Direct Test of the Power Function for Loudness

Abstract. *The loudness of a sound was made to grow as a function of time along a variety of paths. Observers judged the degree of apparent deviation from linearity of the various paths of loudness growth. The growth of loudness was judged to be least nonlinear when the sound pressure was made to increase as the 1.67 power of time. It follows, therefore, that loudness must be related to sound pressure by a power function with exponent 0.6 (reciprocal of 1.67). This outcome shows that the exponent of the psychophysical power function may be verified by a procedure that does not require the assumption that numerical estimates are proportional to subjective magnitudes.*

A large number of experiments have demonstrated that sensory magnitude (Ψ) is related to stimulus intensity (ϕ) by a power function

$$\Psi = k\phi^{\beta} \quad (1)$$

where k is a unit constant. The work of S. S. Stevens and others (*1*) has shown that the power function describes the input-output relation for most if not all sensory systems. Each sensory modality has its characteristic exponent β, although parameters such as adaptation may alter the value of an exponent. The power function also seems to describe the data for individual observers, as well as group averages (*2*).

Some of the procedures used to scale sensory magnitudes, in particular those procedures that require observers to assess sensory magnitudes numerically, have met with objections (*3*). As a result, other procedures have been proposed and tested. A notable success was the direct cross-modality matching of one sensory continuum to another (*4*).

The aim of this paper is to test the psychophysical power function by means of a rather different procedure, one that allows observers to compare various time-dependent growth functions. The procedure was explored earlier by Lee McMahon, but it becomes a more practicable method with the generation of signals by a computer. An observer can be presented stimuli whose intensity increases as some function of time. It is hypothesized that there will be one intensity-time function that gives rise to a sensation whose magnitude appears to increase linearly with time, and that that function will be the inverse of the psychophysical function. In other words, it should be possible to discover the intensity-time function that just cancels out the nonlinear transformation produced by a given sensory system.

If stimulus intensity grows as a power function of time (t),

$$\phi = a(t+b)^{\alpha} \quad (2)$$

Then, if Eq. 1 represents the psychophysical function,

$$\Psi = k[a(t+b)^{\alpha}]^{\beta} \quad (3)$$

If $\beta = 1/\alpha$

$$\Psi = ka^{\beta}(t+b) \quad (4)$$

Thus if the so-called psychophysical law is a power function, then sensory magnitude will grow linearly with time provided the stimulus intensity increases with time according to a power function whose exponent is the reciprocal of that of the psychophysical function.

The earliest function proposed as the psychophysical law was Fechner's logarithmic function

$$\Psi = c\ln\phi \quad (5)$$

If that is the correct function, then it can be shown that sensory magnitude will increase linearly with time, provided the stimulus intensity grows as an exponential function of time

$$\phi = ae^{bt} \quad (6)$$

In an experiment designed to test the foregoing hypotheses, 21 observers listened to buzzes of 160 pulses per second whose intensity increased as various functions of time. The observers judged the degree of nonlinearity of the increase of loudness with time. For seven of the stimuli the sound pressure increased as a power function of time according to the equation $\phi_i = a(t+b)^{\alpha}$, with α ranging from 0.5 to 5.0 (see Table 1); for the eighth stimulus the sound pressure increased as an exponential function of time according to the equation $\phi_8 = 0.0898e^{1.84t}$, where ϕ is in dynes per square centimeter. The functions were computed so that all of the stimuli began at the same sound pressure level (53 db) and all ended at 93 db sound pressure level 2.5 seconds later. It was assumed that 1 volt across a pair of PDR-8 earphones connected in series gave 100 db sound pressure level.

The stimuli, generated digitally by a PDP-4 computer, were converted to analog signals by a digital-to-analog converter that was an integral part of the display device. The analog signals were fed to the PDR-8 earphones.

Table 1. Constants of the equation $\phi_i = a(t+b)^{\alpha}$, where ϕ is in dynes per square centimeter. The constants a and b were calculated so that the stimulus values ϕ would be the same at the start ($t = 0$) and at the end ($t = 2.5$ seconds) of each temporal growth function. Between the two limits the form of the growth functions varied.

i	a	b	α
1	5.68	2.5×10^{-4}	0.50
2	4.52	5.4×10^{-3}	.75
3	3.19	4.0×10^{-2}	1.11
4	1.72	1.7×10^{-1}	1.67
5	5.91×10^{-1}	4.7×10^{-1}	2.50
6	1.61×10^{-1}	8.4×10^{-1}	3.33
7	7.23×10^{-3}	1.66	5.00

The observer was asked to assign numbers proportional to the apparent nonlinearity of the growth in loudness of the stimuli. Through written instructions, each observer was told to assign a number to represent the apparent nonlinearity of the growth in loudness of the first stimulus; then, for each succeeding stimulus, he was to assign a number whose proportion to the first number reflected the ratio of the apparent nonlinearities. Just prior to the experiment, the observer was given a brief training session in which he judged the nonlinearity of various curved lines.

Each sound stimulus lasted 2.5 seconds and a half-second pause separated repetitions. The observer could listen to each stimulus as many consecutive times as he wished. After four practice stimuli, the eight experimental stimuli (intensity-time growth functions) were presented twice, in an irregular order, and after each stimulus the observer announced his numerical estimation of the apparent nonlinearity.

The geometric means of the 42 numerical estimates made for each stimulus are plotted as circles in Fig. 1. The exponential function is plotted as a power function with an exponent of

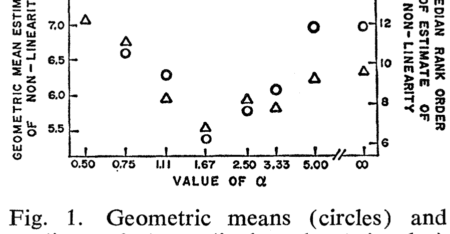

Fig. 1. Geometric means (circles) and medians of the ordinal ranks (triangles) obtained when 21 observers assigned numbers to represent the apparent nonlinearity of the increase of loudness with time. The abscissa gives the exponent of the power function that describes the increase in sound pressure with time. The exponential function is plotted as a power function with an exponent of infinity.

infinity, since the limit approached by a power function as the exponent approaches infinity is an exponential when the function is plotted between fixed limits. Although the differences among the means were not large, the judgment of nonlinearity was clearly minimal for the intensity-time function with an exponent of 1.67. This outcome suggests that the psychophysical function for loudness is a power function with an exponent of 0.6, the reciprocal of 1.67. As shown in Fig. 1, the inverse of the logarithmic function—the exponential function—was judged to be among the most nonlinear of the functions examined.

On the basis of a weaker assumption, namely, that the observers' estimates merely reflect rank-order (judgments of "greater" or "less"), the median ordinal rank assigned to each stimulus was calculated and plotted as a triangle in Fig. 1. The numerical estimates by each observer were ranked, and the median rank was taken for each stimulus. Since the eight stimuli were presented twice to each observer, the rank-orderings went from "1" (least nonlinear) to "16" (most nonlinear). The rank-order measures accord with the geometric means in showing that the power function with an exponent of 1.67 resulted in the most linear increase of loudness with time. For various observers, the estimates of apparent nonlinearity were minimal for different exponents of the intensity-time growth functions. The minimum was at 1.11 for three observers, at 1.67 for seven observers, at 2.50 for five observers, and at two other values for three observers. The minimum was indeterminate for three observers. Thus the psychophysical functions for 15 of the 21 observers were power functions with exponents between 0.4 and 0.9 (the reciprocals of 2.50 and 1.11).

Although the foregoing results do not determine a precise value for the loudness exponent, they are consistent with the value 0.6 which has been recommended for engineering purposes by the International Standards Organization. The results also accord with those of experiments in which observers assigned numbers to represent loudness rather than the growth of loudness (*5*).

LAWRENCE E. MARKS *
Center for Cognitive Studies, Harvard University, Cambridge, Massachusetts
A. WAYNE SLAWSON †
Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge

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6. Supported by grant MH-08083-03 from the National Institute of Mental Health to Harvard University, Center for Cognitive Studies. We thank Dr. Donald Norman and Janice March for making the facilities of the PDP-4 computer available.
* Present address: John B. Pierce Foundation Laboratory, and Department of Psychology, Yale University, New Haven, Conn.
† Present address: Speech Transmission Laboratory, Royal Institute of Technology, Stockholm, Sweden.

30 September 1966

Inactive Alkaline Phosphatase in Duodenum of Nursling Mouse: Immunological Evidence

Abstract. *A preparation of duodenal alkaline phosphatase from 11-day-old mice contains an enzymatically inactive material that cross reacts with antiserum prepared against one isozyme of 20-day alkaline phosphatase. After precipitation of this material from solution, the antiserum cannot precipitate the 20-day enzyme.*

Differentiation is often characterized at the chemical level by the appearance of or increase in enzyme activity that is proper to a type of cell or tissue under investigation. Although such an increase in activity has, in a few cases, been shown to be due to *de novo* synthesis (*1*), an alternative mechanism is the activation of a previously existing inactive form of the enzyme. In adult animals, enzymes such as phosphorylase (*2*) and tryptophan pyrrolase (*3*), which are subject to sudden metabolic stimulation, may be present in an inactive form. In developmental stages there is little evidence for similar phenomena, although grasshopper eggs are known to contain a protyrosinase that can be activated by heat (*4*) and fetal rat liver has an inactive *p*-hydroxyphenylpyruvate oxidase that may be activated by glutathione (*5*). However, no enzyme has been shown to be preceded in development by an immunologically similar precursor.

During postnatal development, the alkaline phosphatase of mouse duodenum exists in several forms that are characterized by differences in electrophoretic mobility, elution pattern from DEAE-cellulose (diethylaminoethyl) columns, and ratio of activity with phenylphosphate to that with β-glycerophosphate (*6*). A form of the enzyme with ratio of phenylphosphate to β-glycerophosphate of about 0.7 is found up to about 12 days of age. It then begins to be replaced by two new isozymes, one of which has a ratio above 6.0 and the other has a ratio of about 2.0 (*6*). The new isozymes reach their peak at 20 days. Since the administration of inhibitors of RNA and protein synthesis to young mice accelerate the appearance of the high-ratio activity, it has been suggested that the new forms are the result of conversion of precursor molecules of different catalytic properties (*7*). We have now investigated this possibility by an immunological method.

An antiserum was produced against high-ratio phosphatase (HR) that was prepared by passing a butanol extract of 20-day duodenum through a column of DEAE-cellulose (*6*). Such an antiserum can precipitate the greater part of added HR from solution (Table 1, lines 1–3). However, when the same amount of antiserum was mixed with an 11-day column eluate of low-ratio phosphatase (LR), a precipitate was observed. However, all but a negligible amount of the phosphatase activity remained in the supernatant (Table 1, lines 4–6).

All enzyme and antiserum preparations were centrifuged before combination in order to eliminate any possibility of nonspecific precipitation. No precipitate formed in the combination of LR with normal serum from the same rabbit. Therefore, the precipitation in the second case must have been caused by a serological reaction of something other than LR with the antiserum.

There are two possibilities: (i) the enzyme preparation contained a contaminating molecule that reacted with another antibody in the antiserum; or, (ii) the preparation contained an enzy-