Reports

Stratigraphic Sections, Bedding Sequences, and Random Processes

Abstract. *Both dependent and independent random processes can be used to study and synthesize stratigraphic sections and bedding sequences. Synthetic stratigraphic sections can be modeled to fit any geologic problem with a good correspondence between the synthetic and actual stratigraphic sections. To synthesize a sedimentary sequence, one needs only a transition procedure to go from one lithology or bedding type to another, and frequency distributions of thickness for the various lithologies. A sedimentation process with a "memory" is termed Markovian; if the past has no influence on either present or future sedimentation, it is called an independent trials process.*

It is probably fair to say that the first major progress in understanding sedimentary rocks occurred only after geologists began to measure and describe stratigraphic sections in the field in the middle and late 18th century. Today, the measurement, use, and interpretation of stratigraphic sections and bedding sequences remain fundamental to the vast majority of modern and ancient sedimentary and stratigraphic studies. The use of random processes to examine transitions of stratigraphic sequences offers the possibility of looking at sedimentation processes in a different way.

Published literature on sedimentary applications of random processes is scant. In 1949, Vistelius (*1*) counted transitions from one bedding type to another in a flysch sequence in the Caucasus, and published a transition matrix. Later, Allègre (*2*) and Pettijohn *et al.* (*3*) outlined some elements of random processes and their relation to stratigraphic and sedimentary studies. Recently, Vistelius and Faas (*4, 5*) used random processes to study a flysch sequence in the southern Urals. About the same time, Vistelius and Feygel'son (*6*) used a Markov model to analyze the transition of strata resulting from turbidity current deposition. A review which illustrates the use of Markov processes in the geosciences is presented by Griffiths (*7*). An important step forward has been made by Harbaugh (*8*) in simulating facies maps with a computer.

An investigation was undertaken to discover the use of random processes in synthesizing a sedimentary sequence. Random processes are appropriate for such synthesis because (i) the depositional process commonly involves many events for which it is difficult to write exact functional relationships; (ii) there usually is a large amount of interdependence among the variables of a sedimentary system; and (iii) nearly every sedimentary sequence can be considered a product of some type of sedimentary cycle based on transitions from one lithology or bedding type to another.

Random processes can be used to synthesize stratigraphic sections and bedding types, and thus model actual measured sections. Good correspondence between synthetic and actual stratigraphic sections—in terms of length of stratigraphic section, kinds of lithologies, thickness of lithologies, and proportions of bedding or lithologic types—suggests that the factors used in the model closely approximate those in nature. With synthetic sections, one can construct such varied sedimentary deposits as a carbonate bank, a fluvial sand body, or even an entire sedimentary basin by varying the different factors in the model. Behind such an effort is the idea that improved prediction will result, either vertically or laterally within the stratigraphic section, if the depositional process is better understood.

Any sedimentary section can be synthesized by random processes provided (i) there is a transition procedure to go from one lithology or bedding type to another, and (ii) the thickness distributions of the different lithologies or bedding types are known.

The transition procedure may be an independent process (a series of independent trials) wherein past deposition has no influence on either present or future deposition, or it may be a dependent process (in particular, a Markov process) with a one-step "memory" wherein present deposition is influenced by the past. Dependence may extend backward one or more steps in the depositional process. For example, the probability of deposition of a coal bed may depend only on the presence or absence of an immediately underlying underclay, or dependence may extend two steps backward to the lithology that preceded the underclay. In this investigation the concern was with one step, Markov dependence. Whether a particular sedimentary sequence is generated by an independent trial or by a Markov process is essentially determined by how much memory is in the process.

The memory of a sedimentary process can be linked to the lateral distribution of environments in a basin, particularly for regressive and transgressive sequences. Suppose that a series of marine depositional environments is related to water depth which deepens off-shore in a systematic manner, and that in each environment a distinctive sediment is deposited. Ideally, a slow transgression will produce a stratigraphic sequence wherein all, or nearly all, the environments are represented. (Conversely, if the environments have no systematic lateral relationships to one another, a slow transgression will produce a stratigraphic section with only a weak or nonexistent memory.) Thus, in a vertical sequence, memory may be useful in assessing the extent to which one can predict the lateral position of a lithology (compare *9*).

Memory is specified by a transition matrix. For example, for a three-state system of sandstone (S_1), shale (S_2), and limestone (S_3) the transition matrix for all the elements p_{ij} is:

$$\begin{array}{cc} & \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array} & \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \end{array}$$

where p_{11} is the probability of a sandstone following a sandstone, p_{12} is the probability of a shale following a sandstone, p_{23} is the probability of a limestone following a shale, p_{32} is the probability of a shale following a limestone, etc. Each row of this matrix sums to one, because something must follow each lithology. The transition procedure is based on counting the lithologies that succeed each other as deposition proceeds.

For example, in a Pennsylvania cyclothem the probability of a coal bed, a gray shale, or a marine limestone following an underclay differs strikingly, and could be estmaited by counting actual transitions. Similarly, one could determine the probability with which the different bedding types of the turbidite cycle succeed one another and thus determine an empirical transition matrix for a turbidite sequence.

If all the elements p_{ij} of some power of the transition matrix are positive, one can obtain the long-term equilibrium proportions of the different lithologies from the transition matrix by either iteration or by the solving of a set of simultaneous equations (*10*). Usually matrix iteration of 64 or 128 times is sufficient.

Synthetic stratigraphic sections can be altered by changing their length (number of beds deposited), by modifying the transition matrix so that the probability of going from a particular state i to j is different, or by changing the distributions of bed thickness.

To better understand dependent processes, a stratigraphic section was synthesized with the Mississippian Chester Series of the Illinois Basin as a model. The transition matrix was:

$$\begin{bmatrix} 0.1 & 0.8 & 0.1 \\ 0.4 & 0.2 & 0.4 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$$

where the p_{ij}'s are defined as they were previously. These particular p_{ij}'s seemed especially reasonable for the cyclic deposition of the Chester Series because iteration of 128 times yielded the following equilibrium proportions: .25, sandstone; .50, shale; and .25, limestone, which closely approximate those of the Chester Series.

We specified the thickness of each of these three different states by determining the thickness distributions for sandstone, limestone, and shale from electric logs in Posey County, Indiana. We used thickness (which is tangible, and more easily measured than time, in stratigraphic sections) to specify the duration that the depositional process remained in a given state. The distributions from Posey County are strongly negatively skewed, because sedimentation processes almost always produce more thin beds than thick beds. Thickness distributions of sedimentary beds generally are approximated by log normal, negative exponential, or chi-square distributions.

Generating a stratigraphic section, given a transition matrix and either empirically determined or assumed frequency distributions of the different thickness units, is relatively simple with the following procedure. (i) Choose randomly an initial state, i, which then specifies a particular row of the transition matrix. (ii) Choose randomly the thickness of the ith lithology from its frequency distribution. (iii) Select the following lithology, j, in accordance with the probabilities of the p_{ij}'s of the ith row, all of which sum to one. (iv) Select randomly a thickness from the frequency distribution of the jth lithology. (v) Repeat the above process starting with a new ith row of the transition matrix, where i is determined by the preceding jth lithology obtained in step (iii).

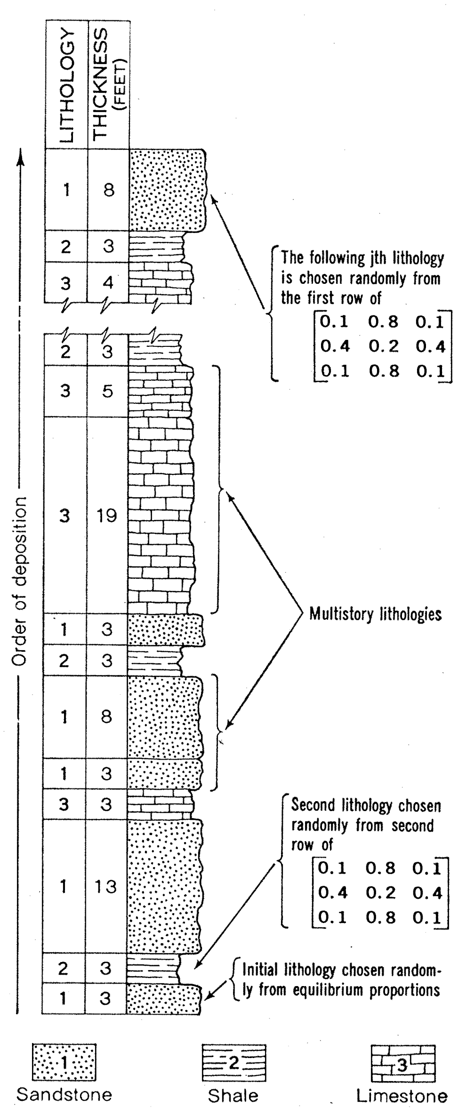

Fig. 1. Synthesized stratigraphic section.

A table of random numbers was used to select, for a given lithology, a thickness from the frequency distributions. For a given i (shale, for example) the jth lithology was selected by randomly choosing, with replacement, one of ten balls which were of three different colors. The proportions of the three colors corresponded to the probabilities of the p_{ij}'s of a particular row. Because $i = 3$, three sets of ten colored balls were necessary. Computer techniques are readily available for this purpose and would be used for a research problem. However, for class demonstration, the above method is excellent because it so graphically illustrates random processes.

Two hundred lithologic units representing 3093 feet of section were generated. Figure 1 shows a portion of the section which resulted. The empirical transition matrix was determined to be:

$$\begin{bmatrix} 0.09 & 0.79 & 0.12 \\ 0.44 & 0.26 & 0.31 \\ 0.16 & 0.68 & 0.16 \end{bmatrix}$$

This matrix is close to the generating one and upon iteration (128 times) yielded the equilibrium proportions: .278, sandstone; .499, shale; and .222, limestone.

Stratigraphic sections can also be generated by independent trials processes choosing lithologies randomly from the equilibrium proportions. With an independent trials process a lithology is unaffected by what preceded it, so $p_{ij} = p_j$. Hence, if the sedimentary process remains at equilibrium, and if sufficient numbers of observations are made, the rows of the empirical transition matrix should all be approximately the same; the p_{ij}'s of each row will approximate the equilibrium proportions of the different states.

One can determine a transition matrix for a section without assuming a mathematical model of any kind. The resulting matrix yields an empirical description of how the different lithologies succeed one another. If one can distinguish multistory lithologies (for example, a sandstone following a sandstone), a test can be made to determine whether the sequence was an independent trials process or whether it had a memory of one or more steps (*11*). However, if one cannot distinguish multistory lithologies, the empirically determined transition matrix will have zeros down its main diagonal, as follows:

$$\begin{bmatrix} 0 & p_{12} & p_{13} \\ p_{21} & 0 & p_{23} \\ p_{31} & p_{32} & 0 \end{bmatrix}$$

The zeros in the above matrix could arise from a random process in which all repetitions were lumped together, simply because multistory lithologies could not be distinguished.

Empirically determined transition matrices, because they summarize all the probabilities of going from one lithology to another, establish actual sedimentary cycles. However, before modeling can become fully effective, geologists must obtain data for empirical transition matrices such as that obtained by Vistelius (*1*) and implied by de Raaf *et al.* (*12*). Such matrices will permit us to determine, by statistical test, whether or not the depositional processes that produce stratigraphic sections have a memory.

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14. We thank K. Alam, R. F. Blakely, C. R. Blyth, and J. L. Doob for their assistance.

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Magnetic Anomalies over the Pacific-Antarctic Ridge

Abstract. *Four magnetic profiles across the Pacific-Antarctic Ridge reveal magnetic anomalies that show trends parallel with the ridge axis and symmetry about the ridge axis. The distribution of bodies that could cause these anomalies supports the Vine and Matthews hypothesis for the generation of patterns of magnetic anomalies associated with the midocean ridge system. The geometry of the bodies accords with the known reversals of the geomagnetic field during the last 3.4 million years, indicating a spreading rate of the ocean floor of 4.5 centimeters per year. If one assumes that the spreading rate within 500 kilometers of the ridge axis has been constant, reversals of the geomagnetic field during the last 10.0 million years can be determined. This new, detailed history of field reversals accords with observed anomalies over Reykjanes Ridge in the North Atlantic if a spreading rate of 1 centimeter per year is assumed there.*

Many have recognized the pattern of magnetic anomalies associated with midocean ridges: Heezen *et al.* (*1*) first described the anomaly associated with the Mid-Atlantic Ridge; Ewing *et al.* (*2*) and Heezen *et al.* (*3*) showed the association of this anomaly with the axial region or rift valley; Vine and Matthews (*4*) and Heirtzler and Le Pichon (*5*) have shown generalized linearity of the pattern along the ridge parallel with the axis, and some degree of symmetry across the axis of the ridge. More recently Heirtzler *et al.* (*6*), using data from an aeromagnetic survey of Reykjanes Ridge, have demonstrated more clearly the linearity and symmetry of the anomaly patterns.

To explain the magnetic pattern of the ridges, Vine and Matthews (*4*) hypothesized that material wells up from the ridge axis, becomes magnetized in the direction of the geomagnetic field at the time of cooling below its Curie temperature, and spreads laterally from the axis. Thus the magnetization of the material is parallel or antiparallel to the present direction of the geomagnetic field, depending upon its distance from the ridge axis and the spreading rate. Vine and Wilson (*7*), using the magnetic-reversal history of Cox *et al.* (*8*), have applied the hypothesis to the Juan de Fuca Ridge.

During 1965 U.S.S. *Eltanin* made systematic traverses of the Pacific-Antarctic Ocean between New Zealand and Chile. We now report magnetic measurements from four important traverses of the Pacific-Antarctic Ridge between 40° and 55°S (Fig. 1); the data were acquired with a proton-precession magnetometer system. In Fig. 2 the magnetic-anomaly and bathymetry profiles are projected along an azimuth normal to the axis of the ridge.

The two most striking features of the magnetic profiles are the linearity of the pattern from profile to profile, and the symmetry of the anomaly pattern about the axis of the ridge. To illustrate the linearity, several of the anomalies are assigned numbers (Fig. 2). The axial anomaly 1 to 1′, with its adjacent small anomalies, is prominent in all the profiles and strikingly resembles the axial patterns of the Juan de Fuca Ridge and the Pacific Antarctic [the latter from the *Vema* 16 crossing; see Vine and Wilson (*7*)].

On the eastern ends of the *Eltanin*-21 and northernmost *Eltanin*-20 profiles, anomalies 3′ and 4′ are difficult to identify in a pattern that is quite persistent elsewhere. This distortion may reflect the proximity of the Chile Rise, which can be seen joining the ridge in the northeast corner of Fig. 1.

Lines connect the numbered features

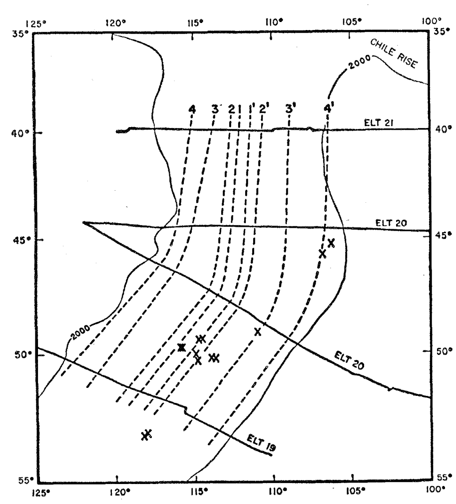

Fig. 1. Ship's tracks in the Pacific-Antarctic Ocean; bathymetric contours in fathoms. The numbers refer to specific magnetic anomalies, and the dashed lines connect similar anomalies. The crosses indicate positions of earthquake epicenters south of 40°S between 1 June 1954 and 31 May 1965 (*18*). *Elt, Eltanin.*