived analytically some working formulas for trip lengths, flow, speed, and density relationships. He found a remarkable consistency of some of these formulas when he tested them numerically in various cities having a great variety of configurations of street networks. In this way, Smeed can give, for example, the total number of cars which can be accommodated, at a given average speed, in the central business district of a city, in many parts of the world.

I believe that we have barely seen the beginning of the potential benefits of network theory in traffic planning and management. More benefits are to be derived as our system design capability increases and the use of computers moves past the present stage of signal manipulation into a stage at which it will be possible to instruct drivers on how they can best reach their destination. "Route control" is a yet untapped possibility, which is likely to considerably augment our means for combatting congestion, once we have learned how to use it.

Concluding Remarks

I have by no means covered all the important contributions to the theory of traffic flow. In particular, I have said nothing about the very important work on human factors, sponsored by such institutions as Michigan State and Ohio State universities and the Bureau of Public Roads. Neither have I covered adequately the contributions of the British Road Research Laboratory (see *79*).

I close with a comment on the future role of traffic theory. I have no doubt that this theory, which is already part of the traffic-engineering curriculum in major universities here and abroad, will influence the thinking and techniques of the coming generation of traffic engineers. Some benefits can be immediately derived from work in this area. Others will come indirectly through better understanding of traffic phenomena. For example, the research workers of the Port of New York Authority have credited the flow theories with having provided the basic knowledge required for the development of their heuristic control schemes (*80*) for obtaining optimum flow through the tunnels of New York City. Many problems in traffic control cannot yet be subjected to rigorous analysis and may be attacked only by heuristic schemes. It is very likely, however, that better heuristic solutions will come only after better analysis has been achieved.

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Psychiatry and International Affairs

Psychiatric approaches to analysis of international transactions will require professional innovation.

Bryant Wedge

Albert Einstein wrote to Sigmund Freud in 1932 that "It would be of the greatest service to us all were you to present the problem of world peace in the light of your most recent discoveries, for such a presentation might blaze the trail for new and fruitful modes of action." Freud answered that he saw " . . . no likelihood of our being able to suppress humanity's aggressive tendencies" (*1*). This reply failed to undermine Einstein's hope that psychiatry, the profession most practically concerned with disorder and conflict within and between human individuals, might help in the management of relations among nations. That hope is still with us but continues to be disappointed. Psychiatry has failed to provide practical assistance in the management of international conflict, though such conflict has become vastly more dangerous to mankind since the time of Einstein's appeal.

The idea that "wars begin in the minds of men" and that "it is in the minds of men that the defenses of peace must be constructed" is as old as the history of relations between organized societies. It has been restated most recently and authoritatively in the constitution of UNESCO. Why has the scientific profession most concerned with helping the individual with the troubles of his mind failed to contribute to solving the most significant problem in all human behavior? What could psychiatry contribute? How should the profession make the contribution to better management of international affairs which it is theoretically capable of making?

Albert Einstein was certainly correct when he observed that "Peace cannot be kept by force. It can only be achieved by understanding." A psychiatry of international affairs can, I think, contribute to such understanding.

Demands of and Responses by Psychiatry

Einstein was neither the first nor the last to call on psychiatry to contribute its services to the solution of problems in international affairs, nor was Freud unusual in his willingness to respond and to assert that the discipline has something useful to say on the subject. A number of substantial reviews have documented these claims (*2*).

Most psychiatric statements on international affairs can be classified as diagnostic, prescriptive, or inspirational; few have addressed themselves to problem-solving at the international level, either in theory or in practice.

When William Alanson White, for example, suggested that "War removes cultural repressions and allows the instincts to come to expression in full force," he was making a dynamic-diagnostic statement which is somewhat more sophisticated than the frequent statements describing national behavior as "collective psychosis" or nations as "paranoid" (*3*). When Harry Stack Sullivan, one of the few psychiatrists who attempted to cope with the realities of international life on an operational level, called for a "cultural revolution to end war," he spoke more thoughtfully than the recommendation that "universal love must be encouraged from childhood" (*4*).

Inspirational statements have come in waves: after World War I, in the years before and after World War II, and again in the early 1960's. The last wave was apparently released by the thaw in the Cold War and growing awareness of the risk of nuclear annihilation. In 1935, 339 psychiatrists from 30 nations signed a manifesto on war prevention, declaring that "we psychiatrists declare that our science is sufficiently advanced for us to distinguish between real, pretended, and unconscious motives, even in statesmen." In 1941 George H. Stevenson's presidential address to the American Psychiatric Association claimed that "we as psychiatrists are able to evaluate [psychopathological factors constantly determining toward war] more adequately than other groups" (*5*). Repeated calls to the psychiatric profession from

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