loss mechanism. However, the total effect on the thin (10^6 atom/cm^2) radon atmosphere proposed here, over its 3.8-day half-period, is the loss of at most only a few percent of the atmosphere by direct or charge-exchange collisions.

Some qualitative remarks may be made concerning the alpha spectra which might be observed from the lunar surface. The predicted alpha intensity is sufficient to produce clear spectra in a practical low- or medium-background alpha-ray spectrometer which could, for example, be orbited about the moon. Although the Tn activity is much less than the Rn activity, it has been shown that a large contribution should be observed by decay of Tn in trajectory. This will appear as a "thin-source" line, sharply peaked. The spectra produced by decay of Rn and Rn-daughter products may not be truly "thin-source" spectra (*9*) since impacting and surface roughness may produce low-energy tails on "thin-source" peaks when observed with an uncollimated spectrometer.

We have tacitly assumed in all of the preceding discussion that the lunar surface is electrostatically neutral. As the recoil ion following alpha decay is charged, the details of deposition in a vacuum are affected by lunar surface or space charges. No estimate has been made of the consequences of this effect.

The surface activities attributable to released Rn and Tn contain gamma and beta activities comparable to the alpha activities discussed.

The effects of sintering of lunar surface materials by energetic solar protons has been discussed by Smoluchowski (*10*). The predicted alpha fluxes at the surface cause sintering effects comparable to those predicted for energetic solar protons.

An astronaut on the moon will acquire a surface alpha activity in two ways. The decay series of Rn includes RaA (Po^{218}) and RaC (Bi^{214}), with relatively short half-periods, and 19.4-year RaD (Pb^{210}), the long-lived parent of RaF (Po^{210}). The short-lived daughters RaA and RaC will reach equilibrium concentrations by direct recoil deposition on his exposed suit area. Long-lived activity will be acquired by pick-up of lunar dust. In either case, if the astronaut presents an exposed suit area of 1 m^2, from the predicted rates he would acquire an activity of about 1 μc. Thus, possible contamination of the interior of a spacecraft or of returned samples should be considered.

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References and Notes

1. Robley D. Evans, *The Atomic Nucleus* (McGraw-Hill, New York, 1955).
2. H. Israel and A. Krebs, *Nuclear Radiation in Geophysics* (Academic Press, New York, 1962), pp. 76–86; H. W. Kraner, G. L. Schroeder, R. D. Evans, A. R. Lewis, *Rev. Sci. Instr.* **35**, 1259 (1964).
3. H. W. Kraner, G. L. Schroeder, R. D. Evans, in *The Natural Radiation Environment*, J. A. S. Adams and W. M. Lowder, Eds. (Univ. of Chicago Press, Chicago, 1964), pp. 191–215.
4. G. L. Schroeder, H. W. Kraner, R. D. Evans, *J. Geophys. Res.* **70**, 471 (1965).
5. This idea has been considered independently by others, including J. R. Arnold in *Proc. Lunar and Planetary Exploration Colloquium* (North American Aviation, Inc., Downey, Calif., 1958), vol. 1(3), pp. 28–31.
6. S. Hayakawa and M. Matsuoka, *Report of Ionosphere and Space Research in Japan* **16**, No. 3 (1962); R. D. Evans and R. W. Riatt, *Phys. Rev.* **48**, 171 (1935).
7. J. W. Salisbury and P. E. Glaser, *Studies of the Characteristics of Probable Lunar Surface Materials*, Air Force Cambridge Res. Labs. Report No. 64-970, January 1964.
8. F. C. Michel, *Planetary Space Sci.* **12**, 1075 (1964).
9. G. J. Stephenson, R. D. Evans, H. W. Kraner, "TID-16349" (Radioactivity Center, Massachusetts Institute of Technology, Cambridge, 1962), pp. 55–56.
10. R. Smoluchowski, *Science* **150**, 1025 (1965).
11. We thank Professor Robley D. Evans for several very helpful discussions. The work of H. W. Kraner and G. L. Schroeder was supported in part by AEC contract AT (30-1)-952.
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Comet Tempel-Tuttle: Recovery of the Long-Lost Comet of the November Meteors

A comet, not quite bright enough to be seen with the unaided eye, was discovered by W. Tempel at Marseilles, France, on 19 December 1865. It was found independently by H. P. Tuttle at Washington, D.C., some 2 weeks later. The comet, which was under observation only until 9 February 1866, was named after its discoverers and received also the designation 1866 I. The orbit of the object was found to be an ellipse with an orbital period of approximately 33 years (*1*). Since the comet was under observation for only a short time, the period was uncertain by perhaps 2 years. The comet was not seen again at its subsequent returns to the vicinity of the earth and sun in 1899 and 1932. However, it was recognized that the comet shared a common orbit with the Leonid meteor stream which produces the November meteors. It had turned out that the Leonid meteors also had a period of 33 years, and the maxima were found to coincide with times when the comet was close to the sun. It was also pointed out by Hind (*2*) that the comet seen in 1366 by Chinese astronomers may have been an earlier appearance of the 1866 comet. Observations of the 1366 comet have been described in a recent translation (*3*) and permit the determination of an approximate orbit (*4*). The proposed identity is favored by this orbit and also by the fact that a strong shower of meteors was reported at almost the same time (*5*). Although the identity offered a possibility for correcting the uncertain orbital period in 1866, no successful use of this could be made in former times on account of the enormous amount of computation needed.

Nowadays the numerical integration of the system of differential equations, which contains the attractive forces of the major bodies in the solar system, can be done on an electronic computer. So it is possible to compute the motion of a small body over a long time interval. Brian Marsden (*6*) made use of this; his predictions of the returns of seven long-lost comets have already led to two successful recoveries.

B. G. Marsden drew my attention to the 1965 return of Comet Tempel-Tuttle and to the opportunity for using the supposed identity for a reliable prediction. The problem consisted in selecting from the permissible values of the orbital period for 1866 the correct one, which represents not only the observations of that year, but also the Chinese observations of 1366. Several values had to be adopted for the period, and for each of them a backward computation by numerical integration was necessary over the interval of five centuries. A special computer program had already been provided by

P. Stumpff and me at the Astronomisches Rechen-Institut, Heidelberg. This program is independent of tables for the planetary coordinates. A description of this program, which led to the successful recovery of the comet, is now presented.

In a program for the n-body problem only the starting values of the rectangular coordinates and velocities of the planets must be known for some moment. Corresponding values are also needed for the one or several small bodies being considered. The coordinates of the large and small bodies, the sun being one of the former, can then be evaluated by numerical integration over any given interval of time. The basic equations used are differential equations of the second order, and they do not contain the time and the velocities explicitly. They have a common form for both the large and small bodies, as only Newton's law of gravitation is considered. No use is made of the approximately elliptic motion of bodies in the solar system. The problem of predicting the motion of n bodies in a case like this over any length of time is called the n-body problem. There is no general solution known for $n > 2$, whereas for $n = 2$ Kepler's laws hold. The mass values of the small bodies will be taken as equal to zero in our case. All the bodies are treated as mass points in an n-body problem.

The integration method used to obtain the solution of the problem numerically consists in stepping forward by small time intervals in the positive or negative direction of time. Thousands of steps are necessary to integrate over a century. The powerful Adams-Störmer extrapolation method for second-order differential equations (*7*) can be used here according to the special form of the basic equations. A method for numerical integration like this makes use of a given number, m, of preceding steps, for which the coordinates and attractions already have become known, to obtain these values for the next step. Formulas of this type have been used in astronomical problems for many years. A special method had to be developed to obtain the m first steps of the integration, in order that the main calculation could be started. A twofold integration of a Lagrange-type interpolation formula and an iterative process were used for this. A disadvantage of the method is the use of a constant step length in the time. But a great advantage is that the attractions are computed from the differential equations only once per step.

The numbers m and n and the step length h are optional in the program. Values of h such as 1, 2, 5, 10, or 40 days were used in applications generally, with m equal to 11 or 12. If h is chosen sufficiently small, a high accuracy will be obtained, and there is no danger of instability of the method (see Rutishauser, *8*). Effects of instability will only show up after a great many steps, but they grow exponentially. A step of integration is connected with a smaller error if m is increased while h is kept at a small value. But m must not be too large, as otherwise the method will become unstable for reasonable values of h.

A detailed explanation of the method and of the program has been made (*9*), together with starting values of the major planets for an epoch in the middle of this century. One set of starting values was derived for the five outer planets, beginning with Jupiter, as these were treated earlier as an n-body problem (*10*). The sum of the masses of the sun and the inner planets is assumed to be concentrated at their barycenter. A second set of starting values was prepared for all the major planets except Mercury.

In the application of the program to Comet Tempel-Tuttle only the outer planets (excluding Pluto) were taken into account. Thus it was possible to use a step length of 5 or 10 days. But it was to be expected that neglect of the attraction of the earth in particular during close approaches would cause some uncertainty in the results.

The n-body problem given by the sun, the planets from Jupiter to Neptune, and the set of possible orbits of the comet was then treated. The motion of the bodies was computed backward from 1866 to 1366 on the IBM 7090 computer of Deutsches Rechenzentrum, Darmstadt. An orbit could then be interpolated that agreed with the observations in both 1866 and 1366. The ancient observations were not accurate enough, however, to supply a final proof of the supposed identity. Thus it was important that the comet be recovered in 1965. Before this was done, however, it became evident that the comet would have been in a favorable position for observations from the earth in 1699.

I have noted an observation by G. Kirch in Guben, Germany, of a comet on the morning of 26 October 1699 (*11*). Within the limits of the expected uncertainty, this observation can be well represented by the orbit of Comet Tempel-Tuttle.

Finally the comet was recovered on plates taken in the summer of 1965 at the Boyden Observatory in South Africa and with the 48-inch (120-cm) Schmidt telescope at Mount Palomar in California. The comet did not come very close to the earth at this time, so that only faint images could be expected. (At great distance the deviations from the predicted position in the sky would also be small.) The comet was detected 3° northeast of its predicted position, and it was fainter than predicted. Positions of the comet were measured on the recovery plates (see *12*).

Now Comet Tempel-Tuttle is on its way back to the outer parts of the solar system. It will not come in for another visit to the sun before the last years of this century. Although the comet has gone, increased activity of Leonid meteors was observed in November 1965, and it may be expected that this will be repeated to some degree in 1966.

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References and Notes

1. T. Oppolzer, *Astron. Nachrichten* **68**, 241 (1867).
2. J. R. Hind, *Monthly Notices Roy. Astron. Soc.* **33**, 48 (1873).
3. Ho Peng Yoke, *Vistas Astron.* **5**, 127 (1962).
4. S. Kanda, *Japan. J. Astron. Geophys.* **10**, 106 (1932).
5. v. Boguslawski, *Astron. Nachrichten* **17**, 103 (1840); A. v. Humboldt, *Kosmos* **1**, 398 (1845); *ibid.* **3**, 621 (1850); H. A. Newton, *Amer. J. Arts Sci., Ser. 2* **37**, 377 (1864); *ibid.* **38**, 53 (1864).
6. B. G. Marsden, *Astron. J.* **68**, 795 (1963).
7. L. Collatz, *Die Numerische Behandlung von Differentialgleichungen* (Springer, Berlin, ed. 3, 1960).
8. H. Rutishauser, *Z. Angew. Math. Phys.* **3**, 65 (1952).
9. J. Schubart and P. Stumpff, in preparation.
10. W. J. Eckert, D. Brouwer, G. M. Clemence, *Astron. Papers Washington* **12** (1951).
11. C. Kirch, *Miscellanea Berolinensia (Akad. Wiss. Berlin)* **5**, 49 (1737).
12. Central Bureau of Astronomical Telegrams, Intern. Astron. Union, Circulars No. 1926 and 1936 (1965).
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