Since the size of the Virgo cluster is comparable to the 10° by 10° beam of the horn antenna, a correction to Eq. 1 is needed because gas in the outer part of the antenna beam makes a smaller contribution to the antenna temperature than that in the center. This correction is determined from de Vaucouleurs' (*10*) plot of 212 galaxies in the region of the cluster and from a superimposed, calculated antenna pattern. If the distribution of gas is the same as that of the galaxies, then the sum of the relative antenna power responses for each galaxy divided by the number of the galaxies is the fractional reduction in antenna temperature that would be produced by moving the gas from the center of the antenna beam to the positions of the galaxies. The fractional reductions obtained at the three right ascensions are 0.284, 0.470, and 0.373. The right-hand side of Eq. 1 must be multiplied by the mean of the reciprocals of these fractional reductions or 2.78. Applying this correction and substituting the values $r = 10^7$ parsecs and $A_a = 1.1$ m^2 gives the formula

$$M = 3.47 \times 10^7\, T_a\, \Delta\nu \qquad (2)$$

which is applicable to the present observations of the Virgo cluster. If the hydrogen atoms have the same velocity spectrum as the galaxies in the cluster, the observed optical red shifts serve to estimate the form of the velocity spectrum. The measurements of Humason, Mayall, and Sandage (*11*) contain velocities for 45 galaxies within 5° of the adopted center $12^h\,28^m$, +13°. For these 45 galaxies the mean velocity of recession is 1130 km/sec or 5.4 Mc/sec while the effective width of the velocity spectrum is 1400 km/sec or 6.7 Mc/sec. Figure 2 does not show the presence of line emission from the cluster. The rms deviation of the nine points from a least-squares straight line is 0.012°K. I adopt twice this value as an upper limit to the antenna temperature caused by line emission from the cluster. Substitution of 0.024°K for T_a and 6.7 Mc/sec for $\Delta\nu$ into Eq. 2 yields 5.6×10^{12} solar masses as an upper limit to the atomic hydrogen in the Virgo cluster. For comparison, Holmberg (*2*) has obtained 3.1×10^{13} solar masses for the galaxies in the cluster, an estimate based on luminosities.

In conclusion, if the excitation temperature of the neutral hydrogen atoms in the Virgo cluster is much greater than the background continuum radiation, and if the angular distribution and velocity spectrum of the hydrogen are similar to those of the galaxies, then the mass of optically-thin neutral hydrogen does not make an important contribution to the dynamics of the cluster. If one takes the Virgo cluster to be a sphere whose center is 10^7 parsecs away and whose radius subtends an angle of 5°, a uniform density of 8.1×10^{-5} hydrogen atoms/cm^3 is equivalent to the upper limit found above. This density is about one order of magnitude higher than upper limits (*12*) that have been found for the density of neutral hydrogen atoms in the general intergalactic medium.

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Gravitational Lens Effect: An Observational Test

Abstract. *The large proper motion of the nearby star 40 Eridani-A will pass a distant star in 1988. It should be possible to observe the gravitational lens effect of this eclipse by existing photoelectric methods.*

In 1936, Einstein, in response to a suggestion by R. W. Mandl, reported that a nearby star passing in front of a distant one could create a lens-like effect which would intensify the image of the distant star (*1*). Some recent review papers by Liebes (*2*), Refsdal (*3*), and others have reopened discussion of the direct observation of this effect. Refsdal states that such events should occur rather frequently, and Liebes calculates that "as a consequence of the gravitational lens-like action of one star, it is possible for the image of another more distant star within our galaxy to be intensified by a factor in excess of 1000." However, there seems to be no record of any such event having been actually observed.

Because of the possibility that I (or the observatory) may not be around in 1988, it is the purpose of this report to draw attention to a possibly observable stellar eclipse that would be suitable for verifying the gravitational lens effect. During a study of double stars with large proper motions, carried out over the past 5 years with the 30-inch (75-cm) refractor at Allegheny Observatory, a star of about magnitude 15 was found directly in the path of 40 Eridani-A. The system 40 Eri consists of three stars—the pair BC has a period of about 250 years and revolves around A in about 8000 years (*4*). The proper motion of 40 Eri-A is 4.082″ in position angle 212.6°. If the background star X has no appreciable proper motion of its own (whether it does can be checked easily during the next 10 years), then the motion of 40 Eri-A will take it directly over the position of X in 1988; the resulting eclipse may be utilized to determine the gravitational lens effect (Fig. 1).

The distance and spectral type of star X are not known, but comparison of blue and red plates indicates that it is a normal star. For the rest of the discussion it is assumed to be a solar-type star at a distance of 1000 light years. The choice of 40 Eri-A is fortuitous since the parallax, spectral types, and masses of the three components are accurately known. [It is interesting to note but of no direct concern in this discussion that star B is the best-observed white dwarf for which the relativistic red shift has been accurately determined by Popper (*5*).]

Liebes shows that when a distant object star, O, lies directly in the path of a nearer star, D, called the deflector, the image of O undergoes a transformation, splitting into two ellipses when D approaches the cone of inversion whose angular radius is θ_o. For an impact parameter $\psi_o = 0$, that is, for perfect alignment of D and O, the ellipses grow into crescents on either side of D and eventually join to form a ring whose diameter is $2\theta_o$ and whose thickness is approxi-

Table 1. Data for pairs of stars. For explanation of symbols, see text.

Star	Mass (10^{33}g)	Distance (light year)	φ (radians)	θ_0 (radians)	Intensity gain A
O_1	1.99	4.0	1.84×10^{-8}	2.79×10^{-7}	60 (max)
D_1	1.99	8.0	0.92×10^{-8}		
O_2	1.99	0.5×10^{4}	1.47×10^{-11}	7.90×10^{-9}	2150 (max)
D_2	1.99	1.0×10^{4}	7.36×10^{-12}		
X	1.99	1.0×10^{3}	7.36×10^{-11}	1.63×10^{-7}	2* 10† 4430‡
40 Eri-A	1.4	16.3	4.52×10^{-9}		

* $\psi_0 = \theta_0$. † $\psi_0 = 10^{-1}\theta_0$. ‡ $\psi_0 = 0$.

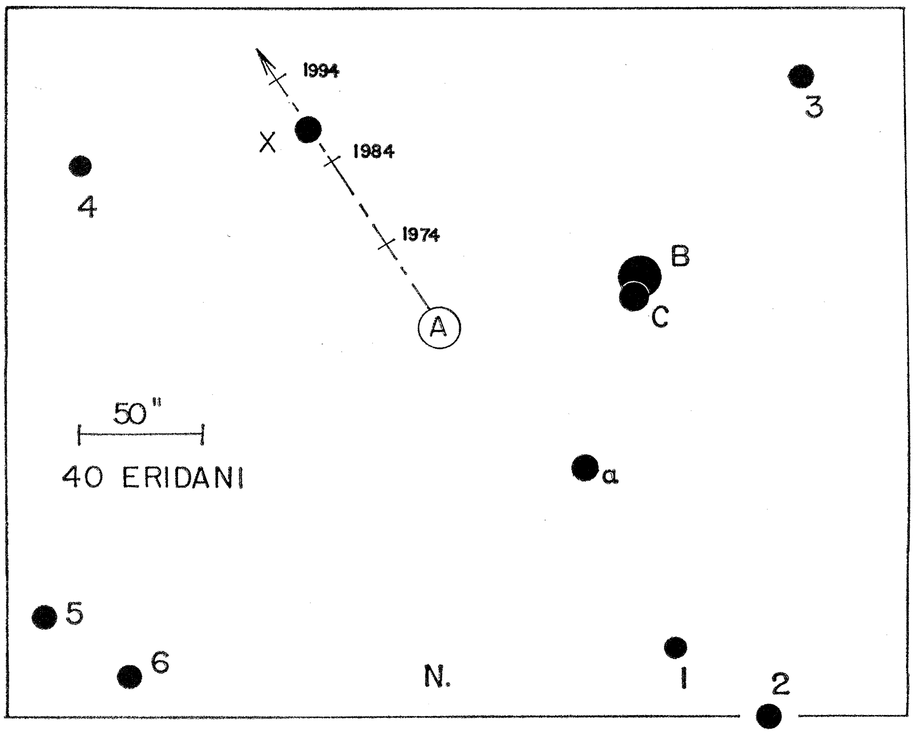

Fig. 1. Region of 40 Eridani. The arrow indicates proper motion of 40 Eridani-A from 1964 to 1994, crossing star X in 1988. Notation of stars A, B, C, and a is that of the Aitken double-star catalog (7); the other stars are not listed in the catalog.

mately φ_0, the angular apparent diameter of O. Since the surface brightness of the ring is the same as that of the star, the much greater area of the ring makes more photons available for observation. This increase of photons gives rise to an intensity gain A, whose maximum value for perfect alignment is approximately $2\theta_0/\varphi_0$. The value of θ_0 is calculated from the expression

$$\theta_0 = (4GM/\mu\, l_D\, c^2)^{\frac{1}{2}}$$

where G is the gravitational constant, M is the mass of the star, μ is $1 + (l_D/lO_D)$, l_D is the distance from the observer to the deflector star in light years, lO_D is the distance from the deflector to object star in light years, and c is the velocity of light.

It is essential that φ_D be smaller than θ_0. For all cases considered here this criterion is met. (None of the planets meet this criterion; thus we can eliminate them as possible deflectors. In addition, of course, their masses are several orders of magnitude too small.)

Liebes has given two illustrative examples for pairs of stars: (i) the relatively unfavorable condition of two stars, O_1 and D_1, 4 and 8 light years distant, respectively, and (ii) a pair, O_2 and D_2, located at 0.5×10^4 and 1.0×10^4 light years. Solar masses and solar radii have been assumed for the stars as well as for star X. The mass of 40 Eri-A is known to be 0.7 solar mass, and the distance is 16.3 light years. The resultant expected intensity gain A for these three cases and varying impact parameters for the last case are given in Table 1.

Refsdal estimates that, for two 14th-magnitude, solar-type stars with a mean distance interval of 325 light years, the time-symmetric pulse of light should be of the order of 20 days. For the case of 40 Eri-A it is of the order of 3.5 days.

Visual or photographic observations of the gravitational lens effect will probably be impossible because of the large difference in magnitudes of the stars. Photoelectric measurements, however, are accurate enough even today. As a rule of thumb, Felgett (6) gives a flux of 10^6 photons per square centimeter per second at the surface of the earth for a star of zero magnitude. The visual magnitude of 40 Eri-A is about 5, corresponding to about 10^4 photons cm^{-2} sec^{-1}, while the 15th-magnitude star gives about 10 photons cm^{-2} sec^{-1}. By monitoring the brightness of the two stars separately several years before and after eclipse it should be possible to detect an increase in brightness over the sum of the separate measurements at the time of eclipse, even if the gravitational lens effect amplifies light by a factor of less than 1000.

The data show that the stars have to be aligned to within 1.63×10^{-7} radian, or about 0.04 second of arc, for the effect to be observable. This is indeed a stringent requirement, but the relative ease with which these observations can be carried out with available equipment favors a concentrated effort in 1988 by several observatories to study what may be a unique and leisurely eclipse.

It is difficult to estimate now the exact time and minimum distance of approach for two reasons: (i) the large annual parallax (0.200") of 40 Eri-A, and (ii) uncertainties about the 8000-year period of its orbit. Presumably these uncertainties can be cleared up by 1988, but perhaps the best solution is simply to observe and see what happens. Even a negative result may yield an upper limit to the gravitational lens effect, which so far has eluded observational checks on solar eclipses.

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