these and observations of sudden magnetic storm commencements, geomagnetic bays, and solar flare effects, it should be possible to study the distribution of conductivity within the earth and thereby "to say something about the temperature distribution and the phase transition supposed at a depth of a few hundred kilometers." However, Rikitake "is of the opinion that in view of the resolving power of existing analyses of transient geomagnetic variations, we can only say that the increase in the conductivity at a depth of several hundred kilometers is very steep."

The last three chapters deal with the effect of the ocean on geomagnetic variations, the advantages and limitations of the "magneto-telluric" method, and local anomalies of geomagnetic variation that indicate lateral inhomogeneities of electrical conductivity within the earth's crust and mantle.

The book can be highly recommended as a clearly presented, comprehensively documented, and well-illustrated summary of the results of the current research in geomagnetism and electromagnetism relating to the interior of the earth.

JAMES R. BALSLEY
Department of Geology,
Wesleyan University,
Middletown, Connecticut

Studies in Mathematics Series

Volume 1 in the Mathematical Association of America's "Studies in Mathematics" series, *Studies in Modern Analysis*, was published in 1962. Volume 3, **Studies in Real and Complex Analysis** (Prentice-Hall, Englewood Cliffs, N.J., 1965. 221 pp., $4), edited by I. I. Hirschman, Jr., might be considered a continuation of volume 1, for it makes contact with the subject matter of that book at several points. The second volume, *Studies in Modern Algebra*, was concerned with quite different mathematical material.

In the introduction to volume 1, it was stated that the purpose of the series is "to bring to the general mathematical community expository articles at the collegiate and graduate level on recent developments in mathematics and the teaching of mathematics" in order to help overcome "the communication barrier which has arisen as a natural consequence of the tremendous acceleration in mathematical development that has taken place especially in the last twenty-five years."

If "mathematical community" is interpreted as referring solely to pure mathematicians and collegiate and graduate students of pure mathematics, then these volumes are a great success. The exposition is quite uniformly clear, occasionally delightfully exciting (to a mathematician), and excellent in lending historical orientation and showing the remarkable interplay that frequently exists between seemingly different branches of mathematics.

There is no reason why a particular series of mathematical expositions *should* be addressed to the wider community of scholars and scientists concerned with the applications of mathematics. Pure mathematicians do indeed have a communication—and subject-matter—barrier to overcome, even among themselves. I only want to inform the applied mathematician that he should expect to find applicable, but only very slightly applied, mathematics here.

The editor of the present volume has wisely left to the contributors decisions about how much mathematical background should be assumed, the extent to which details should be given, and so on. Accordingly, there is a wide variation in the length, depth, and style of the articles. Each is useful in its own way and should be judged for what it is, not for what it is not.

H. J. Bremermann traces the interesting and sometimes surprising history of the theory of functions of several complex variables from its inception at the turn of the century. Lawrence M. Graves deals with nonlinear functions from one Banach space to another, in particular with the implicit-function theorem; writers of texts on advanced calculus should perhaps take note. Einar Hille gives a brief introduction to semigroups, with some indication of their many mathematical and physical applications. In a joint article, I. I. Hirschman, Jr., and D. V. Widder discuss the genesis of the real inversion formulas of the Laplace and Stieltjes transforms and show their relation to totally positive matrices and variation-diminishing transforms. H. H. Schaefer gives a detailed treatment of the Lebesgue-Stieltjes integral, in which the theory of measure appears as a byproduct at the end rather than as a basic tool at the beginning. Guido Weiss presents the main aspects of classical harmonic analysis and some of the modern theory. Finally, Harold Wisdom discusses the history of a rather special problem, the inversion of semi-infinite Toeplitz operators, showing incidentally how several branches of analysis can become involved in the solution of a single problem.

E. F. BECKENBACH
Department of Mathematics,
University of California, Los Angeles

Ionospheric Research

The literature dealing with ionospheric research is so large and continues to grow at such a rapid rate that monographs summarizing our knowledge must be gratefully welcomed. R. C. Whitten and I. G. Poppoff have performed a useful service in providing this compact book, **Physics of the Lower Ionosphere** (Prentice-Hall, Englewood Cliffs, N.J., 1965. 240 pp., $7.50). By restricting their volume to that portion of the atmosphere within the height range from 50 to 150 kilometers, they have avoided treatment of many complex studies of the ionospheric F-region and the magnetosphere, and in considering the lower ionosphere, their point of view leans heavily towards a description of the physical and chemical properties of the atmosphere and of the ionization and recombination processes that occur in response to solar radiations.

A summary of the nature of the solar ionizing emissions precedes chapters that treat the pressure and density of the atmosphere and its chemical properties. The production and recombination of ionization are then considered, and material on measurements of atmospheric parameters and ionization profiles is presented. Electromagnetic propagation is discussed in terms of the conductivity equations, with emphasis on collision frequencies. The final chapter provides a brief description of sudden ionospheric disturbances, polar cap absorption, auroral effects, and the perturbations produced by nuclear bursts. Perhaps the following technical words chosen at random from throughout the book will give an impression of the contents: auroral temperature; bombardment; conductivity; photoionization; emission; polarizability; recombination; concentration; particle; collector plates; affinity; data; flux; and photons.

The list of references includes more than 400 items, most concerned with topics close to the author's own in-

terests in reaction rates and ionization processes. The coverage of areas outside of their interests is sparse. The value of the book lies mainly in the treatment of radiations, ionizing processes, and closely related subjects. On peripheral topics, such as atmospheric motions, the treatment tends to be less penetrating. The typography, illustrations and style of writing are clear, and the monograph will be a useful introduction and reference work in the field.

L. A. MANNING
Electronics Research Laboratory,
Stanford University

Computability Theory

The present book, **Enumerability, Decidability, and Computability** (Academic Press, New York; Springer, Berlin, 1965. 255 pp., $9.75) is a translation by G. T. Herman and O. Plassmann of a work by Hans Hermes which was originally published in German in 1961. Hermes is well known for the lucidity of his exposition, and as one would expect, his book is a masterly introduction to the subject. It does not go far into the remoter reaches of its field, but what it does it does with great clarity and thoroughness. A beginning student will find most developments well motivated, and many points that the original papers tend to take for granted are explained in detail; the expert will find it useful for recalling details concerning fundamental theorems.

The book begins with a general discussion of algorithms and formal methods. The next three chapters deal with μ-recursiveness—that is, with functions generated in standard fashion by the primitive recursive operations and the μ-operation—and general recursive functions as defined by systems of equations. The fundamental theorems for these kinds of functions, due mostly to Kleene, are proved, including the equivalence of each type of computability with the preceding type. However, only everywhere-defined functions are treated; partial recursive functions are barely mentioned. There follows a chapter on the application of these results to the undecidability theorems which have become more or less standard. The final chapter treats a number of supplementary topics, including the universal Turing machine, the basic properties of enumerable and arithmetical predicates, brief sketches of alternate formulations of computability, and recursive analysis.

The chapter on general recursiveness is capable of improvement in one respect. There are some advantages to stating the rule of replacement without involving substitution (thus emphasizing that it deals only with constants), and allowing only one occurrence, and that on the right, to be replaced. This gives the evaluation of a function an algorithmic character. The equation scheme given for $\mu y(g(x, y) = 0)$, on page 126, seems clumsy, and in any event could not give an algorithm in the sense just explained, because two different equation schemes have the same left side. It is possible, however, to satisfy that requirement as well as those stated in the book; this is done by the following equation schemes, where h can be, for example, the h^* of the book (or g if one is not concerned about having Φ a total function), and δ, Φ are new auxiliary functions:

$$\delta(x,y,0) = x, \qquad \delta(x,y,z') = y,$$
$$\Phi(x,y) = \delta(y,\Phi(x,y'), h(x,y)),$$
$$f(x) = \Phi(x,0)$$

The translation has been very well done. However, there are a few Germanisms that might bother a reader who knows no German. One example is the translation of the German word "Paragraph" as "paragraph," whereas the German term, as used by Hermes, indicates a much larger section of the text than what we customarily call a paragraph. There are other such cases, and a number of misprints, but these should not bother a reasonably alert reader.

H. B. CURRY
Department of Mathematics,
Pennsylvania State University

Organic Synthesis

The author, Robert L. Augustine, states that his book, **"Catalytic Hydrogenation** [Dekker, New York, 1965. 200 pp., $8.75] is designed as a guide for those interested in using hydrogenation as a synthetic tool." For this purpose it is excellent.

The book presents (i) discussion of different types of apparatus available in many organic chemistry laboratories and provides operating instructions for this equipment; (ii) discussion of catalysts, the effect of variables such as temperature and pressure on the outcome of a hydrogenation, the choice of catalyst for the specific hydrogenation to be carried out, and the preparation of the common catalysts and (iii) recommended procedures for the hydrogenation and hydrogenolysis of functional groups. Although many references are included at the end of each chapter, this book is designed to provide sufficient information so that in most cases it will not be necessary to consult and study these articles before carrying out the hydrogenation experiment. I believe that the author has accomplished his objective.

Chapter 2, on apparatus and techniques, is limited, and the section on high pressure equipment somewhat out of date. Several equipment manufacturers have available stirred autoclaves which may be operated up to 7000 to 8000 pounds per square inch and which have better agitation and heat control than the rocker-type high pressure unit. Neither of the units described has any provisions for cooling. Because almost all hydrogenations are exothermic, in the apparatus described it is often necessary to carry out the reactions at the lowest possible temperature range in which the reaction can be obtained in order to control the temperature. In selective hydrogenations, I have found that this is neither convenient nor desirable. In addition, lengthening the hydrogenation time from minutes to hours may cause undesirable side reactions, particularly that of the hydrogenation product with the reactant.

As is customary in academic articles, safety is treated lightly and inadequately. Instead of merely mentioning that "some hydrogenations are highly exothermic," it would have been preferable to point out, in the safety section and in the detailed section on hydrogenation of functional groups, the places where special precautions should be taken. For instance, it should have been emphasized that catalytic hydrogenation of nitro groups must be carried out in a unit provided with cooling and at as low a pressure and temperature as possible, or a runaway reaction will probably occur. Other systems such as those in which telomerization as well as hydrogenation may occur also require special precautions.

In the discussion of catalyst and conditions (chapter 3), there is no mention of the effect of agitation on either rate or selectivity of the hydro-