Rotation Technique in Radially Symmetric Electron Micrographs: Mathematical Analysis

Abstract. *The rotation operation of order* m *operating on a signal removes all spatial frequencies not multiple to* m. *Anomalous reinforcements, causing misinterpretation, can be discounted without following previously suggested restrictions. The improvement in signal-to-noise ratio is less than* m, *in contrast to signal averaging of electrophysiology. A dove prism and stroboscope can implement the technique.*

Markham *et al.* (*1*) have described a technique of rotation photography to enhance image detail in electron micrographs which have radial symmetry. However, the technique is prone to misinterpretation; Agrawal *et al.* (*2*) show the need for caution in the interpretation of photographs made by this technique and suggest restrictions on its use to avoid incorrect interpretations.

The rotation technique may be considered as a mathematical operation performed on a signal. I now describe the nature of the anomalous reinforcements, which give rise to the misinterpretations, and offer a method for determining whether a reinforcement is anomalous. The effect of the technique on noise is discussed, and a simple mechanism is described for conveniently performing the rotation technique.

Let us consider the data as represented by the light intensity transmitted through a circular photographic transparency or reflected from a print and described in polar coordinates, $f(r,\theta)$, where $0 \leq r \leq R$, R is the radius of the photograph, and $f(r,\theta) = f(r,\theta+2\pi k)$ for any integer k. To simplify the mathematical treatment, since all operations deal only with rotation, the dependence on r is omitted from the equations. That is, the following analysis applies separately to each annulus, and the problem reduces to that of analyzing a periodic function, $f(\theta)$, with period 2π.

The rotation technique of order m can be simply described as an operation, R_m, operating on the original data to produce a new image, $F_m(\theta)$, as defined by

$$F_m(\theta) = R_m[f(\theta)]$$

$$= \frac{1}{m}\sum_{k=1}^{m} f(\theta + \frac{2\pi k}{m}) \qquad (1)$$

That is, the figure is rotated $1/m$ of a full revolution and photographed, rotated another $1/m$ of a revolution and rephotographed, this exposure being superimposed onto the previous one, and so on, until the figure has been rotated one full revolution, and m exposures have been made. A normalizing factor of $1/m$ is introduced so that the total cumulative exposure is independent of m.

To obtain the relationship between $f(\theta)$ and $F_m(\theta)$, the Fourier series expansion is used. The index, n, of the Fourier coefficient shall here be called frequency; the term "order" shall be reserved for the rotation order, m. By definition

$$c_n = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)\exp(-in\theta)\,d\theta \quad (2)$$

$$c_{m,n} = \frac{1}{2\pi}\int_0^{2\pi} F_m(\theta)\exp(-in\theta)\,d\theta \quad (3)$$

so that

$$f(\theta) = \sum_n c_n \exp(in\theta)$$

$$F_m(\theta) = \sum_n c_{m,n} \exp(in\theta)$$

Substituting Eq. 1 into Eq. 3, and integrating as indicated by Eq. 2

$$c_{m,n} = \frac{1}{m}\sum_k c_n \exp[i(n/m)2\pi k]$$

$= c_n$ (if n is an integral multiple of m)
$= 0$ (if n is not)

Thus R_m removes all Fourier coefficients from the signal that are not harmonics of the order of rotation, leaving the harmonics unchanged.

To illustrate, consider a signal with sixfold symmetry, consisting of six small points located at $0, \pm\pi/3, \pm 2\pi/3$, and π radians. This signal, in the Fourier expansion, has equal coefficients for all frequencies that are multiples of six. All other frequencies have a coefficient of zero. Just as in the case of the equivalent time signal the Fourier expansion contains a zero frequency or d-c term representing the average intensity of the signal, a fundamental of frequency six, and higher harmonics of frequencies that are multiples of six.

If R_2 is performed on this signal, all odd frequencies are eliminated and all even frequencies left unchanged. But the original signal had no non-zero components at odd frequencies so $F_2(\theta) = f(\theta)$. Similarly $F_3(\theta) = F_6(\theta) = f(\theta)$. But if R_4 is performed then c_6 is eliminated, c_{12} kept, c_{18} eliminated, and so on; only frequencies that are multiples of 12 are retained. Hence $F_{12}(\theta)$ has 12 points, equally spaced around the circle. Similarly R_5 produces a figure with 30 points. The production of higher order patterns in this way is well illustrated by Agrawal *et al.* (*2*, Fig. 3) in which a figure of fivefold symmetry produces patterns of 10, 15, 30, 40, and 50 lobes after application of R_2, R_3, R_6, R_8, and R_{10}, respectively.

The number of lobes, L, in the rotated picture is, in general, the smallest multiple common to both the order of rotation, m, and the degree of symmetry in the original, x. It follows, then (*3*), that $x = L/k$, where k is a divisor of m which is relatively prime to L/m, that is, k and L/m have no common divisor other than 1. This limits the possible values of x to a rather small set. Repeating the rotation technique for several values of m, judiciously selected, will provide the correct value of x. For example, if $L = 12$ after application of R_{12}, then $L/m = 1$, and all the divisors of 12 are relatively prime to 1. Hence k can be 1, 2, 3, 4, 6, or 12, yielding $x = 12$, 6, 4, 3, 2, or 1, respectively. Suppose R_6 also produces $L = 12$. Now, of the divisors of 6, only 1 and 3 are relatively prime to $L/m = 2$ so that $x = 12$ or 4. Application of R_4 will thus determine the value of x. If clear pictures can be obtained for prime values of m, only two rotation orders are needed to specify x exactly, since in this case $x = L$ or L/m.

To illustrate the effect of the rotation technique on noise, consider the simplest case in which the data can be represented by the superposition of signal and noise, where the signal and noise are uncorrelated, and where the noise at one point is uncorrelated with the noise at any other point. In this case, white noise, the data power spectrum (*4*) is equal to the sum of the signal power spectrum plus the noise power spectrum and, furthermore, the noise power spectrum has coefficients of equal amplitude for all frequencies. The rotation operations, being linear, operate independently on the signal and noise, and have the same effect on the power spectra as on the regular Fourier coefficients. Thus the application of R_m will remove $m - 1$ out of every m noise components, leaving only those that are multiples of m. The noise power, the sum of the coefficients in the noise power spectrum, is thus reduced by a factor of m; and the average noise amplitude, the square root of the noise power, is reduced by a factor of $m^{\frac{1}{2}}$.

The effect of R_m on the signal power cannot be determined without a detailed knowledge of the spectrum of the signal. If the signal contains only elements of m-fold symmetry then all signal components are of frequencies harmonic to m and the signal, hence the signal power, is left unchanged; the signal-to-noise power ratio is increased by a factor of m. If the signal contains components at other frequencies, they will be eliminated and the signal power will be decreased, resulting in an improvement in signal-to-noise ratio by a factor less than m. Alternatively, if the noise is not white, the noise power will be reduced, in general, by a factor less than m, and again the improvement in signal-to-noise ratio will be less than m.

The rotation technique, therefore, is not completely equivalent to the signal-averaging technique used in electrophysiology because of the possible presence of signal frequencies not harmonic to the order of rotation, anomalous reinforcements and a loss in noise reduction resulting.

The presence of heavy noise in the original data will tend to produce reinforcements for all rotation orders, because the noise does contain components of all orders of symmetry. However the number of lobes in the patterns produced by noise will be equal to the rotation order, and application of the rules previously described produces the fundamental degree of symmetry $x = 1$.

A convenient mechanism for performing the rotation operation can be constructed along the lines suggested by the stroboscopic method of Markham *et al.* (*1*). The photograph (or transparency) is held stationary and (trans) illuminated with a stroboscopic light source. The image is rotated optically, a dove prism (*5*) driven by an electric motor being used. The stroboscope may be triggered by a photoelectric commutator attached to the rotating prism. This method provides the convenience of having a stationary photograph, for easy centering and adjusting, as well as having the advantage of immediate viewing of the image.

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References and Notes

1. R. Markham, S. Frey, G. J. Hills, *Virology* **20**, 88 (1963).
2. H. O. Agrawal, J. W. Kent, D. M. MacKay, *Science* **148**, 638 (1965).
3. Proof: $L = mx/(m,x)$, where (m,x) is the greatest common divisor of m and x. Hence $x = L/k$, where $k = m/(m,x)$ is a divisor of m and $(k,L/m) = [m/(m,x),\ x/(m,x)] = (m,x)/(m,x) = 1$.
4. The coefficient of any frequency in the power spectrum is the square of the absolute value of the corresponding coefficient of the regular Fourier expansion.
5. Available from Edmund Scientific Co., 101 East Gloucester Pike, Barrington, New Jersey 08007. Note that the dove prism doubles the angle of revolution; one revolution of the prism rotates the image by two revolutions.
6. Support provided by a NSF predoctoral fellowship.

18 January 1966

Potassium-Argon Geochronology of Deep-Sea Sediments

Abstract. *A potassium-argon dating method applicable to small quantities of volcanic minerals and glass has been developed and used to determine the ages of North Pacific sediments. Tertiary sedimentation rates range from less than 1.0 millimeter per 10^3 years for deep-sea "red clay" to 1 centimeter per 10^3 years for calcareous-siliceous ooze nearer the continent. The potassium-argon ages obtained from different minerals are concordant, and in samples with associated fossils, these ages are compatible with the paleontologic evidence.*

Our present understanding of the time relationships of pelagic sediments is based upon radiometric methods which apply only to the past 500,000 years (*1*) and upon paleontologic studies. If we are to extend our knowledge of open-ocean sediments to the more remote past and to the vast nonfossiliferous areas of the oceans, we shall require new dating techniques. The use of potassium-argon analysis of volcanic debris to date pelagic sediments has been suggested (*2*) as a means of reconciling discrepancies between existing dating techniques. This suggestion led to the development of a K–Ar technique, which is applicable to small quantities of volcanic contributions found in pelagic sediment cores, and has permitted the assignment of ages to several cores from the Pacific.

The usual assumptions of K–Ar dating (a closed system and no argon present at time zero) must be amplified if the age obtained from volcanic materials is to correspond to the time of deposition as follows: (i) The volcanic material must arrive at the depositional site soon after its formation during the volcanic episode. (ii) Continental detritus, submarine erosional products, or authigenic marine minerals must not be present in the analyzed material. By choosing samples from volcanic ash layers, detrital contamination is minimized, and one can be relatively sure of dealing with a short-term volcanic event. The low temperatures of the marine abyssal environment (approximately 1°C) greatly reduce the possibility of argon loss by diffusion.

Cores containing obvious volcanic contributions, either in the form of altered ash beds or concentrations of fresh volcanic glass, were chosen for

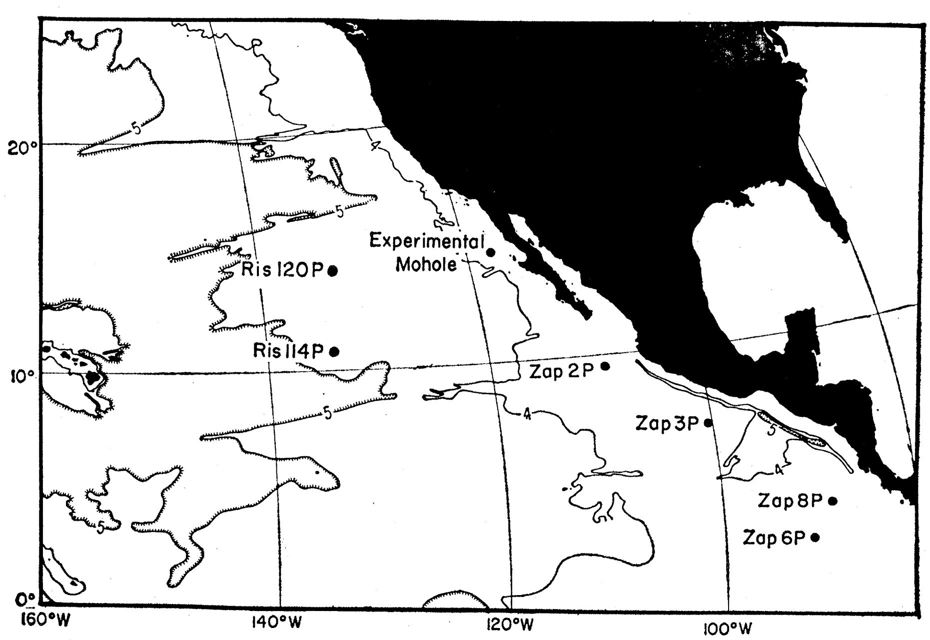

Fig. 1. Location of cores. Contours in thousands of meters.