4. E. A. Johnson, T. Murphy, O. W. Torreson, *ibid.* **53**, 349 (1948).
5. M. J. Keen, *Nature* **187**, 220 (1960); *Deep-Sea Res.* **10**, 607 (1963).
6. C. G. A. Harrison and B. M. Funnell, *Nature* **204**, 566 (1964).
7. M. D. Fuller, C. G. A. Harrison, Y. R. Nayudu, *Amer. Assoc. Petrol. Geologists Bull.* **50**, 566 (1966).
8. F. Rimbert, *Rev. Inst. Franç. Petrole Ann. Combust. Liquides* **14**, 1 (1959); J. A. As and J. D. A. Zijderveld, *Geophys. J.* **1**, 308 (1958).
9. A. Cox, R. R. Doell, G. B. Dalrymple, *Science* **144**, 1537 (1964).
10. R. R. Doell and G. B. Dalrymple, *ibid.*, **152**, 1060 (1966).
11. K. M. Creer, *J. Geophys. Res.* **67**, 3461 (1962).
12. E. Bullard, *Proc. Cambridge Phil. Soc.* **51**, 744 (1955); T. Rikitake, *ibid.* **54**, 89 (1958); D. Allan, *Nature* **182**, 469 (1958).
13. C. G. A. Harrison and M. N. A. Peterson, *Amer. Mineralogist* **50**, 704 (1965).
14. G. Dickson, personal communication.
15. J. D. Hays, *Amer. Geophys. Union Antarctic Res. Ser.* **5**, 125 (1965).
16. ———, in *Progress in Oceanography* (Pergamon, London, in press).
17. D. B. Ericson, M. Ewing, G. Wollin, *Science* **139**, 727 (1963).
18. ———, *ibid.* **146**, 723 (1964).
19. W. R. Riedel, F. L. Parker, M. N. Bramlette, *ibid.* **140**, 1238 (1963).
20. J. R. Connolly and M. Ewing, *ibid.* **150**, 1822 (1965).
21. R. J. Uffen, *Nature* **198**, 143 (1963); in *Proc. Intern. Geol. Congr. 22nd New Delhi* (1964).
22. J. F. Simpson, *Geol. Soc. Amer. Bull.* **77**, 197 (1966).
23. N. D. Newell, personal communication.
24. Work supported by ONR contract Nonr 266(48) and NSF grants GP 4004 and GA-178. We thank the staff of Lamont Geological Observatory who contributed to the success of this investigation. In particular we thank James Heirtzler, Maurice Ewing, Wallace Broecker, and Allen Be for suggestions; N. D. Newell and Roger Batten for reading the manuscript and for comments; and Drs. Larson and Strangway of M.I.T. for allowing G. Dickson of the Observatory to make curie-point determinations on their equipment. Lamont Geological Observatory contribution 956.

What Are Mathematicians Doing?

Their search for abstractions leads to applications.

Bernard Friedman

The past 20 years have seen a great increase in the number of mathematicians and a noticeable flourishing in all branches of mathematics. Many new techniques have been devised, and many important unsolved problems have been settled. Mathematical techniques and ideas have permeated not only the natural sciences but also the social sciences, and even the management of large business organizations. Because of the belief that mathematics is vitally important for our technology and for the development of our society, the government has supported an increase in mathematics research and has sponsored attempts to improve mathematics education.

The past few years, however, have seen a growing disillusion, not so much with mathematics as with mathematicians. The signs are small but significant. A few years ago the employment ads in the New York Sunday *Times* contained many requests for mathematicians. Today they ask for programmers or for systems analysts instead of mathematicians. At many universities the science and engineering departments insist that their students are not being taught the mathematics they need, and they claim that the solution is to have separate mathematics departments in the engineering colleges. Similar criticism comes from my colleagues in physics, who tell me that when they have a differential equation to solve, or a contour integral to evaluate, they know it is useless to go to the mathematics department for help. R. W. Hamming says in *Science* (*1*) "much of modern mathematics is not related to science but rather appears to be more closely related to the famous scholastic arguing of the Middle Ages."

The continual complaint of these people reduces to this: "where is mathematics going and what are mathematicians doing?" This query comes not only from mathematically unsophisticated laymen but also from mathematically sophisticated and technically trained people. I recall a question once asked me by a well-known physicist. In the course of a discussion about mathematics he expostulated, "I have studied and used mathematics all my life. I have known some of the greatest mathematicians, such as John von Neumann and Hermann Weyl. Yet when I try to understand, just in a general way, not in detail, what some of your young colleagues are doing, I find myself baffled. I'm not a mathematical ignoramus. Can you explain to me what they are doing and, more important, how relevant it is to my work in physics?" I had to confess that to explain what my colleagues were doing would take at least a week of preparatory lecturing, that their work is not directed toward physics, and that it probably has in fact no application other than in mathematics itself. To this he said with disgust, "Well if mathematics is now just another discipline like Sanskrit, then why should the university and the nation support mathematics on the scale they have been doing?"

The question is a serious one, and it has disturbed me for a long time. I believe that both the university and the nation are right in supporting and expanding mathematics research and training, but it is not easy to give objective evidence to support my beliefs. Instead, I shall discuss a slightly simpler question: What are mathematicians doing and where are they going?

Struggle for Generality

What are mathematicians doing? They are doing mathematics, and at an ever-increasing rate. Where are they going? They are going in the direction of a more elegant, a more unified, and, necessarily, a more abstract approach to the study of mathematical structures. The important idea here is the emphasis on mathematical structures. We—and by "we" I mean most people in the mathematics profession—are no longer interested in particular problems and their detailed solutions. Instead, we are interested in the method by which the solution was obtained. Thus, a proof of Goldbach's conjecture that every even integer is the sum of two prime numbers would

The author is professor of mathematics, University of California, Berkeley. This article is adapted from his vice-presidential address to Section A of the AAAS, presented 27 December 1965 at the Berkeley meeting.

be of only ephemeral importance, but the method used in the proof would have important consequences. However, even more important would be the discovery of the proper setting for this method—a setting which would provide the proof of Goldbach's conjecture as a special case of a more general result. This proper setting would probably arise in the consideration of a mathematical structure more general than the structure of integers. To investigate such a structure one would have to learn many new concepts and study their properties (a study which in many cases seems tedious and even trivial) and finally develop a whole theory based on these concepts, until, at the end, the difficult problem had become a trivial corollary to a more general result.

This desire for greater generality, and the consequent need for more and more general structures, is the spur for much present-day research in mathematics. In this struggle for generality, concepts are almost daily revised and refined in an effort to attain a formulation which will be as simple and as useful as possible. The revision of old concepts and the introduction of new ones go on at such a rate that even professional mathematicians have a hard job keeping up, not with the substance, but merely with the terminology of mathematics. The scientist or engineer faces a harder problem. He does not have the time to keep up with the changes in mathematical terminology and concepts, but he still needs mathematics as an aid in his work. He has two choices, which may be illustrated by the following analogy.

Suppose you want to read a Russian paper, know nothing about the language, and can get no outside help. The most direct way of translating the paper would be to get a good dictionary and to translate word for word. Another way, less direct but more useful in the long run, would be to get a textbook and learn the alphabet, the vocabulary, and the grammar. This would certainly take much longer, but in the end you could read this paper and any other Russian paper at sight. The scientist confronted by a mathematics problem faces the same dilemma. Either he can use a dictionary—or, in this case, a book of techniques—and hope they will be adequate for solving his problem or he can try to learn the mathematical language underlying it.

Recent Educational Developments

The recent developments in mathematics education at the elementary and high school levels have been attempts to teach the children the language, not merely the methods of using the dictionary. We have tried to show the children that the fundamentals of arithmetic and geometry fit into a larger pattern and that the techniques of computation are simply necessary consequences of the axioms. Of course the attempt has not been completely successful. Naturally, parents and scientists have complained that the emphasis on the "why" of mathematics has caused the children to neglect the "how," and that, as a result, the children's ability to compute is poor. Undoubtedly there is truth in this complaint. However, steps are being taken to remedy this weakness in the mathematics program.

This neglect of computational skills could have been foreseen, because mathematicians are not interested in computation. There is a little story about von Neumann which reveals this fact very clearly. The story goes that a physicist had to evaluate a complicated sum in statistical mechanics. He tried his best, asked all his friends, and finally came to request help from the master. Von Neumann was very busy but he looked at the problem, went to the blackboard, and scrawled busily for about 5 minutes. Turning to the awe-struck physicist, he said, "triple generating functions" and ushered him out of his office. The moral is clear. Despite von Neumann's great interest in applied mathematics and physics, he was basically a mathematician. Once he had realized that the proper technique for evaluating the sum was to use the method for generating functions, his interest waned. The details of the evaluation and the actual answer were of no interest to him.

The Concept of Function

I have said that mathematicians are interested in studying more and more general structures. Let me present a special case which may illustrate what is meant by a mathematical structure. In high school mathematics we study formulas such as $A = s^2$, for the area of a square in terms of its side, or $v = 32t$, for the velocity of a body falling for a given time under the influence of gravity, or $V = \pi r^2 h/3$, the volume of a cone in terms of its height and the radius of its base. We learn how to manipulate these formulas and how to combine them. But the main problems are two. First, what is the value given by the formula for a given value of the argument? Second, for what value of the argument will the formula have a given result?

In more advanced courses we change our viewpoint and start investigating what we have done and what we have been working with. What is a formula? What does it do? It produces an assignment or a correspondence between one number or a group of numbers, which we call the arguments of the formula, and another number, which we call the result of the formula. Thus, if the argument is the side s of a square, the result is the area A, and if the arguments are the height of the cone and the radius of its base, the result is the volume of the cone. Each of these formulas produces an assignment or a correspondence between the arguments and the results. Once this is noticed, the mathematician sets himself the problem of studying the concept of an assignment or a correspondence. He calls this concept a "function," and he says that the formula defines functions and that the result of a formula is the value of the function defined by that particular formula. Notice that here the mathematician has taken a typical leap into a higher level of abstraction, because, instead of considering numbers, such as the length of the side of a square or the time of a falling body or the height of a cone, he considers the function which produces the area from the side of the square, or the function which produces the volume of a cone from its height and radius. The function is not a number; it is an assignment or a correspondence.

This concept of the function being an entity in itself is fairly recent and is only slowly appearing in elementary education. I still remember how my high school teacher would scold his class for writing "$\cos^2 + \sin^2 = 1$." He said that sine and cosine, by themselves, have no meaning; instead we should have written "$\sin^2\theta + \cos^2\theta = 1$," which was a meaningful relation between numbers. Here was a case where the class, even though it did not understand why, was correct, and the teacher was not. Of course, sine and cosine are not numbers, but they are functions, and the relation which we had written down was a correct relation between functions.

Once we have introduced the concept of function, our problems necessarily change. We are no longer interested in the problem of whether a given function has a prescribed value, or in the problem of finding the value of this function for a given value of the argument. Instead, we ask, Is this function increasing or decreasing, and where does it stop increasing and begin decreasing? Is it increasing at an increasing rate? And so on. As you see, we have now become interested in the properties of the function as a whole and not in its individual values. Such questions are obviously of great importance. For example, at present the question of whether the cost of living is or is not increasing at an increasing rate has an obvious political significance.

Functions of Functions

To study these questions in which the function is considered as an entity, we find it useful to assign to each function another function, called its derivative. The values of the derivative function enable us to decide whether the given function is increasing or decreasing. The mathematician, looking at this and trying to analyze what is going on, notices that the concept of an assignment or of a correspondence has appeared again. Since this concept has already been called a function, the mathematician realizes that he can and must consider functions of functions. Thus, differentiation and integration are functions of functions. For semantic reasons we introduce a new word, *operator,* and we say that differentiation and integration are operators of functions. However, *plus ce change, plus c'est la même chose.* An operator is just a function of functions—that is, a correspondence between functions.

Notice that again the mathematician has jumped to a new level of abstraction. He started with numbers, and then considered correspondence between numbers. These correspondences were called functions. Now he considers correspondences between functions and calls them operators.

Again, the kind of questions a mathematician asks changes. He asks: Is the operator linear? Is it bounded? Is it completely continuous? Does it have a spectral representation? All these questions can be asked about the operators of differentiation and integration, but these questions are not immediately relevant to the specific problems that might trouble a user of mathematics—such as, How do I solve this given differential equation? How do I evaluate this given integral? Necessarily, the mathematician knows some general facts about these problems, but to expect him to give detailed technical advice is as unrealistic as to expect a theoretical physicist to be able to fix a malfunctioning television set.

Creation of New Structures

Parallel with the introduction of more complicated concepts there is the creation of new mathematical structures. We know that, with every real number, there is associated a point on a line; with every ordered pair of real numbers, a point in a plane; with an ordered triple of real numbers, a point in space; and so on. The points on a line, in a plane, and in space have associated with them a vector space of one, two, or three dimensions, respectively. It is a natural generalization to consider an ordered set of n numbers as a point in a space of n dimensions and to associate a vector space with this n-dimensional space. When we go higher by one level of abstraction and consider functions as entities, we find that certain sets of functions may be considered as vector spaces of infinite dimension. If, in this vector space, we introduce a generalization of the concept of the length of a vector, we get a mathematical structure called a Banach space. At the next level of abstraction we find that some sets of operators can be considered first as a vector space and then as a Banach space, but then we can go a step further. We realize that operators have a structure richer than that of a Banach space, because, given two operators, we can always apply them successively to get a new operator. If this process of composing two operators to get a third is taken into account, the structure of the set of operators is called a Banach algebra. In recent years, mathematicians have gone up to a still higher level of abstraction by considering a Banach algebra as an entity in itself and by considering different Banach algebras as elements of a category of Banach algebras. Then the properties of this category are studied.

Mathematics and the Real World

I have already mentioned that, with each new level of abstraction, the problems the mathematician considers are changed. Consequently, even though, at the lowest level, the concepts may have been very intuitive and the problems close to reality, at the later levels the concepts and the problems studied have little or no contact with reality. This brings us back to the question posed by the physicist: Why should the government and the university support mathematics on so large a scale? My answer must be based on pragmatic grounds.

Despite the fact that mathematicians, from Grecian times to the present, have tried to avoid having any contact with reality, their work turns out eventually to be intimately connected with, and vitally necessary for an understanding of, the real world. Instances of this are too numerous to mention and probably well known to all of you. I shall try to present a semiphilosophical basis for this interaction between the ivory-tower mathematician and the common-sense man. The mathematician is always trying to find concepts that are interesting and significant. A concept is interesting to the mathematician if it organizes many apparently unrelated facts and if it lends itself to the discovery of new facts. The common-sense man looks at the real world and wants to understand it and manipulate it. Since reality is too complicated and difficult to understand, he must idealize real situations, and he must organize the multiplicity of available facts so that they can be fitted into patterns. Here is where the mathematician comes in. Mathematics provides a set of concepts or a pattern, with which we may try to organize the real world. These concepts are useful because they come provided with a multitude of consequences we mathematicians call theorems. It is only common sense to try to fit these ready-made concepts into the real world. Sometimes they fit and sometimes they do not. The surprising fact is that frequently they do fit very well. Perhaps this fact is the reason for Eddington's belief that God must be a mathematician.

I should like to describe one instance in which mathematicians introduced and studied, for purely mathematical reasons, a concept which much later was found to be of fundamental im-

Table 1. Group of permutations, P, of the roots of the cubic equation (Eq. 2).

P_1	P_2	P_3	P_4	P_5	P_6
$t_1 \rightarrow t_1$	$t_1 \rightarrow t_2$	$t_1 \rightarrow t_3$	$t_1 \rightarrow t_1$	$t_1 \rightarrow t_3$	$t_1 \rightarrow t_2$
$t_2 \rightarrow t_2$	$t_2 \rightarrow t_3$	$t_2 \rightarrow t_1$	$t_2 \rightarrow t_3$	$t_2 \rightarrow t_2$	$t_2 \rightarrow t_1$
$t_3 \rightarrow t_3$	$t_3 \rightarrow t_1$	$t_3 \rightarrow t_2$	$t_3 \rightarrow t_2$	$t_3 \rightarrow t_1$	$t_3 \rightarrow t_3$

portance to our understanding of the elementary particles of physics. The concept I refer to is that of a group. This concept was conceived, more than a hundred years before the discovery of the positron, as a result of intensive study during the 17th and 18th centuries of methods for finding the roots of a polynomial equation. The Greeks had a solution of the second-degree equation. The third-degree equation was solved in the 16th century, and the fourth-degree equation was solved shortly afterward. But that was all. For the next 200 years all attempts to solve the fifth-degree equation failed. It resisted solution until, in 1832, Galois proved that the fifth-degree equation could not be solved by radicals.

Given any equation, there are standard numerical techniques by which the roots of that equation can be found to an arbitrary degree of accuracy. There are techniques available for solving polynomial equations of any degree, and even for solving more complicated equations. What, then, is meant by the statement that Galois proved that it is impossible to solve the fifth-degree equation by radicals? To understand Galois's result, we must realize that the phrase "solving an equation" can mean two different things. Solving an equation can mean finding the roots numerically to a given degree of accuracy, or it can mean what was meant in high school—finding a formula which expresses the roots of the equation in terms of the coefficients. Galois's theorem refers to solution in the latter sense.

The concept of formula needs some further elaboration. Given a third-degree equation, we can find a formula which expresses the roots of the equation as trigonometric functions of the coefficients. This is not the kind of formula I am referring to. By a formula I mean an expression that is formed from the coefficients of the equation by adding, subtracting, multiplying, dividing, and, finally, taking roots, such as square roots, cube roots, and so on. We call such an expression "an algebraic function of the coefficients." Galois's theorem states that the roots of a polynomial equation of degree higher than four cannot be expressed algebraically in terms of the coefficients.

If you take a common-sense and practical view of mathematics, as some mathematicians do, the search for an algebraic formula seems a sheer waste of time. After all, computers can provide us with the roots of any equation with any desired degree of accuracy. What more could a formula do? Also, for the third- and fourth-degree equations, application of the formulas is more difficult than straightforward numerical technique. Then why bother to study the solution of polynomial equations?

The mathematicians of the 17th and 18th centuries were well aware that there was no practical need for algebraic formulas, despite the lack of 20th-century computers. Nevertheless, motivated perhaps by a puzzle-solving attitude, or by a feeling that gaps in knowledge must be filled, or by a need to understand why certain polynomials can be solved and others cannot be, they wasted many hours and many reams of paper trying to solve the fifth-degree equation. It was this waste of time, this impractical pursuit of knowledge for its own sake, that led Galois to introduce the concept of a group, a concept which is fundamental to the understanding of present-day physics.

To see how this concept of a group might have arisen in the study of the solution of polynomial equations, let us consider the third-degree equation

$$t^3 - a_1t^2 + a_2t - a_3 = 0 \qquad (1)$$

and let us try to find its roots. We know that it has three roots—t_1, t_2, t_3—and that each satisfies Eq. 1.

If we write the equation as

$$(t - t_1)(t - t_2)(t - t_3) = 0$$

and equate the coefficients of corresponding powers of t, we find the following system of three equations for the three roots:

$$\begin{aligned} t_1 + t_2 + t_3 &= a_1 \\ t_1t_2 + t_2t_3 + t_3t_1 &= a_2 \\ t_1t_2t_3 &= a_3 \end{aligned} \qquad (2)$$

The fundamental problem of the theory is to distinguish between the roots. Which one is the first? Which is the second? Which is the third?

A glance at Eq. 2 shows that each equation contains all of the roots in a symmetric fashion. Consequently, if the roots are relabeled in an arbitrary way—for example, in such a way that the root t_1 is labeled t_3 (we indicate this by the symbol $t_1 \rightarrow t_3$) and the root t_3 is labeled t_1 ($t_3 \rightarrow t_1$)—each equation in Eq. 2 will remain unchanged. Such a relabeling of the roots is called a permutation. Because any permutation of the roots followed by another permutation is also a permutation of the roots, we say that the set of all permutations of the roots is a "group," called the symmetric group on three objects. For later reference, I list all permutations of this group in Table 1.

The concept of a group can be used to give a precise definition of the concept of a symmetric function. A function of the three roots of Eq. 1 is called symmetric if the function remains invariant no matter how the three roots are permuted. Thus, Eq. 2 shows that each coefficient in Eq. 1 is equal to a symmetric function of the roots of Eq. 1. It is easy to verify the proposition that any function formed from the coefficients by addition, subtraction, multiplication, and division—we call such a function a rational function of the coefficients—must also be a symmetric function of the roots. An important result is the converse (*2*): Any rational symmetric function of all the roots can be expressed as a rational function of the coefficients. Since one root is not a symmetric function of all the roots, we conclude that a rational function of the coefficients cannot give a formula for an individual root.

The only hope for expressing unsymmetric functions of the roots as formulas in terms of the coefficients is to use algebraic functions of the coefficients—that is, functions containing radicals, such as square or cube roots. Let me illustrate by constructing an unsymmetric function D of the roots, where D is defined as follows:

$$D = (t_1 - t_2)(t_2 - t_3)(t_3 - t_1) \qquad (3)$$

Clearly, D is not a symmetric function of the roots because, under the permutation P_4 in which $t_1 \rightarrow t_3$ and $t_3 \rightarrow t_1$, D becomes $(t_3 - t_2)(t_2 - t_1)(t_1 - t_3) = -D$. Consequently, D cannot be expressed rationally in terms of the coefficients. However, the quantity D^2 is invariant under all permutations of the symmetric group, and it is easy to show (*2*) that

$$D^2 = a_1^2 a_2^2 + 18\, a_1 a_2 a_3 - 4a_2^3 - 27a_3^2, \tag{4}$$

a rational function of the coefficients. From Eq. 4 the value of D can be found by taking square roots.

Consider the expression for D in Eq. 3. Even though D is not invariant for the full symmetric group, it is invariant for the subgroup containing the following permutations: P_1, P_2, and P_3. It can be shown (*2*) that any function of the roots which is invariant for this subgroup can be expressed rationally in terms of the coefficients and D, or, because of Eq. 4, in terms of rational functions and square roots of rational functions of the coefficients. For example, if

$$E = (t_1/t_2) + (t_2/t_3) + (t_3/t_1),$$

it is apparent that E is invariant under the permutations of the subgroup but not under P_4, P_5, or P_6. We find that

$$2a_3E = a_1a_2 - 3a_3 \pm D^{1/2}.$$

The arbitrariness in the sign of the square root is due to the fact that the value of E depends on the labeling of the roots. For some labelings the value of E is given by the positive square root, for other labelings by the negative square root.

The next step in the solution of the cubic is to study those functions of the roots which are invariant for still smaller subgroups of the symmetric group and to express such functions in terms of the coefficients. Finally, we should reach a function such as t_1 which is invariant only under the identical permutation P_1, and we would have obtained a formula for the root of the cubic in terms of the coefficients.

An analysis of the method used reveals the following important idea [an idea whose applications and philosophical implications have been studied by Weyl (*3*)]. In studying an object, one considers the changes produced in that object by certain transformations. Those transformations which do not change the object are called symmetries of the object. The set of symmetries of an object always form a group. The study of the group, its subgroups, and those objects which are left invariant by all transformations of the group can be used to obtain useful information about the original object.

I give two examples of the usefulness of the group of symmetries of an object. One example is from elementary geometry, the other from the theory of ordinary differential equations. In elementary geometry, a parallelogram is a four-sided figure with its opposite sides parallel. A little thought shows that the parallelogram has only two symmetries. One is the identity transformation—that is, the transformation which leaves every point unchanged; the other is the reflection of every point in the intersection O of the diagonals, so that a point such as P in Fig. 1 is transformed into Q and Q is transformed into P. We use this center of symmetry to prove the following.

In Fig. 2, if EFGH is a straight line, through O, intersecting AB and DC extended in E and H, respectively, and intersecting AD and BC in F and G, then EF = GH. The proof takes just a few lines. Because reflection in O is a symmetry of the parallelogram, OE = OH and also OF = OG. Therefore (OE − OF) = EF = (OH − OG) = GH.

The next example shows that the study of groups can help in solving differential equations. Consider the differential equation

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x^2}{y}$$

We observe that if we make the transformation $x \to \alpha x$, $y \to \beta y$, where α and β are constants, the equation becomes

$$\frac{\beta}{\alpha}\frac{dy}{dx} = \frac{\beta}{\alpha}\frac{y}{x} + \frac{\alpha^2}{\beta}\frac{x^2}{y}$$

We notice that if $\beta = \alpha^2$, the equation is invariant under the transformation. Since the quantity y/x^2 is also invariant under this transformation, an obvious device is to introduce the new dependent variable $u = y/x^2$. We find that

$$xu' = u^{-1} - u,$$

an equation which can be immediately solved by separation of variables.

Applications of Group Theory

In the late 19th century (*4*) Sophus Lie applied group theory to partial differential equations and showed that the solutions of equations involving the Laplacian must provide a representation of the orthogonal group. The solutions were the well-known ones containing the Legendre functions.

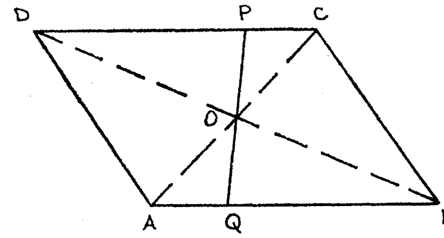

Fig. 1

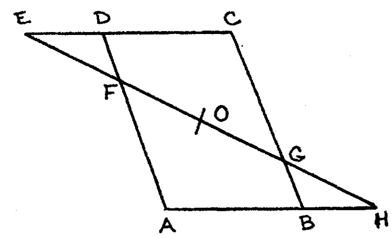

Fig. 2

Finally, we come to physics. When Schrödinger formulated his famous partial differential equation describing the quantum mechanical behavior of a hydrogen atom, the solutions in terms of Legendre functions were available from previous work in partial differential equations. However, when more complicated systems were studied, the relevant differential equations turned out to be too difficult to solve. Wigner (*5*) pointed out that, since atoms have rotational symmetry, the set of their wave functions must be invariant under the orthogonal group. Therefore the Legendre functions come in again. We may even leave out these functions and work with the general properties of the representatives of the orthogonal group.

This introduction of group theory bore unexpected fruit. The first groups considered were those that arose naturally from the fact that space or time is essentially featureless, so that where an observer stands in time and space to look at an event should be irrelevant. However, as physicists explored deeper and deeper into the atom, where partial differential equations were no longer appropriate and where our intuitive concepts of space and time are of doubtful validity, they began to rely more and more on the theory of groups. Thus, Gell-Mann and Neiman recently proposed that certain elementary particle reactions were invariant under the special unitary group in three dimensions. By looking at the representations of this group, they predicted the existence of a new elementary particle, a prediction which was later confirmed experimentally. This and similar successes of group theory have so impressed physicists that any day now we shall hear them say, "The world is just made up of irreducible representations of groups."

Conclusion

Let me emphasize the point I have been trying to make. The mathematician's playing with the roots of equations, a play which had no practical motivations and almost no possibilities of practical application, led to the recognition of the importance of symmetry and groups. The study of theory of groups led to mathematical discoveries in geometry and differential equations, and finally to prediction of the existence of a new elementary particle. Surely a surprising outcome for the ivory-tower speculations of an impractical mathematician!

Despite my professional bias, I must acknowledge that the importance of symmetry was recognized before mathematicians invented the theory of groups. In 1794 William Blake wrote:

> Tiger, Tiger, burning bright
> In the forests of the night,
> What immortal hand or eye
> Could frame thy fearful symmetry?

However, to the mathematicians must be given the credit of recognizing that, to understand symmetry, you must study the theory of groups. I can now answer my original question, What are mathematicians doing? They are trying to make precise the intuitions of poets.

References

1. R. W. Hamming, *Science* **148**, 473 (1965).
2. R. A. Dean, *Elements of Abstract Algebra* (Wiley, New York, 1966).
3. H. Weyl, *Symmetry* (Princeton Univ. Press, Princeton, N.J., 1952); *The Classical Groups* (Princeton Univ. Press, Princeton, N.J., 1953).
4. L. P. Eisenhart, *Continuous Groups of Transformations* (Princeton Univ. Press, Princeton, N.J., 1933).
5. E. P. Wigner, *Group Theory and Its Application to the Quantum Mechanics of Atomic Spectra*, J. J. Griffin, Trans. (Academic Press, New York, rev. ed., 1959).

NEWS AND COMMENT

1966 Nobel Laureates in Medicine or Physiology

Two eminent scientists, Peyton Rous and Charles Huggins, were named last week to share the 1966 Nobel prize in medicine or physiology for their work on cancer. Rous is Member Emeritus of Rockefeller University; Huggins is director of the Ben May Laboratory for Cancer Research at the University of Chicago. The following are descriptions and appreciations of their work by W. Ray Bryan and by Paul Talalay and Guy Williams-Ashman.

Charles Huggins

The ravages of cancer present medicine with one of its most difficult and challenging problems. Cancer research must be concerned not only with understanding of the nature and causes of malignant transformations but also with the development of effective measures to combat the tragic consequences of this disease in man. The award of the 1966 Nobel prize for medicine or physiology jointly to Charles Huggins and Peyton Rous honors two scientists whose investigations have revolutionized both our comprehension of the cancerous process and approaches to the treatment of human cancer, and have served to inspire many aspects of contemporary cancer research.

The Nobel prizes over the past 65 years have served as chronicles of human achievement. With the single exception of a prize given in 1926 for a rather restricted contribution to carcinogenesis, no Nobel award has been made hitherto for work on cancer, a fact which only serves to emphasize the importance of this year's Nobel prizes, and of the researches of Huggins and Rous.

Charles Huggins is director of the Ben May Laboratory for Cancer Research at the University of Chicago. Born in Halifax, Nova Scotia, in 1901, the year of the very first Nobel awards, he was educated at Acadia University, Nova Scotia, and the Harvard Medical School. Following a surgical apprenticeship under Frederick A. Coller at the University of Michigan, he became in 1927 a member of the original faculty of the School of Medicine at the University of Chicago, where he has worked and taught for 40 years. With the encouragement and guidance of his distinguished surgical chief, Dallas B. Phemister, Charles Huggins entered the field of urology, and he headed the urological division of the department of surgery for 25 years. Sent to Europe by Phemister in 1930 for training in clinical urology, Huggins spent several months in the laboratory of Sir Robert Robison at the Lister Institute. Here he became acquainted with the phosphate esters and the phosphatases, which came to play a prominent part in his later work on induction of bone formation and the treatment of prostatic cancer. In that year he also met Otto Warburg, an experience which made a strong impression on Huggins, and which later matured into a long and interesting friendship.

Professional identification with urology gave Huggins an opportunity to concern himself with problems in the physiology and diseases of the male genitourinary system. After several years of novel and important work on the induction, by bladder epithelium, of the transformation of connective tissue elements into bone, he turned his attention to the chemistry and hormonal control of the secretions of male accessory glands of reproduction. It was these studies that formed the basis for Huggins's work on carcinoma of the prostate which has been honored by the Nobel prize. By an ingenious surgical procedure introduced in 1939,