

Spectral Distribution of Solar Radiation at the Earth's Surface

The spectral quality of sunlight, skylight, and global radiation varies with atmospheric conditions.

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The spectral distribution of the intensity of direct sunlight and scattered skylight and the variation of the intensity with time of day, season, latitude, altitude, and atmospheric condition are important for the distribution of plants and animals on earth, the weathering of natural and man-made materials, the climate of the earth, and illumination for human activity. Atmospheric conditions are widely variable; skies range from clear to overcast, and air ranges from clean to polluted or dusty, and from dry to moist. The direct rays of the sun, illuminating the earth's surface, may traverse a long or a short slant path through the atmosphere, depending on season, time of day, and location. It has been 25 years since Moon wrote his excellent paper (1) on this subject. Our knowledge of the extinction coefficients of the atmosphere, of atmospheric conditions, and of the extraterrestrial spectral distribution of sunlight is greater than it was 25 years ago. It is not my purpose here to attempt to express the spectral distribution of the solar radiation reaching the earth's surface for every combination of atmospheric conditions; rather, I shall attempt to give reasonable examples. For cloudless conditions the greatest variation in the spectral quality of direct sunlight is caused by changes in the concentration of aerosol or dust.

The Solar Constant

The total amount of radiation received from the sun, at normal incidence, on a surface outside the atmosphere at the mean solar distance of the earth is called the solar constant. There is little doubt but that this amount of solar radiation is slightly variable over time (the variation is less than 1.5 percent), but whether or not there is any systematic variation has not been determinable. At present the most generally accepted value for the solar constant is 2.00 calories per square centimeter per minute; it is based on the meticulous comparison by Johnson (2) of solar spectral data. The calculations presented here are based on Johnson's data for the spectral distribution of extraterrestrial sunlight. For many years the Smithsonian Institution was the trustee of solar data in the United States and had concluded (3) that the solar constant is $1.9408 \text{ cal cm}^{-2} \text{ min}^{-1}$. Moon (1) had further reduced the value for the solar constant which he used in his work to $1.896 \text{ cal cm}^{-2} \text{ min}^{-1}$; this reduction was based on a correction believed accurate at that time. Thus Moon's values for extraterrestrial sunlight are 5.0 percent lower than those used here.

Large fractional changes occur in the intensity of very-high-frequency solar radiation in the x-ray and far-

ultraviolet regions (4). These fluctuations do not contribute substantially to the total energy radiated, hence the solar constant does not exhibit apparent variations throughout the solar cycle. The extraterrestrial radiation at normal incidence at any instant will differ from the solar constant because of the variation in the earth-to-sun distance throughout the year. This results in a maximum variation of irradiation of ± 3.5 percent.

When observed with very high spectral resolution, the extraterrestrial solar emission is seen to contain enormous detail of emission lines and Fraunhofer absorption lines superposed on an emission continuum. Such detail is of interest to astrophysicists concerned with the solar atmosphere and to geophysicists concerned with the atomic processes of the earth's upper atmosphere and has been adequately discussed elsewhere (5).

Atmospheric Extinction

The attenuation of solar radiation is caused by scattering and absorption by the atmosphere. Molecular or Rayleigh scattering is caused by nitrogen, oxygen, and other molecular components of the atmosphere where the scattering particle is small compared with the wavelength of the radiation. Large-particle or Mie scattering is caused by dust, aerosols, water droplets, and other particles of diameter comparable to the wavelength of the radiation. Absorption in the ultraviolet is caused by ozone, which produces an abrupt termination of the solar energy reaching the earth's surface at a wavelength near 2900 angstroms. The long-wavelength radiation of the solar spectrum is attenuated more gradually by a series of absorption bands in the infrared, caused by water vapor, car-

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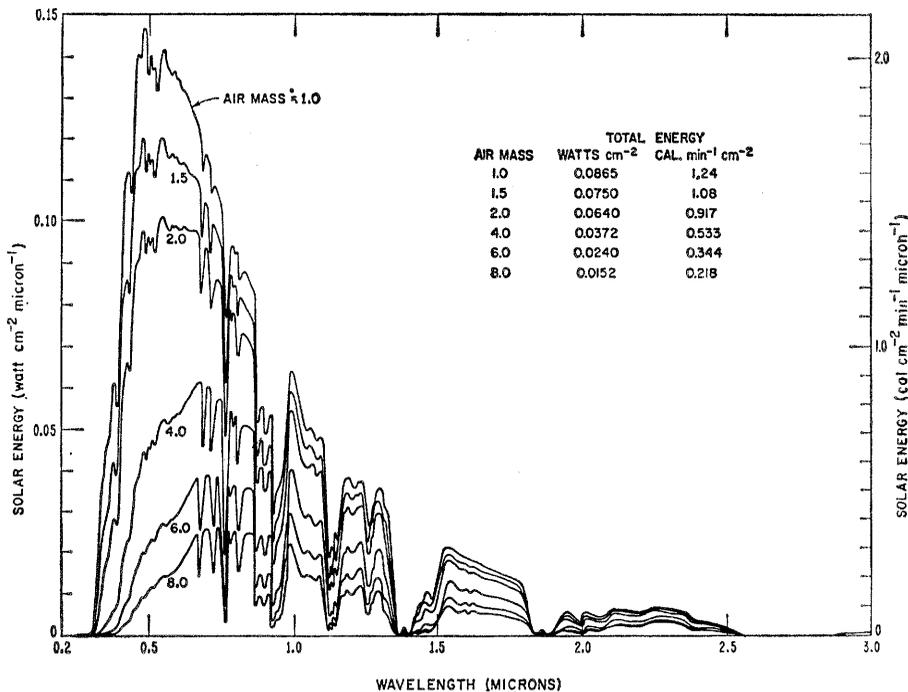


Fig. 1. Spectral distribution as a function of wavelength of direct solar radiation incident at sea level on a surface perpendicular to the sun's rays for slant paths of air mass 1.0 to 8.0. Concentration of precipitable water, 10 millimeters; of aerosol, 200 particles per cubic centimeter; of ozone, 0.35 centimeter.

bon dioxide, ozone, and other minor constituents. The transparent windows of the atmosphere, or spectral intervals where there is little absorption, constitute an extremely narrow slot in the entire electromagnetic spectrum and exist because of a distinct frequency separation of the absorption processes of nature. The short-wavelength, high-frequency absorption bands are associated with electronic transitions, and the long-wavelength, low-frequency absorption bands are associated with transitions in vibrational and rotational states of atoms within molecules. These different transitions seldom overlap in energy, and for the relatively simple gases comprising the earth's atmosphere a convenient frequency gap in the visible region exists through which sunlight streams to warm and illuminate the earth's surface. Through gaps in the absorption band associated with vibrational and rotational transitions in atmospheric gases at infrared wavelengths, the surface radiates energy to space.

The ozone-produced absorption in the ultraviolet consists of a broad system of bands made up of the Hartley bands, from 1800 to 3400 angstroms, and the overlapping Huggins bands, from 3200 to 3600 angstroms. In the visible spectrum ozone produces a very weak absorption from 4400 to 7400

angstroms (the Chappuis bands). Molecular oxygen produces a strong narrow absorption band in the red at 7596 angstroms and one in the infrared at 1.27 μ . Molecular oxygen dissociates when irradiated with wavelengths shorter than 2400 angstroms and produces complete absorption by the atmosphere. The complete absorption found between 2400 and 2000 angstroms and known as the Herzberg bands is caused by the forbidden transitions in molecular oxygen. The very intense Schumann-Runge bands found below 2000 angstroms are produced by molecular oxygen. Thus, ozone and oxygen share the burden of completely absorbing the solar ultraviolet radiation of wavelength shorter than 2900 angstroms before it reaches the lower atmosphere.

The term *transmission* is used here to mean the fraction of energy incident at the outer extremity of the earth's atmosphere which penetrates the atmosphere to the earth's surface along a slant path. The monochromatic slant-path transmission is written, from Lambert's law, as follows:

$$\lambda T = e^{-\lambda \tau m} \quad (1)$$

where $\lambda \tau$ is the "monochromatic extinction optical thickness" (defined below) at wavelength λ along a vertical path from the earth's surface to space,

and m is the air mass, or ratio of the length of the slant path to the length of the vertical path. For solar zenith angles, z , of less than 72 degrees, the air mass is given adequately by $m = \sec z$. For larger zenith angles, correction for atmospheric refraction of the ray must be made according to the values given by Bemporad (6).

The monochromatic extinction optical thickness is the sum of the optical thickness due to Rayleigh scattering, $\lambda \tau_r$, to Mie scattering, $\lambda \tau_a$, to selective absorption by ozone, $\lambda \tau_o$, and to selective absorption by water vapor, carbon dioxide, and other minor constituents, $\lambda \tau_m$. Hence:

$$\lambda \tau = \lambda \tau_r + \lambda \tau_a + \lambda \tau_o + \lambda \tau_m \quad (2)$$

Each of these optical thicknesses may be written as the product of the absorption or scattering coefficient and the amount of the particular constituent in the path integrated from the surface to space, as follows.

1) Rayleigh scattering due to air molecules:

$$\lambda \tau_r = \int_0^{\infty} \sigma_r N_r(h) 10^5 dh \quad (3)$$

where $N_r(h)$ is the molecular number density (per cubic centimeter) at altitude h and σ_r is the Rayleigh scattering cross section (in square centimeters) expressed as:

$$\sigma_r = \frac{8 \pi^3 (n_s^2 - 1)}{3 \lambda^4 N_s^2} \frac{6 + 3\delta}{6 - 7\delta} \quad (4)$$

where λ is the wavelength (in centimeters); n_s is the index of refraction of air at standard conditions of temperature and pressure (15°C and 1013 millibars); N_s is the molecular number density (per cubic centimeter) for a standard atmosphere; and δ is the depolarization factor caused by the anisotropy of the air. Values of σ_r have been calculated by Penndorf (7) for atmospheric conditions and for wavelengths from 0.2 micron to 20.0 microns.

2) Mie scattering due to dust, aerosols, water droplets, and so on:

$$\lambda \tau_a = \int_0^{\infty} \beta_a \frac{N_a(h)}{N_a(O)} dh \quad (5)$$

where $a(O)$ is the aerosol attenuation coefficient (per kilometer) at sea level for a meteorological range of 25 kilometers. The range of 25 kilometers is recommended by Elterman (8) in his definition of a "clear standard atmo-

sphere." $N_a(O)$ is the aerosol number density (per cubic centimeter) at sea level for a meteorological range of 25 kilometers. $N_a(h)$ is the aerosol number density (per cubic centimeter) as a function of altitude h .

3) Attenuation due to ozone:

$$\lambda \tau_3 = \int_0^{\infty} \alpha_3 D_3(h) dh \quad (6)$$

where α_3 is the absorption coefficient (per centimeter) due to ozone and $D_3(h)$ is the ozone equivalent thickness or concentration (in centimeters per kilometer).

4) Attenuation due to water vapor, carbon dioxide, or other molecular constituents:

$$\lambda \tau_m = \int_0^{\infty} \alpha_m D_m(h) dh \quad (7)$$

where α_m is the selective absorption coefficient (per centimeter) due to water vapor, carbon dioxide, or other constituents and $D_m(h)$ is the concentration of these constituents, in centimeters per kilometer.

The values used here for Rayleigh scattering cross section are those calculated by Penndorf (7), and the values used for molecular density with altitude, $N_r(h)$, are those listed for the U.S. Standard Atmosphere, 1962 (9). The ozone absorption coefficients used here are those established by Vigroux (10). Recently Elterman (8) has compiled the attenuation data appropriate to a model termed the "clear standard atmosphere." The ozone and aerosol distributions used here are based on Elterman's report. He recommends use of the value 0.35 centimeter at normal temperature and pressure for total ozone distribution. This is the annual mean value at midlatitudes. In order to establish the variation in direct solar radiation at the earth's surface caused by changes in the total amount of ozone, a calculation was made for a total of 0.20 centimeter, which is approximately the minimum amount found at midlatitudes. At equatorial latitudes total amounts of ozone are sometimes as low as 0.15 centimeter (see 11). The aerosol distribution recommended by Elterman is based on the observations of Curcio, Knestruck, and Cosden (12), Penndorf (7), and Chagnon and Junge (13). Two concentrations are used for the troposphere: one for a surface concentration of 200 particles per cubic

Table 1. Listing of air masses used for the calculations on which Figs. 1 and 2 are based; the corresponding angles, including a correction for atmospheric refraction; and the cosine of the angle.

Air mass	Angle	Cosine
1.0	0	1.000
1.5	48°11'	0.667
2.0	60°	.500
4.0	75°31'	.250
6.0	80°42'	.167
8.0	83°12'	.125
10.0	84°48'	.091

centimeter and one for 800 particles per cubic centimeter. Elterman (8) has determined the aerosol attenuation coefficients for the ultraviolet and visible regions on the basis of the observations of Baum and Dunkelmann (14). For an aerosol concentration, at the surface, of 200 particles per cubic centimeter, aerosol scattering predominates over molecular scattering and molecular scattering becomes negligible at red and infrared wavelengths. This is only true near the surface on very clear days. At an altitude of 2 to 4 kilometers above the surface, molecular scattering dominates, because of the rapid decrease in aerosol particle density with height. However, as both the molecular and the aerosol densities

diminish with height, the ozone density increases and dominates the attenuation at a height of 22 kilometers and above (15).

In the near-infrared, absorption by water vapor and absorption by carbon dioxide dominate the attenuation of radiation. These absorptions, which collectively form absorption bands, are the result of quantum transitions in vibrational and rotational states of the molecules. Each absorption band is comprised of hundreds of narrow individual absorption lines, which merge to form the band. Even in the windows between the bands, the wings of these numerous lines contribute a definite amount to the continuum extinction. The extinction coefficients for the infrared absorption bands and for the windows between the bands have been derived by Gates and Harrop (16) and by Gates (17) from direct observation of the solar spectrum. These values are expressed as a function of the total amount of water vapor, in the zenith direction, between the surface of the earth and space, known as the "precipitable water." The amount of precipitable water, in centimeters, is the thickness of liquid water which would be formed if all the water in the zenith

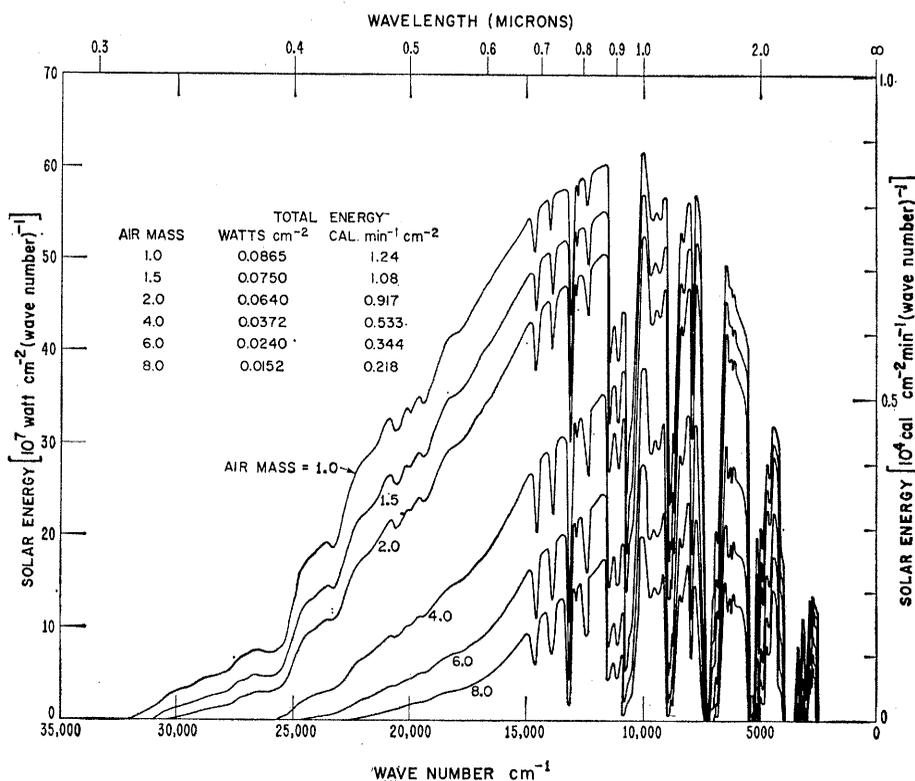


Fig. 2. Spectral distribution as a function of wave number (frequency) of direct solar radiation incident at sea level on a surface perpendicular to the sun's rays for slant paths of air mass 1.0 to 8.0. Atmospheric conditions same as for Fig. 1.

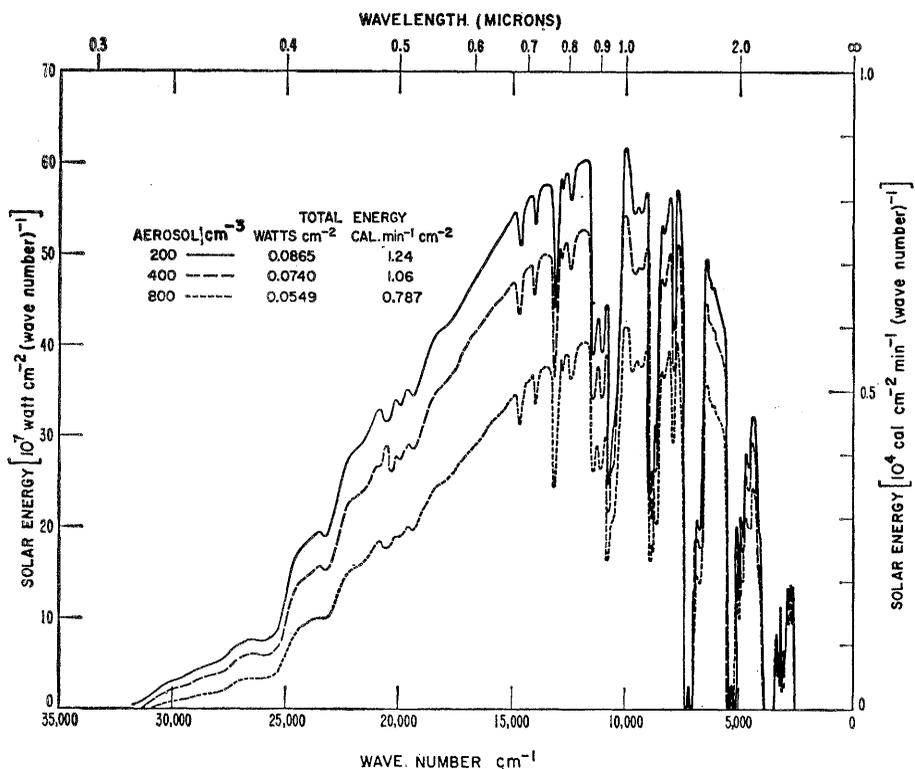


Fig. 3. Spectral distribution as a function of wave number (frequency) of direct solar radiation incident at sea level on a surface perpendicular to the sun's rays for various amounts of atmospheric aerosol concentration at the ground, for sun at the zenith. Air mass, 1.0; concentration of precipitable water, 10 millimeters; of ozone, 0.35 centimeter.

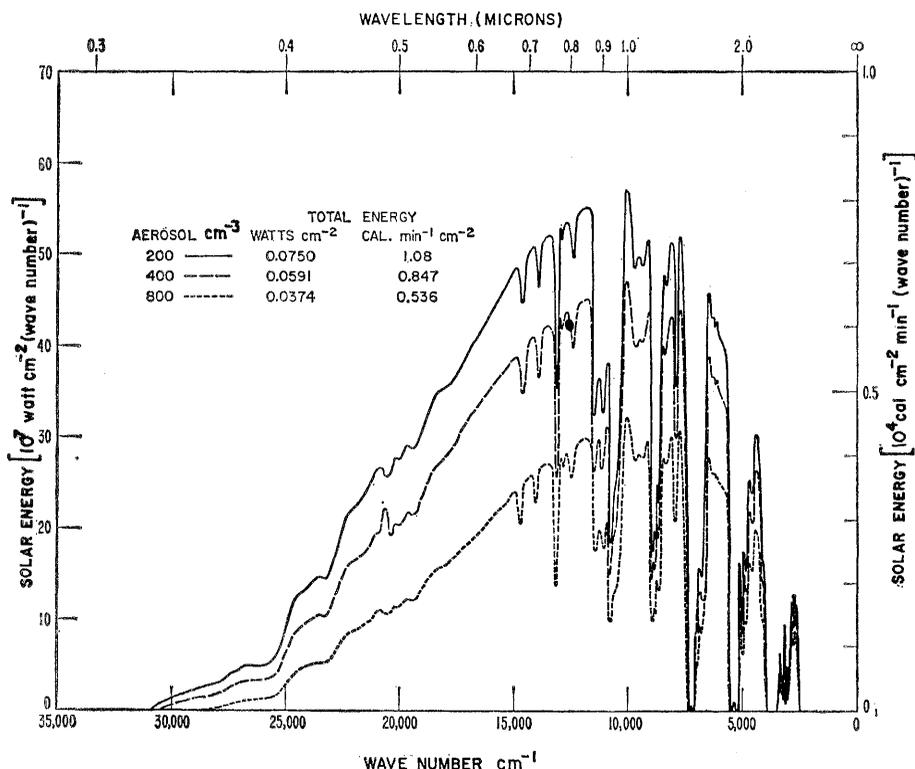


Fig. 4. Spectral distribution as a function of wave number (frequency) of direct solar radiation incident at sea level on a surface perpendicular to the sun's rays for various atmospheric aerosol concentrations at the ground for a slant path of air mass 1.5 (zenith angle, $48^{\circ}11'$). Concentrations of precipitable water and of ozone, same as for Fig. 1.

direction were condensed at the surface. Gates (18) has reported on the amounts of precipitable water found above a few localities on the earth's surface. An extremely dry atmosphere may contain as little as 0.1 centimeter of precipitable water, and a very humid atmosphere, as much as 3.0 centimeters. A very frequently occurring amount is 1.0 centimeter at sea-level locations. The amount of water vapor in the sky is one of the most variable of all the atmospheric constituents, and therefore the infrared solar radiation at the surface varies considerably. The infrared extinction is comprised of two parts: a continuum extinction at all frequencies, caused by scattering and the wings of water-vapor and carbon dioxide absorption lines, and a selective absorption within each individual line contributing to the total absorption band. For a very clear atmosphere the continuum extinction values obtained by Gates (17) are 1.21×10^{-3} per centimeter at 1.54μ and 1.075×10^{-3} per centimeter at 11.08μ , expressed in terms of the amount of precipitable water, in centimeters.

The selective absorption coefficients, C_1 , used for the calculations are based on the following approximation to the absorption law for atmospheric water vapor:

$$\ln T = -C_1 w^3 \quad (8)$$

where T is the fractional transmission and w is the amount of water vapor, in millimeters. This absorption law is not strictly valid throughout the infrared, as there are narrow regions where absorption by gases other than water vapor predominates. When this is true, another absorption law and appropriate coefficients are used for the calculations (19).

Spectral Distribution of Direct Solar Radiation

The spectral distribution of solar radiation may be presented in two forms — monochromatic intensity (i) versus wavelength or (ii) versus frequency or wave number. The wavelength plot (see Fig. 1) is the more familiar form, but it has an inherent difficulty in that the scale gives disproportionate value to the infrared, where the energy is diminishing but where the wavelength is continuing

to larger numbers. The frequency or wave-number plot (see Fig. 2) is a much more self-contained plot, in which the infrared wavelengths rapidly converge toward zero frequency. They are readily contained within the graph, and the ultraviolet or high-frequency end of the scale is terminated by the complete absorption due to the atmosphere. On the wavelength plot the monochromatic intensity is given in watts per square centimeter per micron, or in calories per square centimeter per minute per micron. The unit of wave number is the reciprocal of the wavelength and therefore is expressed in cm^{-1} . A wave number is proportional to the frequency, the two being related by the velocity of light. The wavelength, λ , the frequency, ν , and the velocity of light, c , have the relationship $\lambda\nu = c$, and $\lambda = 1/\bar{\nu}$ where $\bar{\nu}$ is the wave number. Hence $\bar{\nu} = \nu/c$. It is important to understand the procedure for converting from a wavelength scale to a wave-number scale, or vice versa. The quantity λE represents the flux density, F , per unit increment of wavelength, and $\bar{\nu} E$, the flux density per unit wave-number increment. A wavelength increment, $\Delta\lambda$, is related to the wave-number increment, $\Delta\bar{\nu}$, as follows:

$$\Delta\lambda = -\frac{1}{\bar{\nu}^2} \Delta\bar{\nu} \quad \text{or} \quad \Delta\bar{\nu} = -\frac{1}{\lambda^2} \Delta\lambda \quad (9)$$

Then, since

$$\lambda E = \frac{F}{\Delta\lambda} \quad \text{and} \quad \bar{\nu} E = \frac{F}{\Delta\bar{\nu}} \quad (10)$$

it follows that

$$-\frac{1}{\bar{\nu}^2} \lambda E = \bar{\nu} E \quad \text{and} \quad \lambda E = \frac{1}{\lambda^2} \bar{\nu} E \quad (11)$$

The shape of the two displays of spectral energy are very different. Whereas in the wavelength plot the monochromatic intensity appears to peak in the green at 0.55μ , on the wave-number plot it peaks in the vicinity of $10,000 \text{ cm}^{-1}$, or 1.0μ . A broad maximum in the spectral distribution of solar energy depends on the method of plotting. The important quantity is the amount of energy within a wavelength or frequency interval. The area between two wavelength limits in the wavelength plot and the area between the corresponding wave-number limits in the wave-number plot must represent the same total amount of energy.

The spectral distribution of the di-

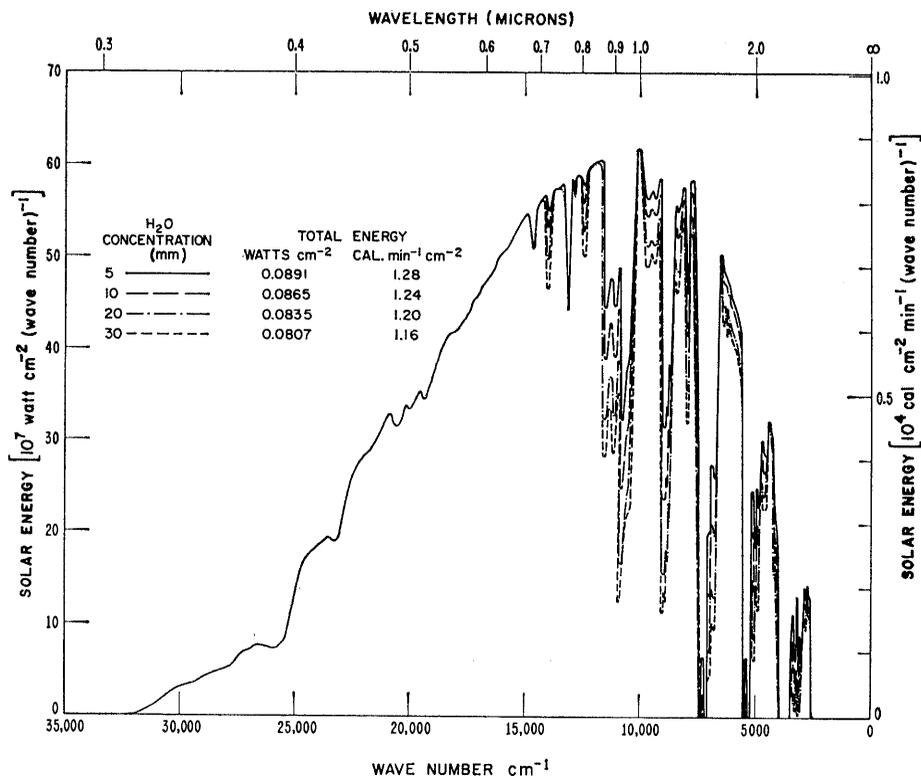


Fig. 5. Spectral distribution as a function of wave number (frequency) of direct solar radiation incident at sea level on a surface perpendicular to the sun's rays for various total amounts of atmospheric water vapor, for sun at the zenith. Air mass, 1.0; concentrations of aerosol and ozone, same as for Fig. 1.

rect solar radiation incident at sea level on a surface perpendicular to the sun's rays is given in Fig. 1 as a function of the wavelength for air masses from 1.0 to 8.0. The total amount of precipitable water vapor in the zenith direction is 10.0 millimeters, the aerosol

concentration at the surface is 200 particles per cubic centimeter, and the total amount of ozone is 0.35 centimeter. The total amount of direct solar radiation received is given by the area under each curve (the corresponding values are shown in Fig. 1 under "total

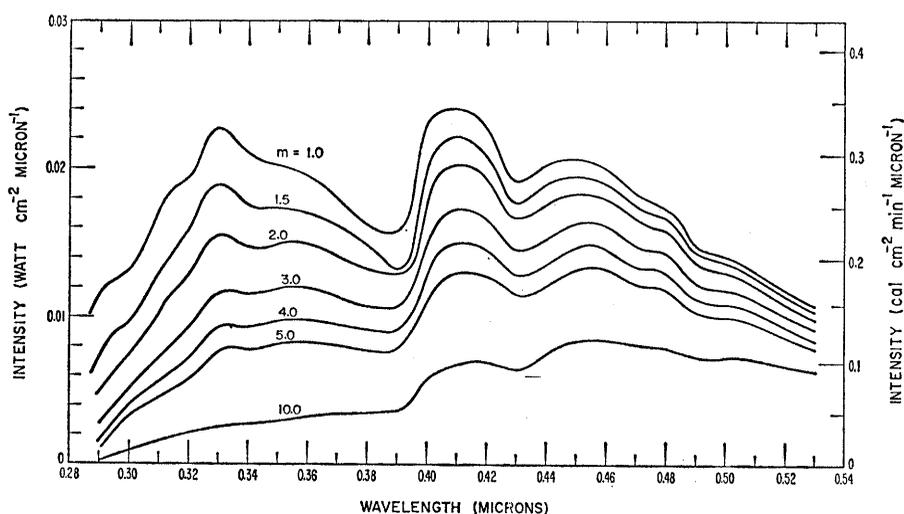


Fig. 6. Monochromatic intensity of skylight as a function of wavelength, for a sky in which Rayleigh scattering typically occurs, for solar-radiation slant paths corresponding to air mass 1.0, 1.5, 2.0, 3.0, 4.0, 5.0, and 10.0. Ground reflection with albedo = 0. [Data from Deirmendjian and Sekera (20)]

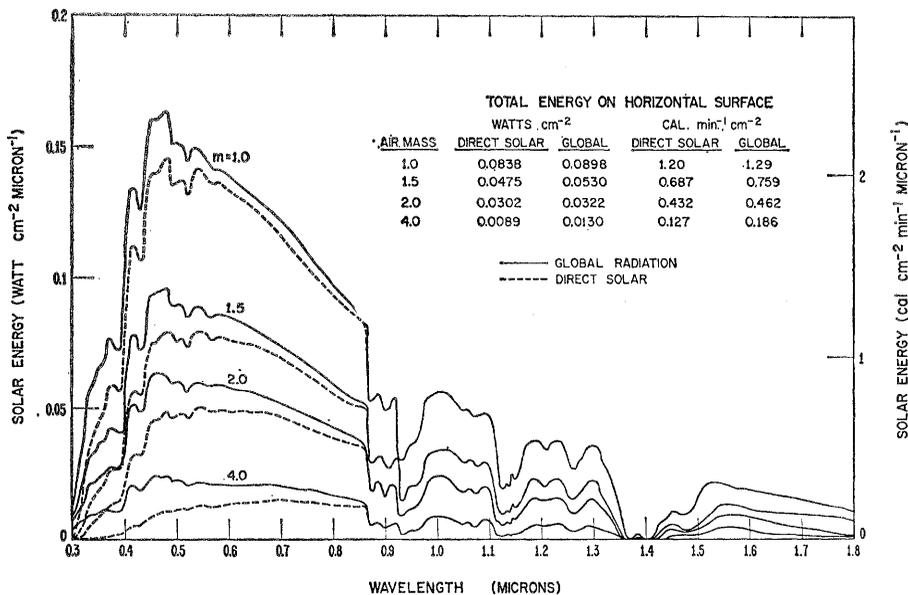


Fig. 7. Spectral distribution as a function of wavelength of the global and direct solar radiation incident at sea level on a horizontal surface, for various slant paths corresponding to air mass 1.0, 1.5, 2.0, and 4.0.

energy"). The spectral distribution of direct sunlight is shown in Fig. 2 as a function of the wave number for air masses from 1.0 to 8.0.

In order to calculate the amount of direct solar radiation received on a horizontal surface, one would multiply the values for "total energy" given in Figs. 1 and 2 by the cosine of the angle between the direction of the sun's rays and the zenith direction. For the air masses given in Figs. 1 and 2, Table 1 lists the corresponding angles and their cosines, with allowance made for atmospheric refraction.

The variation of the spectral distribution of direct solar radiation with aerosol and dust content is shown in Fig. 3 for the sun in the zenith, and in Fig. 4 for the sun at a zenith angle of 48°11'. The attenuation of solar radiation due to scattering depends very much on the particle size distribution. If the atmosphere is very dusty and filled with large particles, then the attenuation will be very much greater than it is for the essentially clear-sky conditions of Figs. 3 and 4.

Figure 5 shows the variation of the spectral distribution of direct solar radiation on a surface perpendicular to the sun's rays as a function of the total amount of water vapor in the path when the sun is in the zenith. The amounts of total precipitable water vapor will vary, normally, between 1.0 millimeter for very dry conditions and over 30.0 millimeters for a warm humid atmosphere. If the rays of the sun traverse a slant path of air mass 2.0, then a zenith-sun amount of precipitable water vapor of 20.0 millimeters will give a slant-path amount of 40.0 millimeters.

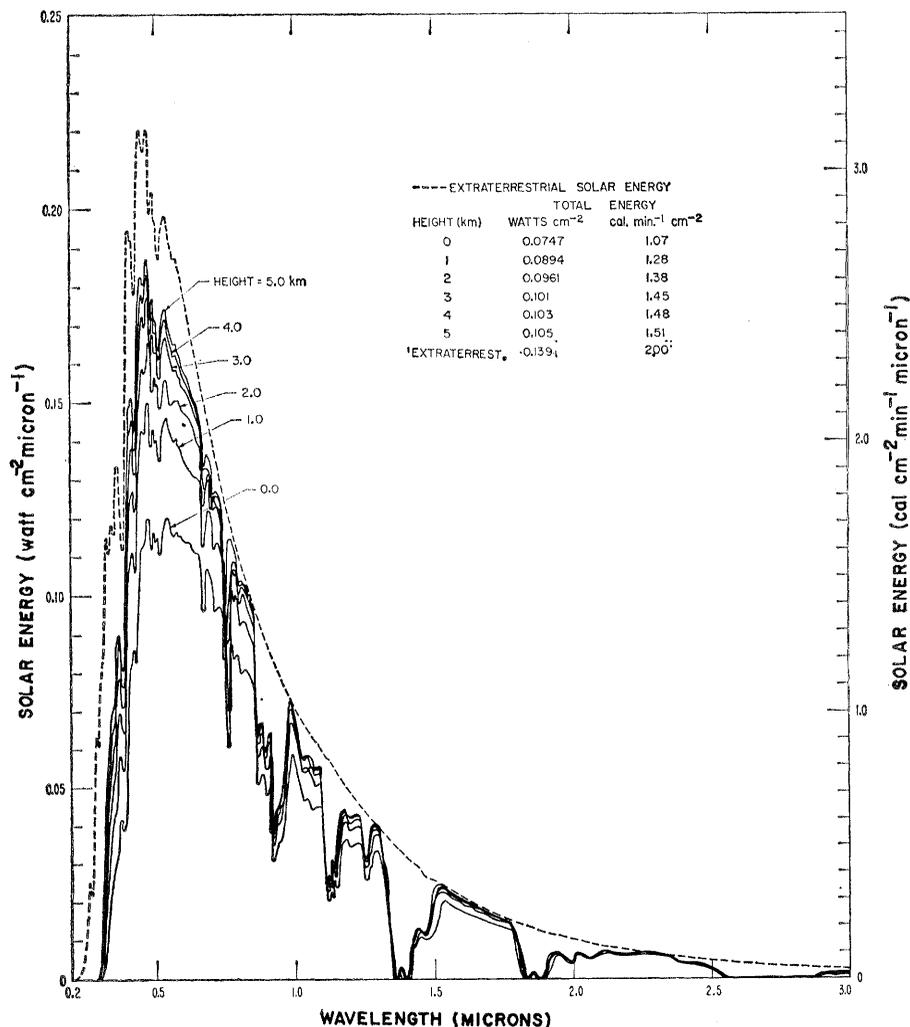


Fig. 8. Variation, with altitude, of spectral intensity versus wavelength for direct solar radiation perpendicular to the sun's rays for an air mass of 1.5. Concentration of precipitable water, 5.0 millimeters; of aerosol and ozone, same as for Fig. 1.

Spectral Distribution of Skylight

A very thorough analysis of the spectral distribution of scattered skylight for molecular (Rayleigh) scattering has been given by Deirmendjian and Sekera (20). The computation of multiple scattering of sunlight by a molecular atmosphere is mathematically very difficult, and precise results could not be obtained until Chandrasekhar (21) had developed his mathematical solutions. The diffuse sky radiation is partially polarized and illuminates a surface from all directions. The results obtained by Deirmendjian and Sekera for the amount of skylight

are used here. The intensity of diffuse sky radiation received on a horizontal surface from a perfectly scattering Rayleigh atmosphere is shown in Fig. 6. It may be seen that the maximum flux of sky radiation shifts toward longer wavelengths with increasing distance from the solar zenith. It is evident that sky radiation during most daylight hours is relatively rich in ultraviolet and blue wavelengths, with peak values occurring at 0.33 and 0.41 micron. A Rayleigh atmosphere occurs only on the clearest days, and the presence of aerosols and dust greatly enhances the scattering in the red. Deirmendjian and Sekera (20) show the effect of ground reflection on the intensity of the sky radiation. A surface reflectivity of 0.25 increases the intensity of scattered skylight by approximately 35 percent relative to the intensity of skylight above a black surface. This means that the quality of the surface—its being covered with vegetation, soil, or sand—strongly influences the downward flux of skylight.

Global Radiation

The sum of the direct-beam solar radiation and the scattered skylight received on a horizontal surface is referred to as the global radiation. If the direct-beam solar radiation incident on a surface perpendicular to the sun's rays is converted to that incident on a horizontal surface and combined with the flux of sky radiation, the curves shown in Fig. 7 result. Although the direct-beam solar radiation incident on a horizontal surface diminishes rapidly with increasing air mass, the flux of scattered skylight adds considerable energy in the ultraviolet and blue. The total energy associated with each curve is given in Fig. 7. These values for total energy include the small amounts in the long-wave infrared not shown in Fig. 7.

Altitudinal Change

The earth's surface ranges in altitude from below sea level to 29,028 feet (8848 meters) above sea level. It is of interest to discover the extent by which the spectral distribution of direct sunlight changes with altitude. As the altitude above sea level is increased, the total amount of atmosphere in the path of the sunlight diminishes and the total number of molecules of air, water vapor, carbon dioxide, or aerosol decreases. In our computations, the values for altitudinal distribution of ozone and aerosol concentrations and for molecular densities were taken from the report by Elterman (8). Since water-vapor concentration in the troposphere is so widely variable, a fixed amount of 5 millimeters of precipitable water was used in the computations. If lesser amounts of water vapor exist above any particular altitude, then Fig. 5 may be used for estimating the amount of attenuation. The variation of the spectral distribution of direct solar radiation with altitude is shown in Fig. 8.

It would be desirable to present the variation of skylight and global radiation with altitude; however, our calculations did not provide this information.

Summary

The spectral distribution of direct sunlight at the earth's surface at sea level is calculated as a function of the air mass and of the concentrations of aerosol, ozone, and water vapor. A solar constant of $2.0 \text{ cal cm}^{-2} \text{ min}^{-1}$ is used for these calculations. The spectral distribution of sunlight is presented on a wavelength basis and also on a wave-number basis. For sea level the spectral distributions of skylight and of global radiation incident on a

horizontal surface are given. Finally, the variation with altitude of the spectral distribution of direct sunlight incident on a surface perpendicular to the sun's rays is presented.

Information concerning the spectral distribution of sunlight is important for understanding the physiological response of plants and animals to sunlight, the climatology of the earth's surface, the weathering of natural and man-made surfaces, the utilization of solar energy, and many other processes. Not all atmospheric conditions could be considered, but it is hoped that the curves of Figs. 1–8 will provide a general guide as to the spectral distribution of sunlight for most clear-sky conditions.

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22. I thank Elaine Deutschman for programming the computations and for assisting with the manuscript.