of the trials and to the direction of the CS in only 20 percent.

So, as Lawicka discovered, animals trained in a differentiation procedure requiring two different instrumental responses to two auditory stimuli mainly use directional cues; they are almost unable to learn the task when confronted with purely qualitative cues. On the other hand, in a go–no go differentiation procedure based on reinforcement-versus-nonreinforcement of responses to two auditory stimuli mainly utilize qualitative cues.

These facts have been tentatively interpreted in detail (*2*). It is notable that monkeys (*3*) also can establish without difficulty a go–no go differentiation between two different tones emanating from the same point, while their go left–go right differentiations between these stimuli are as difficult as they are for dogs; in contrast, the go right–go left differentiation between directional cues is easy.

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Higher Education: A Population Flow Feedback Model

During the last two decades attention has been turned to the production and retention, within the higher educational sector of the economy, of people holding the doctorate degree. It is generally recognized that the production of doctorates depends to a large degree on the doctorate-holding faculty. Because the doctorate holders, especially in the sciences, are in great demand by the other sectors of the economy, a circular or a "feedback" situation exists. The problem is further complicated by the availability of developed student talent and by various socioeconomic conditions existing at different periods of time within two or three decades prior to the time a study is made. Various studies of these problems have been initiated by agencies of government and by privately financed foundations.

In order to attempt to rectify an existing situation or provide for future needs, initiators of programs are in need of a rational methodology to evaluate the effects of their programs. Historical data and judgment have not always yielded satisfying bases for decisions. Both the military and industry are recognizing, to an increasing degree, the need for quick and economical ways of evaluating the various effects of their decisions. For this they have turned to analytical methods. Computer-based simulation is one method successfully used to study the dynamics of complex and nonlinear problems.

I now offer a conceptual and a mathematical model to study the production of doctorate, master's and baccalaureate degrees and their feedback into higher education (see Fig. 1).

This model breaks the educational sector up into four segments: undergraduate programs, master's programs, doctoral programs, and post-doctoral programs. It breaks the other sectors of society employing college- or university-trained people into segments according to the highest degree earned by those within the segment. Furthermore, it shows the retirement and other attrition sectors more or less as a sink outside of either of the above two sectors. The model delineates the various possible paths for population shifts between the segments. This work is an extension of the model developed and studied by Bolt, Koltun, and Levine (*1*), and it derives from my doctoral dissertation (*2*; see also *3*). In a manner similar to the afore-mentioned works and to many other works in the physical sciences, it accounts for all the net flows to a segment and the rate of accumulation of people within the segment.

The equations recognize the fact that degrees, especially at the doctorate level, are not earned at a given time of the year throughout society. Some schools operate on the semester basis, others operate on a trimester, and yet others on the quarter system. When aggregated, an assumption of continuity in flows seems a little more realistic than an assumption of discreteness. Thus, the equations offered are differential equations. Depending on the postulates regarding lead-lag relationships in the production of the various degrees these equations may be nonlinear. Because of the relative complexity of the model, the final set of equations to be solved simultaneously may be in the order of 40 or 50. However, this should not cause great concern in the age of computers. Back in 1956 I analyzed a hydraulic system which was described by a set of 28 nonlinear differential equations (*4*). These equations were then solved on an electronic analog computer. Certainly the state of computer art has progressed some since.

The basic advantages of this model may be described in outline as follows:

1) It recognizes the input of students into the higher education sector.
2) In the educational sector, it distinguishes between persons who have recently become engaged in the educational function and persons who have worked in education for many years.
3) It recognizes the nonlinearities of the situation studied.
4) It distinguishes between the use of doctorate holders in education at the doctorate, master's and undergraduate levels.
5) It considers the effect of the rates of production of high school, bachelor and master's graduates in successively preceding years.
6) It considers postdoctoral university programs and the interrelated flows from them to teaching at the various levels and to the other sectors employing doctorates.
7) Inasmuch as the rate of doctorate production has changed drastically during the last three decades, because of the depression of the 1930's, World War II, and the post war and the cold war periods, it emphasizes that the age mix is not a uniform one. Thus, the number of degrees granted some years prior (say 30 or 35 years) is the independent variable used in calculating attrition in this model.
8) Concern for economic, social, and physical influences in doctorate production and shifts in employment may be built into it.

Basic balance equations. The basic balance equations of this model follow; symbols are explained in Table 1. The equations state that the total rate of flow into a category less the rate of outflow is equal to the rate of accumulation or growth of a given category.

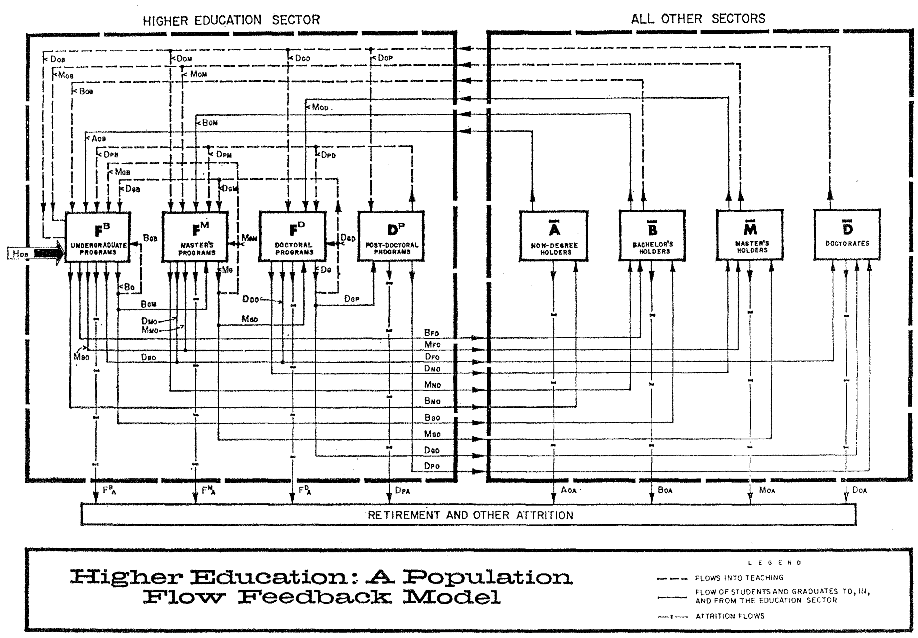

Fig. 1. Population flow feedback model for higher education. Solid lines: flow of students and graduates to, in, and from the education sector; dashed lines: flow into teaching; attrition is shown by bars interrupting the lines.

Postdoctoral programs:

$$D_{GP} - D_{PA} - D_{PO} - (D_{PD} + D_{PM} + D_{PB}) + D_{OP} = dD^P/dt \quad (1)$$

Faculty at doctorate level:

$$D_{GD} + D_{PD} + D_{OD} - D_{DO} - F_A{}^D = dF^D/dt \quad (2)$$

Faculty at master's level:

$$M_{GM} + D_{GM} + D_{PM} + D_{OM} + M_{OM} - M_{MO} - F_A{}^M = dF^M/dt \quad (3)$$

Faculty at bachelor's level:

$$B_{GB} + M_{GB} + D_{GB} + D_{PB} + D_{OB} + M_{OB} + B_{OB} - D_{BO} - M_{BO} - B_{BO} - F_A{}^B = dF^B/dt \quad (4)$$

Doctorates in other sectors:

$$D_{GO} + D_{PO} + D_{FO} - D_{OA} - D_{OP} - D_{OD} - D_{OM} - D_{OB} = d\bar{D}/dt \quad (5)$$

$$D_{FO} = D_{BO} + D_{MO} + D_{DO} \quad (5a)$$

Masters in other sectors:

$$M_{GO} + M_{FO} - M_{OA} - M_{OD} - M_{OM} - M_{OB} + D_{NO} = d\bar{M}/dt \quad (6)$$

Bachelors in other sectors:

$$B_{GO} + B_{FO} - B_{OA} - B_{OM} - B_{OB} + M_{NO} = d\bar{B}/dt \quad (7)$$

Nondegree dropouts in other sectors:

$$B_{NO} - A_{OA} - A_{OB} = d\bar{A}/dt \quad (8)$$

Bachelor's production:

$$B_G = B_{GM} + B_{GB} + B_{GO} \quad (9)$$

Master's production:

$$M_G = M_{GD} + M_{GM} + M_{GB} + M_{GO} \quad (10)$$

Doctor's production:

$$D_G = D_{GP} + D_{GD} + D_{GM} + D_{GB} + D_{GO} \quad (11)$$

Faculty degree mix relations. In the following equations we recognize the fact that faculty members, especially those teaching at the predoctoral levels, do not all hold a doctorate. Thus faculty teaching at the undergraduate levels represents, in general, a mixture of all three degrees.

$$F^B = a^B D^F + b^B M^F + c^B B^F \quad (12)$$

Inasmuch as the above mixture is not necessarily fixed,

$$D^F da^B/dt = D_{GB} + D_{PB} + D_{OB} - D_{BO} - D_{BA} \quad (13)$$

$$M^F db^B/dt = M_{GB} + M_{OB} - M_{BO} - M_{BA} \quad (14)$$

and

$$B^F dc^B/dt = B_{GB} + B_{OB} - B_{BO} - B_{BA} \quad (15)$$

Also

$$F_A{}^B = D_{BA} + M_{BA} + B_{BA} \quad (16)$$

Assuming for the purpose of this presentation that the number of bachelor-only holders teaching at the master's level is insignificant, then we can write similar mix relations for the master's faculty mix.

$$F^M = a^M D^F + b^M M^F \quad (17)$$

and

$$D^F da^M/dt = D_{GM} + D_{PM} + D_{OM} - D_{MO} - D_{MA} \quad (18)$$

while

$$M^F db^M/dt = M_{GM} + M_{OM} - M_{NO} - M_{MA} \quad (19)$$

Moreover

$$F_A{}^M = M_{MA} + D_{MA} \quad (20)$$

It will be assumed here that all faculty teaching at the doctorate level do in fact hold a doctorate. The model can be easily modified for those disciplines, as say engineering, where this assumption will introduce a significant error. Thus,

$$F^D = D^F$$

and

$$F_A{}^D = D_{DA}$$

Lead-Lag Relations. The next set of equations represents a set of postulates regarding the lead-lag relationships between various rates. Because of the nature of their origin they can easily be replaced in the model by statements of other postulates; however, in concept, they cannot be eliminated.

We postulate that the rate of attrition within a given group is a function of the rate of generation (graduation) of members of that group at some time τ prior.

Doctorates in other sectors:

$$D_{OA} = \alpha_O D_G(t - \tau) \quad (23)$$

Doctorates in academic sector:

$$F_A{}^D = \alpha_F D_G(t - \tau) \quad (24)$$

Equation 25, which is typical of the next four, claims that the attrition, other than to education, of Master's holders from other sectors, is a function of the difference between the rate at which masters were graduating τ' years prior and that at which doctors were being produced τ years prior. Of course $\tau < \tau'$

$$M_{OA} = \alpha_O[M_G(t - \tau') - D_G(t - \tau)] \quad (25)$$

Masters in academic sector:

$$F_A{}^M = \alpha_F[M_G(t - \tau') - D_G(t - \tau)] \quad (26)$$

Bachelors in other sectors:

$$B_{OA} = \alpha_O[B_G(t - \tau'') - M_G(t - \tau')] \quad (27)$$

Bachelors in academic sector:

$$F_A{}^B = \alpha_F[B_G(t - \tau'') - M_G(t - \tau')] \quad (28)$$

Bachelor's production. The rate at which baccalaureate degrees are granted in any given year will be a factor, among others, in the rates at which master's and therefore doctor's degrees will be granted some years hence. In turn, the number of bachelors graduating in a given year is a function of (i) the number of students graduating from high-school 4 to 5 years before, (ii) the availability and accessibility of space in our schools of higher learning, and (iii) the economic and intellectual climate in the society—that is, the current relative earning potential of college-educated people versus those in the organized trades or in business, the public image of the intellectual, and other factors.

Items (i) and (ii) represent some of the tangible factors influencing the rate of entrance and dropout at the undergraduate level. These are the items that societies can do something concrete about. Item (iii), on the other hand, represents a group of intangibles or perhaps pseudotangibles. Factors which fall into this group do change with time. Some of these are characterized by long lead times, such as resistance to change, yet some, such as the public's image of the intellectual, can change fairly rapidly.

No postulates are made in this paper regarding the relation between these factors and the rate of bachelor's production. All of these factors are lumped into a vector quantity **V** which is considered to be an exogenous input into the model and a variable in a function in need of establishment.

Thus the number of baccalaureate degrees granted is a weighted function of the number of graduates from high school who chose to go on to college, H_{OB} predominantly, 4 to 5 years before, as well as some function of the availability of faculty, physical plant, and cost.

$$B_G = [\beta_4 H_{OB}(t-4) + \beta_{4.5} H_{OB}(t-4.5) + \beta_5 H_{OB}(t-5) + \beta_n H_{OB}(t-n)][f(F^B, \mathbf{V}, t)] \qquad (29)$$

Table 1. Glossary of Symbols used in Eqs. 1 to 31. Subscripted symbols represent rates; all others represent levels.

	Subscripts and superscripts
A	Attrition
B	Bachelor's degrees
D	Doctorate degrees
F	Faculty
G	Graduation
M	Master's degrees
N	Nonattainment of objective
O	Other sectors
P	Postdoctoral program
	Main symbols
A	College-educated but non-degree-holding people
B	Holders of bachelor's degree only
D	Holders of the doctorate
F	Faculty
H	High school students entering college
M	Holders of a master's degree as the highest degree earned
V	A vector quantity
a	Ratio of doctorate-holding faculty
b	Ratio of master's only holding faculty
c	Ratio of Bachelors only holding faculty
f	Function of
n	Number of years required to achieve a goal other than what is considered to be within the normal range.
t	Time
α	Coefficient of attrition
β_i	Weighting factor indicating the fraction of students achieving a specified goal in *i* years.
Φ	Function of
τ	Fixed time interval where $\tau < \tau' < \tau''$

For any given year, of course,

$$\beta_4 + \beta_{4.5} + \beta_5 + \beta_n + \beta_N = 1 \qquad (30)$$

is a constraining equation in which β_N indicates the fraction of nonachievers in the class and β_n indicates the fraction of same class which will graduate at some time other than 5 years after graduation from high school. To avoid double counting, β_n should not consider those who have at one time dropped out as nonachievers, entered some form of employment in category $\overline{A}$, and later returned for a successful completion of their study. β_n thus is primarily concerned with those who stay on campus and for various reasons take longer than 5 years to graduate. My impression is that $\beta_n <<< 1$. β_N, on the other hand, is a function of the total number of students that are enrolled in the bachelor's program, the number and quality of faculty available for undergraduate teaching, and many of the factors comprising the vector **V**. Thus

$$\beta_N = [H_{OB}(t-5) + H_{OB}(t-4) + H_{OB}(t-3) + H_{OB}(t-3) + H_{OB}(t-1)][\Phi(F^B, \mathbf{V}, t)] \qquad (31)$$

Master's production. The number of master's degrees granted in a given year is a function of the same or similar factors and of the rate at which bachelor's degrees have been granted in the recent past. Moreover, the changing demands of the employment sector both for the present and for the future is a factor of great influence. The great strides made in the development of some of the sciences and of technology have encouraged a greater percentage of graduates to go on to graduate schools.

Equations similar to 20 through 31 can be written for the production of master's degrees.

Doctorate production. Comments made at the previous two levels apply to the doctorate level as well. However, the great emphasis placed on original research at the doctorate level and the inherent per capita costs makes the availability of outside support most influential at this level. Emphasis on university-based research and development by the various agencies of the federal government has greatly accelerated the rate of doctorate production granting during the last decade. The demand for Ph.D.'s in the various sectors of the economy, including the academic, is a factor not to be overlooked. Thus it quickly becomes apparent that one cannot meaningfully study the doctoral feedback into higher education without considering the various economic, social, and physical influences encouraging pursuit of the doctorate. However, to examine completely and thoroughly the problem set forth may require complete analysis of our economy and its feedback interrelationships with all the nations of the world. A study of this kind for the problem at hand is, of course, not justifiable. Thus we are left with a decision that most system analysts of socioeconomic systems sooner or later must face. The decision is two-fold. First, it is concerned with what portion of the universe one ought to subject to study—that is, where he should place his system boundaries. Second, it is concerned with the level of aggregation to be used both within the system's boundaries and within that portion of the remaining universe with which the system communicates. Both parts of the decision ought to be dictated primarily by the questions posed: the objectives of the study. For instance, the model as presented here did not specify whether it will deal with the education of engineers or of scientists, including engineers, or whether it will deal with all people having or acquiring higher education. The model as set down can treat any of these cases. However, the level of aggregation will be somewhat different for each of the cases. If, however, one is interested in considering the question of all the sciences and in studying the interrelationships between them—for example, the flow of physicists into engineering or engineering education—then the model will have to be further deaggregated and hence made much more complex in magnitude, if not in concept.

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