

background data from the reference regions observed around each of the 29 sources investigated that day. The raw data obtained from the record is the difference in temperature between the reference termination and the antenna terminal for each direction.

The data were first corrected for variations in antenna temperature, with elevation, due to both atmospheric absorption and antenna feed polarization changes. This latter effect is the result of the receiver being stationary as the antenna elevation changes. At the time of these measurements the effective temperature of the two principal polarizations of the antenna differed by about 1°K. This effect was due to higher horn loss (later corrected) and higher back lobe levels in the longitudinal polarization. The constraints of scheduling observations caused us to observe several high-declination sources at low elevations, and thus our results are not critically dependent on the parameter used in making these corrections.

When these corrected temperature differences were plotted as a function of time of observation, we found, as expected, a linear variation in their mean corresponding to a "warming" of the reference termination of 0.033°K/hr. The measured points were divided into four classes and plotted on a map of the sky (Fig. 1). Points above the mean line are indicated by plus signs and those below by minus signs. Deviations greater than 0.1°K are circled (5). In only one case (3C410) is the deviation greater than 0.2°K, and that source is near the galactic plane. Its location is marked by a filled circle. From a study of this plot we conclude there is no large-scale deviation from isotropy of more than 0.1°K.

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5. We feel that the possibility of losing a large-scale east-west slope in the process of subtracting the "warming" of the reference termination from the data in this manner is small, since our data cover almost the full range of right ascensions (20<sup>h</sup> to 16<sup>h</sup>). In addition the warm-up rate of the reference termination is in excellent agreement with that found on other occasions when different observations were made.

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## Rotation of the Sun

*Abstract. Dicke has interpreted his recent measurement of the sun's oblateness as implying a fast (1.8-day period) rotation of the solar radiative interior. We find that differentially rotating solar models, such as the one proposed by Dicke, are unstable. The rate of turbulent diffusion in the unstable regions of these models is so rapid that it appears to preclude a fast-spinning solar interior. As a corollary of the stability analysis, we conclude that the loss of a significant fraction of a star's angular momentum must be accompanied by the mixing of material below its convective zone. Such mixing inevitably leads to the depletion of lithium in the star's photosphere.*

Dicke (1) has recently measured a solar oblateness of  $5 \times 10^{-5}$  and has ascribed this observation to a rapidly rotating solar radiative interior (2). In any model of this kind, the angular momentum per unit mass must somewhere decrease with distance from the rotation axis. We have found the flow in a region of this type to be unstable to small perturbations. The physical nature of the instability and its implications are our main concern here.

For simplicity, we focus our attention on a cylindrically symmetric, differentially rotating, compressible fluid. A fixed gravitational field is assumed to act radially inward (toward the rotation axis). In the unperturbed state there is a balance between the gravitational, centrifugal, and pressure forces. We consider an axisymmetric perturbation of the basic state which consists of the interchange of fluid rings initially located at different radii. The displacement of the fluid rings is assumed to take place so slowly that they are always in pressure equilibrium with their surroundings. For the moment, we shall neglect the effects of thermal conductivity and viscosity. Therefore, there is no heat transfer between a displaced fluid element and its surroundings, and each fluid ring retains its initial angular momentum. If energy is released by the exchange of fluid rings, we would expect the motion to be unstable. The two types of energy that need to be considered are the kinetic energy of the fluid, due to its rotational motion, and its potential energy, due to its position in the gravitational field.

In regions where the angular momentum per unit mass decreases outward from the rotation axis, it is well known

that rotational kinetic energy is released by the interchange (3). Where the density field is stably stratified (as it is in the radiative interior of the sun) work must be done to supply the gravitational potential energy needed to interchange the rings. In the solar model proposed by Dicke, the work required to exchange fluid rings adiabatically is always greater than the rotational kinetic energy that can be released by their interchange. This has led Dicke to claim stability for his model (2). However, stellar perturbations are neither adiabatic nor inviscid. The smoothing of temperature fluctuations by radiative transfer will always act to diminish the temperature difference between the displaced rings and their surroundings. This will reduce the work required to overcome the stable stratification in the solar radiative interior. No matter how large a displaced fluid element may be, if it moves sufficiently slowly its temperature will approach that of its surroundings. Thus, finite thermal diffusivity in the solar interior may be expected to produce instability (in regions where the angular momentum per unit mass decreases outward) even though stability is predicted for adiabatic perturbations. We must bear in mind that the release of rotational kinetic energy during the exchange of fluid rings depends upon the retention of angular momentum by the displaced rings. Thus, viscous diffusion will act to destroy the source of energy that drives the instability. In the sun, thermal diffusion is much more rapid than viscous diffusion; the ratio of thermal diffusivity to kinematic viscosity in the solar interior is of order  $10^5$ . Therefore, the temperature of a displaced fluid ring will closely approach the temperature of its surroundings before angular momentum can diffuse either into or out of the ring. Hence, viscosity will not prevent instabilities from arising in regions where the angular momentum per unit mass decreases outward.

Our heuristic discussion of stability, which is based on energetic considerations, does not provide a rigorous proof of instability. Even if an exchange of fluid rings is energetically favorable, the fluid may not possess modes that can release the energy. In other words, the motions that were considered in the heuristic discussion may not be compatible with the fluid equations. For this reason, we have verified the heuristic stability criterion by means of a complete linear perturbation analysis (4).

We have seen that differentially rotating solar models are unstable unless their angular momentum is an increasing function of distance from the rotation axis. This latter condition is violated by all solar models whose interiors rotate sufficiently rapidly to account for Dicke's oblateness measurement. The turbulent motion that would ensue in these unstable models would be characterized by very elongated eddies. Their short dimension (approximately 1 km) would allow rapid exchange of heat with the surroundings, whereas their long dimensions would approach, if not exceed, the local scale height. The circulation time for these energy-containing eddies is on the order of a rotation period. Such eddies would produce very rapid diffusion of angular momentum and complete mixing of the sun would take place in about 10 years.

The sun is subject to a torque by the solar wind (5, 6). This torque will produce differential rotation and therefore turbulence below the solar convective zone. The mixing of material brought about by turbulence will lead to the depletion of lithium and beryllium which are destroyed at temperatures slightly in excess of those at the bottom of the convective layer. From the presence of beryllium (and possibly lithium) in the solar photosphere, we may infer that the sun's angular momentum has not decreased by or-

ders of magnitude since it arrived on the main sequence (unless lithium and beryllium have been formed since then).

The rotational instability that we have described is analogous to a thermohaline instability that was first discovered by Stern (7). This instability arises when a layer of warm salty water lies above cold fresh water. Even if the density increases downward the system can still be unstable because the thermal diffusivity is much greater than the diffusivity of salt. A small rising blob of fluid can lose most of its temperature deficiency, while retaining its initial salinity, and consequently be acted upon by a buoyant force. In this context, salt plays the role that angular momentum did in the rotational instability we have been discussing.

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## Limited Heterogeneity of Gamma Globulin in Hypogammaglobulinemia

*Abstract.* When serums of 11 patients with hypogammaglobulinemia were examined by acrylamide-gel electrophoresis, combined with diffusion analysis, for the gamma chain of immunoglobulin G, the electrophoretic distribution of immunoglobulin G differed from that of normal individuals. The differences consisted of limited heterogeneity and bimodal distributions of the proteins at times. Isolated immunoglobulin G from four of the patients showed similar phenomena. The findings indicate a deficient gene population in these patients.

We investigated the possibility that detectable structural abnormalities of the immunoglobulins might be found in diseases of immunological deficiency. Studies of the electrophoretic characteristics of immunoglobulin G (IgG) in serums of 11 hypogammaglobulinemic patients show that the mobility of the IgG of each of these subjects differs from that of IgG either in a human serum pool or in individual serum samples from a group of normal persons.

Electrophoretic behavior was assessed by diffusion analyses of serums after electrophoresis in acrylamide (1) and by immunoelectrophoretic and acrylamide analyses of isolated IgG. Serum samples containing approximately 150  $\mu$ g of IgG (0.05 to 0.15 ml total) were applied to 7.5-percent acrylamide gels. At the end of electrophoresis (pH 9.5), the gels were sliced longitudinally in half and placed on the surfaces of agar-coated microscope slides. Troughs were cut parallel to the

gel and filled with antiserum specific for gamma chain.

The analyses (Fig. 1) showed that limited heterogeneity was a constant feature in the patients. Bimodal peaks (two arcs) were common. The electrophoretic mobility of most of the IgG's fractionated from samples of whole serum tended to be slow, but peaks of intermediate and fast mobility were also seen. When we increased the smaller volume of control serums (those having normal amounts of  $\gamma$ -globulin) by adding serum albumin (70 mg/ml) or normal saline to make it the same as that of the patients, the diffusion pattern of the normal serums was unchanged. All samples tested were obtained from patients before administration of  $\gamma$ -globulin therapy, except for samples from one patient who had not received any  $\gamma$ -globulin for 1 year before the study. We examined the IgG patterns of two sets of parents of our patients and found them normal. The diffusion patterns of patients with hypogammaglobulinemia of secondary etiology (hypoproteinemia) also do not differ from normal when equivalent amounts of  $\gamma$ -globulin are applied to the gel. Conversely, two patients with hypergammaglobulinemia (chronic infection) also showed normal patterns under these conditions (that is, when serum containing 150  $\mu$ g of IgG was applied to the gel). Similar distributions were also seen when we used antiserum specific for  $\kappa$ - and  $\lambda$ -chains; hence, the restrictions are apparently unrelated to the composition of the light chain.

Immunoglobulin G was isolated from the serums of four male patients; the amounts varied from 85 to 172 mg per 100 ml of serum. None of these samples showed a bimodal distribution; the serum pattern is represented by the lowermost portion of Fig. 1. Fractionation was accomplished by diethylaminoethyl (DEAE)-Sephadex chromatography at pH 6.3 with 0.0175M phosphate buffer (2). When the isolated IgG's were analyzed by immunoelectrophoresis, the samples from the patients tended to have slower mobility and less heterogeneity than the IgG from normal persons (Fig. 2). Analysis in acrylamide gel confirmed the differences between the IgG's of the patients and  $\gamma$ -globulins of normals. After electrophoresis in 7.5-percent neutral gel of equal quantities of proteins from the IgG pools, there was a restricted heterogeneity and slower mobility of the  $\gamma$ -globulin from the