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The Close-Packed-Spheron Theory and Nuclear Fission

Close packing of spherons provides a simple explanation of nuclear properties, including asymmetric fission.

Linus Pauling

Twenty-five years ago a phenomenon of tremendous importance was discovered, the phenomenon of nuclear fission. A striking feature of the fission of the uranium nucleus and other very heavy nuclei is its asymmetry: fissioning to produce a lighter and a heavier nucleus, with mass ratio about 2/3, occurs several hundred times as often as fissioning to produce two nuclei with about equal mass. Various efforts to explain asymmetric fission have had only limited success, including those based on consideration of energy release, of shell effects, of the shape of the energy function of the deformed nucleus in the saddle-point region, and of the penetration through rather than passage over the energy barrier. In this article I describe an extension of the theory of nuclear structure that provides a simple explanation of asymmetric fission and of some other properties of nuclei.

Theories of Nuclear Structure

During recent decades a great amount of knowledge about the properties of atomic nuclei has been gathered. An extensive theory of nucleonic interactions and nuclear structure [liquid-drop theory (*1*), shell theory (*2, 3*), unified theory (*4*), cluster theory (*5–7*)] has been developed that accounts reasonably well for many of these properties: the general dependence of the normal-state energy on the mass number A, nuclear diameter and charge distribution, spin and magnetic moment, energies of excited states, nature of beta decay, rate of nuclear fission and dependence on the energy and nature of bombarding particles, and several others. Some properties, however, including asymmetry of fission, mentioned above, have not hitherto been encompassed by the theory in a satisfactory way.

Recent theoretical treatments of the structure of nuclei have involved the approximate or iterative solution of the eigenvalue equation for each nucleon in a potential determined by its interaction with the other nucleons. The potentials have the Yukawa form beyond internucleonic distance about 0.5 fermi (f, 0.5×10^{-13} cm), corresponding to nucleonic core radius 0.25 fermi, and involve singlet and triplet central forces, tensor forces, and spin-orbit forces. These treatments (*8*) of finite nuclei and infinite nuclear matter (with proton-proton repulsion ignored) have provided calculated values of some properties in reasonably good agreement with experimental results. The proton and neutron densities are given by the theory as approximately constant throughout most of the volume of the nucleus; they then decrease rapidly, the thickness of the skin (90 to 10 percent of maximum) being about 2.4 f. The density of nucleons is about one nucleon per 6 f^3. These properties are essentially as found by electron scattering experiments (*9*).

For H^2, H^3, He^3, and He^4, the nucleons can be described approximately as occupying $1s$ orbitals. The core repulsion suggests that H^3 and He^3 have a triangular structure and that He^4 has a tetrahedral structure. The observed binding energies for $A = 2$, 3, and 4 are 2.2, 8.1 (average of 7.7 and 8.5), and 28.3 Mev, respectively. These are in the ratios 1 : 3.7 : 13, deviating from the ratios 1 : 3 : 6 of the number of internucleonic interactions in the direction corresponding to stronger bonds (smaller internucleonic distance) in the helion (*10*) than in the deuteron. In He^4 the nucleonic valences are nearly saturated: a fifth nucleon (proton or neutron) has negative binding energy, and in the most stable larger nuclei the binding energy per nucleon is only 25 percent greater than in the helion.

The polyhelion model (alpha-particle model) of the nucleus (*7, 11*) has been made the basis of a useful theory. In this theory O^{16} is described as a tetrahedron of four helions (*6*) and Mg^{24} as an octahedron of six helions. (The tetrahedron, octahedron, and icosahedron are the regular triangular polyhedra—their faces are equilateral triangles and their corners are equidistant from a center, which gives them a roughly spherical shape, as shown for the icosahedron in Fig. 1.)

The shell theory has had great success in accounting for many nuclear properties (*3*). The principal quantum number n for nucleons is usually taken to be $n_r + 1$, where n_r, the radial quantum number, is the number of nodes in the radial wave function. (For electrons n is taken to be $n_r + l + 1$; l is the azimuthal quantum number.) Strong spin-orbit coupling is assumed,

The author is a Fellow of the John Simon Guggenheim Memorial Foundation and is Research Professor of the Physical and Biological Sciences in the Center for the Study of Democratic Institutions, Santa Barbara, California.

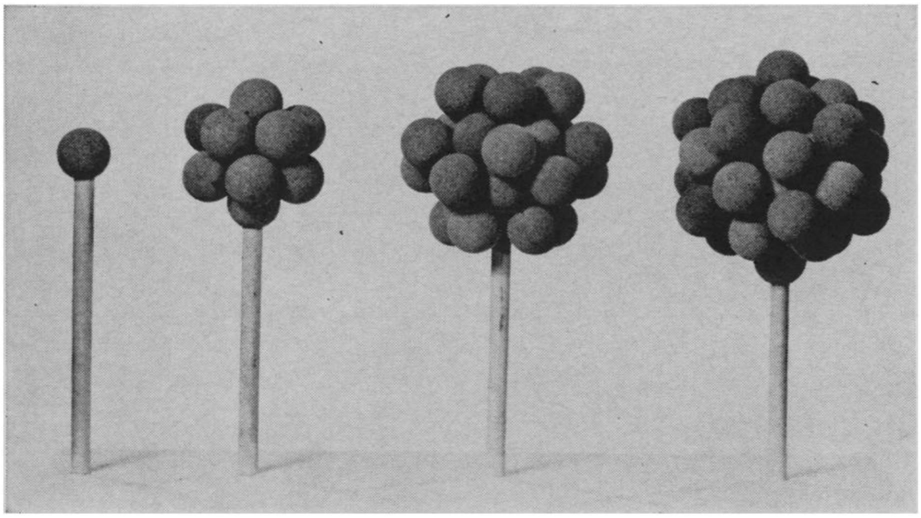

Fig. 1 (left). The arrangement of 45 spheres in icosahedral closest packing. At the left there is shown a single sphere, which constitutes the inner core. Next there is shown the layer of 12 spheres, at the corners of a regular icosahedron. The third model shows the core of 13 spheres with 20 added in the outer layer, each in a triangular pocket corresponding to a face of the icosahedron; these 20 spheres lie at the corners of a pentagonal dodecahedron. The third layer is completed, as shown in the model at the right, by adding 12 spheres at the corners of a large icosahedron; the 32 spheres of the third layer lie at the corners of a rhombic triacontahedron. The fourth layer (not shown) contains 72 spheres.

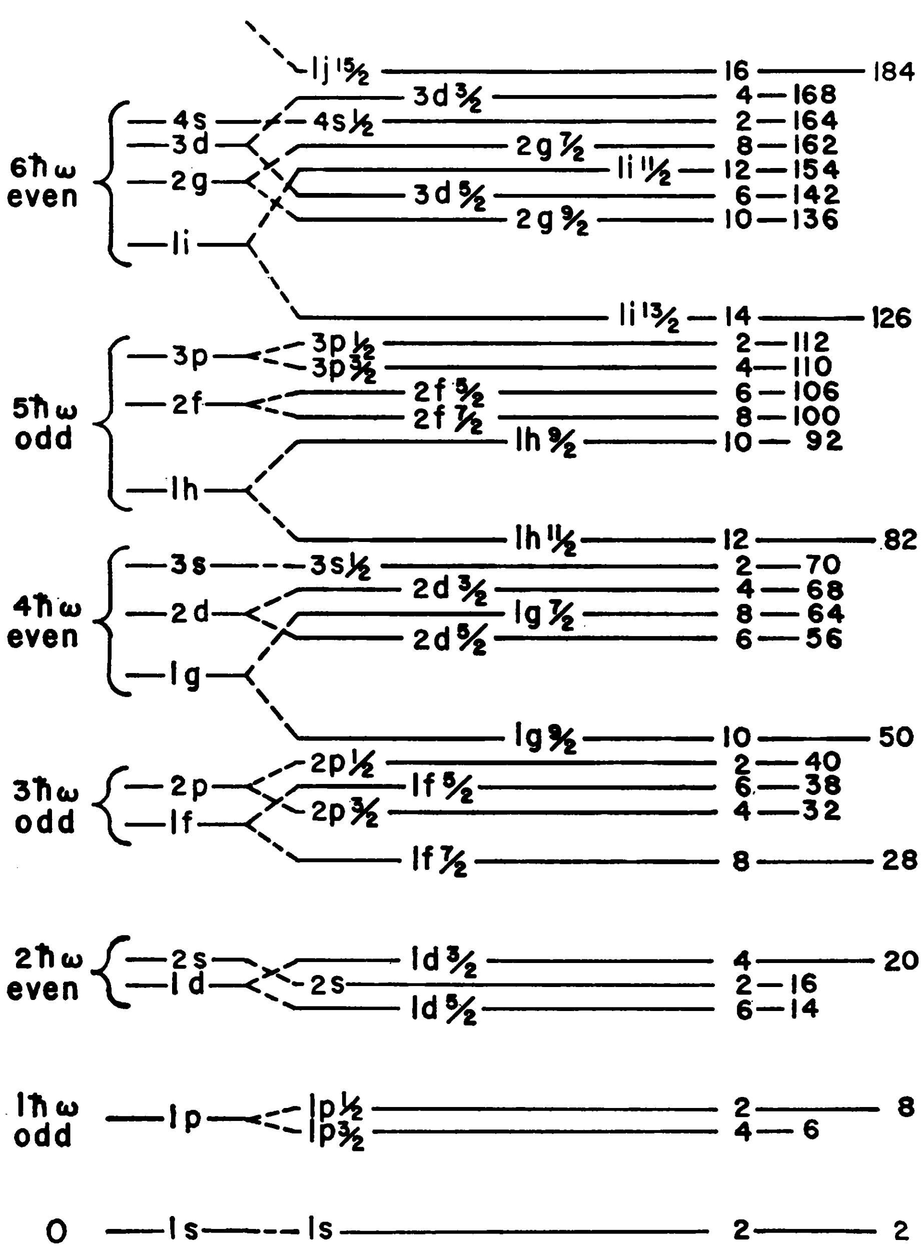

Fig. 2. The sequence of nucleonic energy levels with spin-orbit coupling, redrawn, with small changes, from *3*, p. 58.

splitting each subshell into a more stable subsubshell with $j = l + 1/2$ and a less stable subsubshell with $j = l - 1/2$; j is the total angular momentum number for a neutron or proton. The sequence of subsubshells assigned to both neutrons and protons is given in Fig. 2, as indicated by the observed values of spin and parity of nuclei.

The Close-Packed-Spheron Model

I assume that in nuclei the nucleons may, as a first approximation, be described as occupying localized 1*s* orbitals to form small clusters. These small clusters, called spherons, are usually helions, tritons, and dineutrons; in nuclei containing an odd number of neutrons, an He^3 cluster or a deuteron may serve as a spheron. The localized 1*s* orbitals may be described as hybrids of the central-field orbitals of the shell model.

The close-packed-spheron model differs from the conventional liquid-drop model of the nucleus in having spherons rather than nucleons as the units. This is a simplification; ${}_{64}Gd_{90}{}^{154}$, for example, is described in terms of 45 spherons, rather than 154 nucleons.

The nature of spheron-spheron interactions is such that maximum stability is achieved when each spheron ligates about itself the maximum number of neighbors, to produce a nucleus with a closest-packed structure. A simple argument (*12*) leads to the conclusion that the spherons in a nucleus are arranged in concentric layers. The packing radius of a spheron varies from 1.28 f for the dineutron to 1.62 f for the helion. The radius (to nucleon density half that of the inner region) of the largest nucleus is 6.8 f

(1.07 $A^{1/3}$ f), four times the helion radius. The thickness of the nuclear skin corresponds to the nubbled surface of the outer hemispheres of an outer layer of spherons. In a large nucleus the region within the outer layer would be lined with another layer of spherons, in contact with the outer layer, and within this layer there might be a central spheron or layer of spherons.

In a three-layer nucleus the outermost layer may be called the *mantle* and the other two the *outer core* and the *inner core*, to avoid confusion with the shells of the shell model.

The general geometrical problem of the packing of spheres has not been solved. An example of closest packing of atoms with some variation in effective radius is the icosahedral packing found (*13*) in the intermetallic compound $Mg_{32}(Al,Zn)_{49}$ (Fig. 1). The successive layers in this structure contain 1, 12, 32, and 117 spheres. These numbers are reproduced (to within ±1) by the empirical equation (*12*)

$$n_t = (n_i^{1/3} + 1.30)^3 \qquad (1)$$

in which n_t is the total number of spheres and n_i is the number not including those in the outermost layer. The number 1.30 represents the effective thickness of a layer; the form of the equation corresponds to the assignment of equal volumes to the spheres.

I have assumed that this equation applies to structures with two or more spheres in the central layer (as well as with one, as in icosahedral packing), and have applied it in the calculation of the ranges of values of the neutron number N in which successive subsubshells are occupied (*12*). (In this calculation the difference in radius of the different kinds of spherons is taken into consideration.) The assignment of quantum numbers is made with use of the following assumptions (*14*):

1) Those subshells that occur (are occupied) with only the value 1 for the principal quantum number n contribute only to the mantle.

2) Those subshells that occur with two values of n contribute to the mantle and the next inner layer, and so on.

3) For given n, the subshells are filled in order of increasing l, and for given l in order of increasing n.

Let us again take $_{64}Gd_{90}$ as an example. Its 45 spherons (each containing two neutrons) are expected to be distributed in layers 1, 12, 32, as shown in Fig. 1. There are three layers, and one pair of neutrons in the inner core; hence $1s^2$, $2s^2$, and $3s^2$ occur. To obtain the 12 spherons of the outer core we need $1p^6$ $1d^{10}$ $1f^6$, in addition to $2s^2$. The neutrons in the mantle then are $3s^2$ $2p^6$ $2d^{10}$ $2f^6$ $1f^8$ $1g^{18}$ $1h^{14}$.

The sequence of neutron-occupancy ranges found in this way (*12*) is

Table 1. Nucleon configurations for the magic numbers.

Magic number	Mantle	Core or outer core	Inner core
2	$1s^2$		
8	$1s^21p^6$		
20	$2s^21p^61d^{10}$	$1s^2$	
50	$2s^22p^61d^{10}1f^{14}$ $(1g9/2)^{10}$	$1s^21p^6$	
82	$3s^22p^62d^{10}1f^{14}1g^{18}(1h11/2)^{12}$	$2s^21p^61d^{10}$	$1s^2$
126	$3s^23p^62d^{10}2f^{14}1g^{18}1h^{22}$ $(1i13/2)^{14}$	$2s^22p^61d^{10}1f^{14}$	$1s^21p^6$

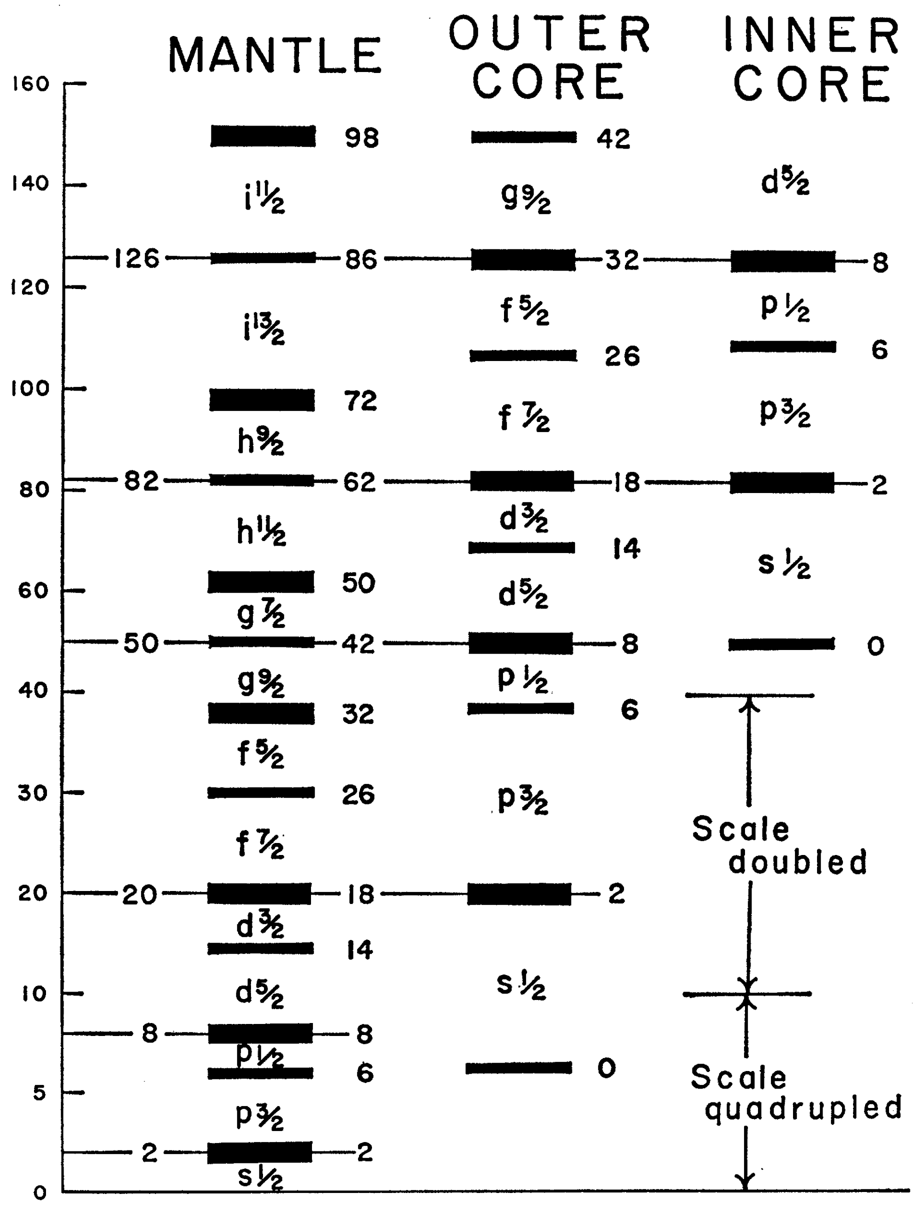

Fig. 3. Diagram showing the ranges of values of neutron number N in which spin-orbit subsubshells of the mantle, outer core, and inner core are occupied, as calculated with use of the spheron-packing equation.

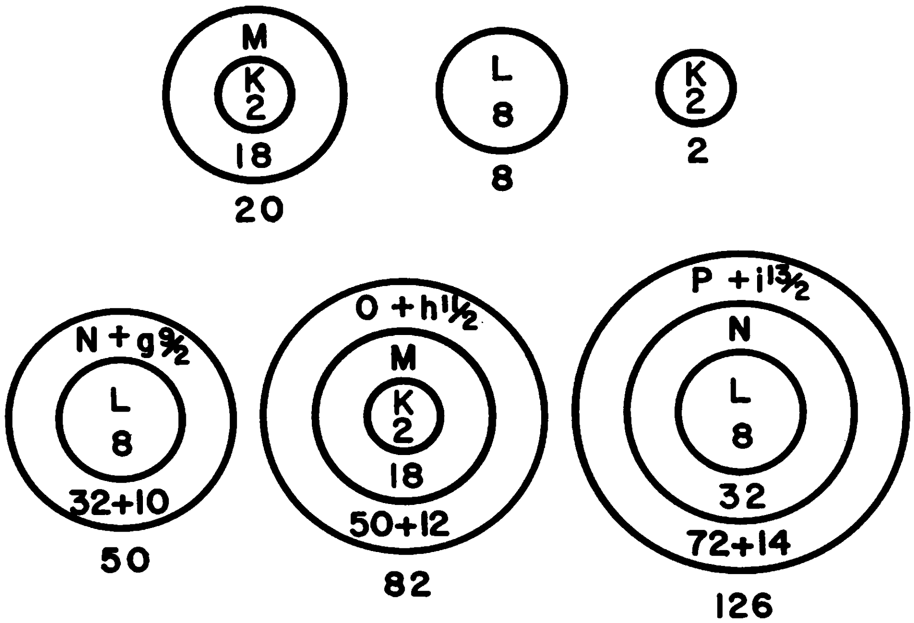

Fig. 4. The magic-number structures of nuclei.

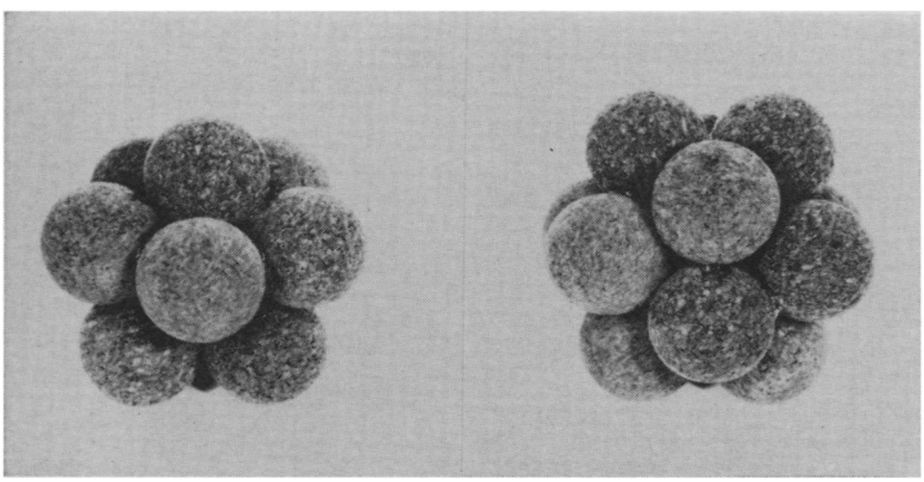
Fig. 5. Limiting stable arrangements of spherons around a central spheron. (Left) Nine outer spherons (*KM* structure); (right) 12 outer spherons (icosahedral structure).

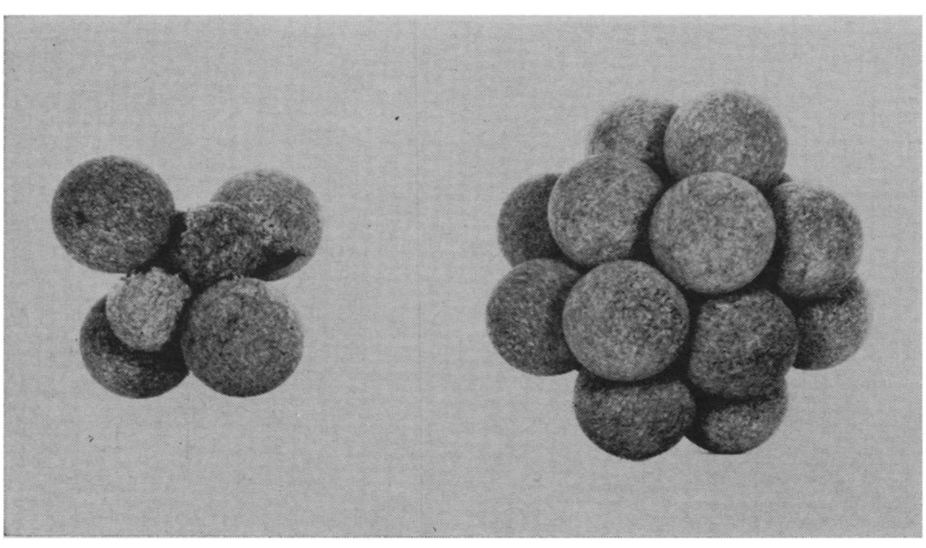
Fig. 6. Arrangement of 16 spherons around four spherons. (Left) The four inner and four of the 16 outer spherons; (right) the completed structure.

shown in Fig. 3. It closely resembles the level diagram found by Mayer and Jensen by analysis of observed nuclear properties, with the help of the calculated level sequences for harmonic-oscillator and square-well potential functions, but it differs from their diagram (Fig. 2) in that subsubshells are assigned to the separate layers and that overlapping ranges of N and Z for their occupancy are shown.

The observed values of spin and parity indicate that Fig. 3 (like the Mayer-Jensen level diagram) applies to protons as well as to neutrons.

Structural Basis of Magic Numbers

Certain numbers of neutrons and protons were recognized by Elsasser (*15*) as conferring increased stability on nuclei. These numbers are 2, 8, 20, 50, 82, and 126. (The set is sometimes considered to include 28 also.) It was in part their effort to account for these numbers that led Mayer and Haxel, Jensen, and Suess to propose their shell model with spin-orbit coupling.

The close-packed-spheron theory leads to a simple structural interpretation of the magic numbers (*16*): they are the numbers at which each layer of the nucleus achieves completion of a shell (K, L, M, . . ., with $2n^2$ neutrons or protons), or at which each core layer achieves completion of a shell and the mantle achieves completion of a shell and a $j = l + ½$ subsubshell.

It is to be expected that the energy gap following completion of a core shell would be greater than that following completion of a mantle shell; hence 50 is a magic number and 38 is not. The magic-number configurations are given in Table 1 and Fig. 4.

The Local Environment of Inner-Core Spherons

The magic number 20 corresponds to the *KM* structure, which is also the structure of the core for magic number 82. This structure, shown in Fig. 5 (left), involves nine spherons ligated about a smaller central spheron. Its stability may be attributed to its double completed-shell character.

As many as 12 spherons can be closely packed about a central spheron. The icosahedral arrangement of 12 about one is shown in Fig. 5 (right).

Two limiting structures with four spherons as core or inner core are shown in Figs. 6 and 7. The structure shown in Fig. 6 has the central tetrahedron of four spherons surrounded by a larger tetrahedron of four and a truncated tetrahedron of 12, a total of 16 spherons in the outer layer. The packing is triangular. This is the structure of the core for magic number 126. It has double completed-shell character, *LN*.

An interesting and possibly significant aspect of this structure is that each of the four spherons of the inner core has ligancy 9 (its neighbors are the three other inner-core spherons, three of the outer tetrahedron, and another three of the outer layer). Each of these four inner-core spherons with its nine neighbors can be described as forming a *KM* complex, and the *LN* core can be described as four interpenetrating *KM* complexes.

The conclusion that each inner-core spheron in a stable core should ligate its neighbors about itself in a way corresponding to local stability is a reasonable consequence of the self-generating character of the potential energy function for nucleons in nuclei (mutual interdependence of structure and potential energy function) and the short range of internucleonic forces.

The arrangement of 22 spherons around an inner tetrahedron of four spherons shown in Fig. 7 involves icosahedral packing: each of the four inner spherons is surrounded by an icosahedron of 12, three of which are the three other inner spherons. This structure (26 spherons, 52 neutrons) with one spheron missing may be assigned to magic number 50. The complete structure, with 26 spherons, corresponds to the stable nucleus $_{44}Ru_{52}$, as discussed in the following section.

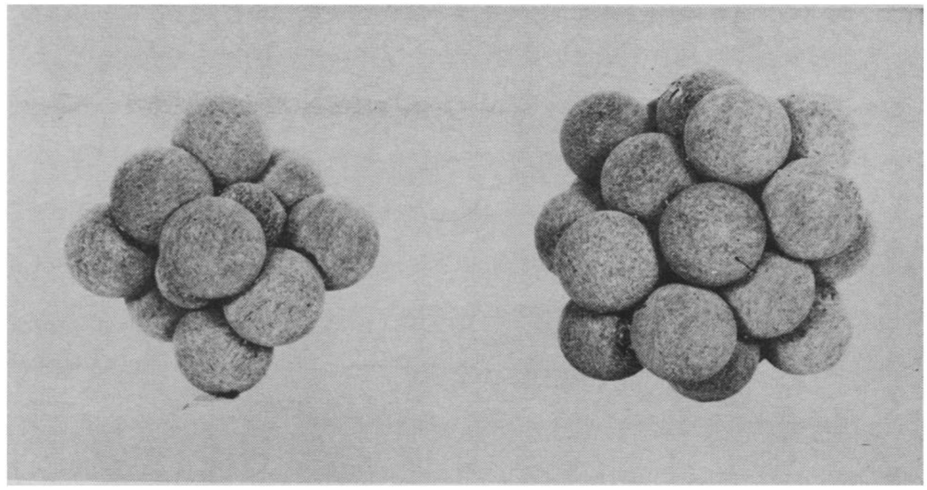

Fig. 7. Arrangement of 22 spherons around four. (Left) The four inner and ten of the 22 outer spherons; (right) the completed structure.

The Proton-Neutron Ratio

The proton-neutron ratio in nuclei has been discussed for over 50 years. Long before the neutron had been shown to exist Harkins (*17*) attempted to draw some conclusions about nuclear structure from the observed excess of neutrons over protons (he used the name neutron for a hypothetical unit of a proton combined with an electron). The course of the proton-neutron ratio is now well understood in relation to the energy change accompanying emission of an electron or positron from the nucleus (that is, conversion of an intranuclear neutron to an intranuclear proton, or the reverse), but no reasonable correlation with the structure of the nucleus has been published.

Let us consider the nucleus $_{44}Ru_{52}$. As discussed above, it is assigned 26 spherons, of which 22 are in the mantle and four in the core. From Fig. 3 we might assign 37 protons to the mantle and seven to the core. This assignment gives a mantle of 11 helions and 11 tritons and a core of three helions and one triton.

However, the Coulomb repulsion of protons may be expected to cause the proton orbitals to have a greater radial extent than the corresponding neutron orbitals, and to overlap spherons of more than one layer. We may consider that for the $_{44}Ru_{52}$ nucleus there is resonance between the helion-triton structure described above and a helion-dineutron structure, with the helions all in the surface layer. This structure has 22 helions in the mantle and four dineutrons in the core.

Structures of this sort, with a mantle of helions and a core of neutrons, have minimum Coulomb energy. We may expect these structures to have the minimum neutron excess compatible with stability; any more protons would be forced from the mantle into the core. In fact, $_{44}Ru_{52}$ has the largest atomic number for which $N - Z$ equals 8 for a stable isotope.

The proton number equal to the

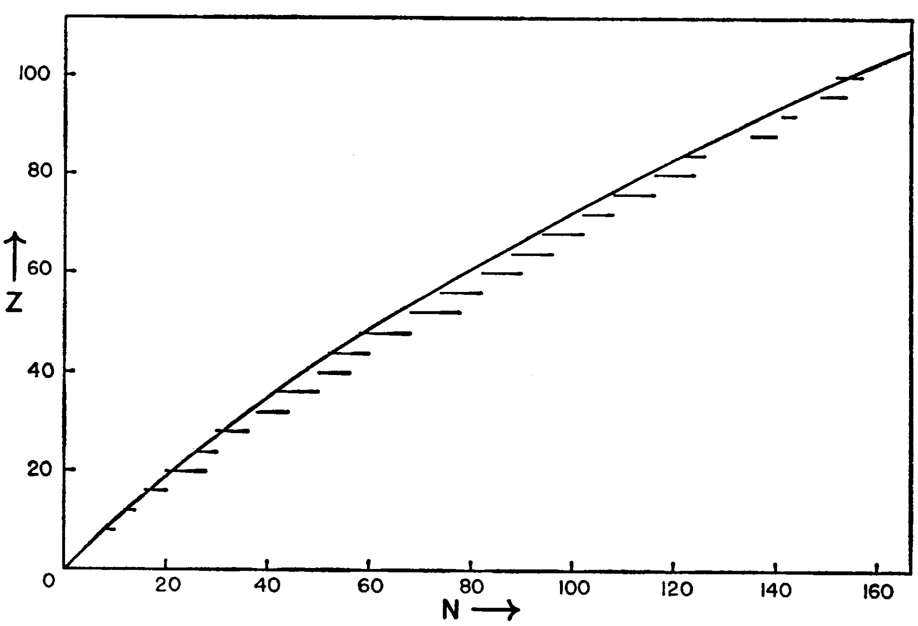

Fig. 8. A curve of proton number Z as a function of neutron number N, calculated as described in the text. The horizontal lines show the ranges of stable isotopes for alternate Z-even elements (for large Z the four most stable isotopes).

number of neutrons in the mantle may accordingly be expected to represent the high-Z boundary of the region of stability of nuclei. The curve shown in Fig. 8 has been drawn through points corresponding in this way to the neutron configurations of Table 1. The horizontal lines are the observed ranges of stability for alternate even-Z sets of isotopes (for large Z the four isotopes with the smallest half-lives).

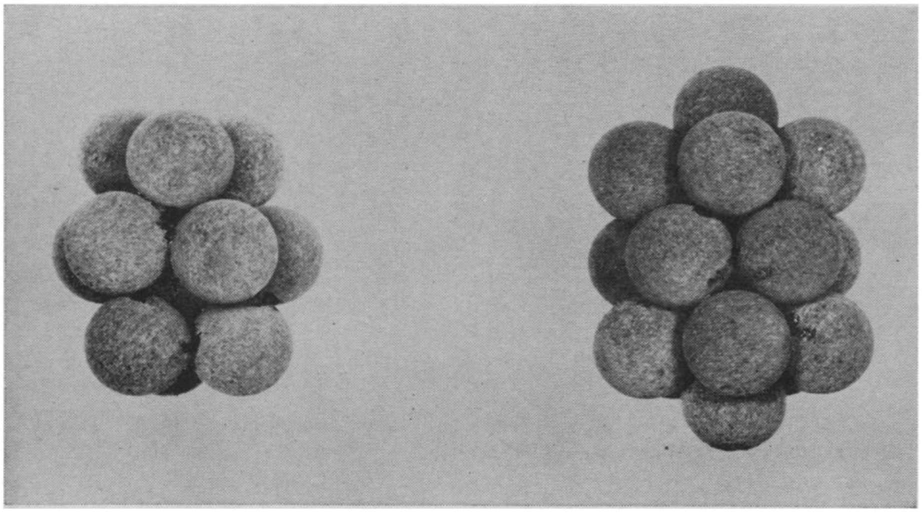

Fig. 9. Arrangement of 11 (left) and 17 (right) spherons about an inner core of two spherons. Each inner-core spheron shows *KM* ligation at left and icosahedral ligation at right.

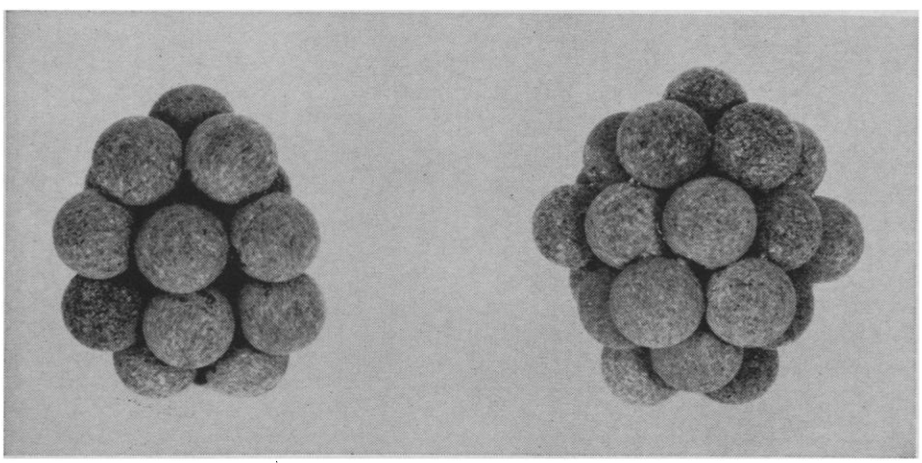

Fig. 10. Arrangement of 18 (left) and 24 (right) spherons about an inner core of five spherons with the configuration of a trigonal bipyramid.

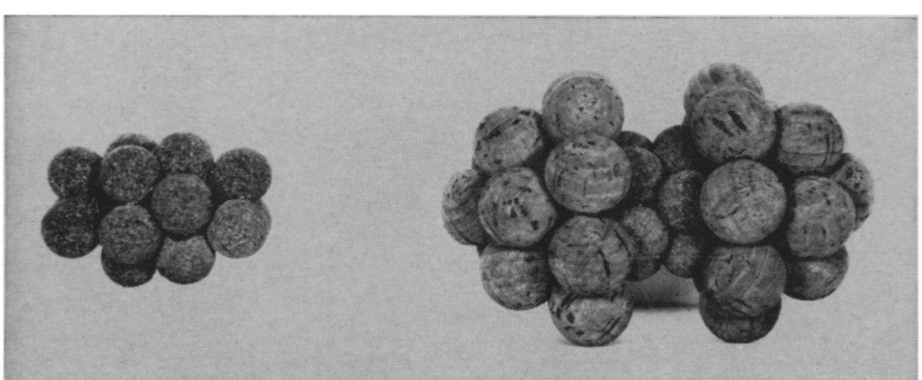

Fig. 11. (Left) An outer core of 16 spherons surrounding an inner core of three, in a linear arrangement. (Right) The same core with a portion of the mantle of the elongated nucleus. (The difference in relative sizes of core and mantle spherons is exaggerated in this and some of the other figures.)

It is seen that, in accordance with the foregoing argument, the low-N ends of these horizontal lines lie close to the curve (mean deviation, 1). The conclusion may be drawn that the structures of stable nuclei involve a significant contribution of structures with a pure neutron core and a pure helion or nearly pure helion mantle.

Permanent Deformation of Nuclei

Observed properties of many nuclei have been interpreted as showing that the nuclei are not spherical but are permanently deformed (*4*). The principal ranges of deformation are neutron numbers 90 to 116 and 140 to 156. Most of the deformed nuclei are described as prolate ellipsoids of revolution, with major radii 20 to 40 percent larger than the minor radii.

A simple explanation of the existence of deformed nuclei in these ranges is provided by the close-packed-spheron theory (*14*); it is that the inner core (of two or five spherons) in these ranges has an elongated structure, and that this elongation is imposed by the inner core on the two surrounding layers.

The most stable core configuration for an inner core of one spheron is *KM*, with an outer core of nine spherons. For icosahedral packing the outer core contains 12 spherons. These structures, which we may take as defining the limits of stability for a one-spheron inner core, have been discussed in the preceding section.

The upper limit of neutron number for one spheron as inner core is 90, corresponding to the icosahedral packing shown in Fig. 1. In the range beginning at about $N = 90$ we assign two spherons to the inner core. With *KM* ligation about each, as shown in Fig. 9, the core contains 26 neutrons; and with icosahedral ligation about each, also shown in Fig. 9, it contains 38 neutrons. These values correspond to the range for N from about 90 to 122 (Fig. 3).

Nuclei with an inner core of three spherons would have oblate deformation, rather than prolate. The packing is somewhat less satisfactory than for one, two, or four spherons, except for ligancy 10 (18 spherons in the outer core), which corresponds to 42 core neutrons, and thus lies in the *LN* core range. We conclude that the inner core contains 1, 2, or 4 spherons, in the range $N = 82$ to 126, and that only prolate deformation should occur.

An ellipsoidal nucleus with two spherons in the inner core has major radius greater than the minor radii by the radius of a spheron, about 1.5 f, which is about 25 percent of the mean radius. The amount of deformation given by this model is accordingly in rough agreement with that observed (*18*). In a detailed treatment it would be necessary to take into account the effect of electrostatic repulsion in causing the helions to tend to occupy the poles of the prolate mantle, with tritons tending to the equator.

In the region $N \geqq 140$ the inner core is a trigonal bipyramid of five spherons, surrounded by an outer core of 18 to 24 spherons (Fig. 10). The deformation is prolate, and about as great as for the region 90 to 120, in agreement with experiment.

The wave functions for the two inner-core spherons can, of course, be described as the symmetric and antisymmetric combinations of $1s$ and $1p_z$ functions. The Nilsson (*19*) treatment of neutron and proton orbitals in deformed nuclei is completely compatible with the foregoing discussion, which provides a structural interpretation of it.

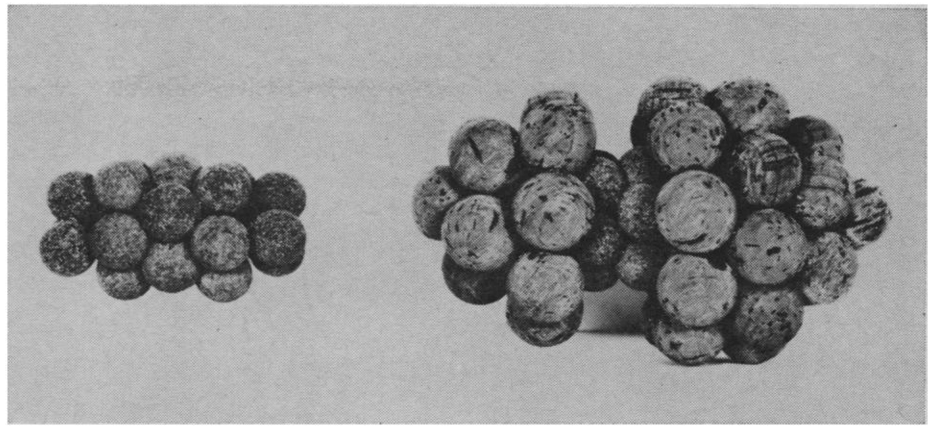

Fig. 12. (Left) An outer core of 21 spherons surrounding a linear inner core of four. (Right) The same core with a portion of the mantle, illustrating asymmetric fission.

Symmetric and Asymmetric Nuclear Fission

The foregoing considerations provide the basis for a discussion of the mechanism of nuclear fission.

Let us consider first the low-energy fission of the lighter fissionable elements, in the neighborhood of Pb^{208}. These elements (gold, thallium, lead, bismuth), when bombarded with particles such as 20-Mev deuterons, undergo symmetric fission, the distribution function of the products having a half width at half maximum of 8 to 15 mass-number units (*20*).

These fissioning nuclei (such as ${}_{84}Po_{127}{}^{211}$, formed by reaction of Bi^{209} and a deuteron) have a nearly spherical normal-state structure, resembling that of the doubly magic nucleus ${}_{82}Pb_{126}{}^{208}$, with an outer core of 16 spherons and an inner core of 4 spherons, shown in Fig. 6. The nucleus is excited, with vibrational energy about 25 Mev (for bismuth bombarded with 20-Mev deuterons). The vibrational deformation involves a change in the structure of the core, from a tetrahedral (nearly spherical) core of 20 spherons to a prolate ellipsoidal core with the same or nearly the same number of spherons.

The principles discussed in the foregoing sections strongly suggest that the deformed core has the structure shown on the left in Fig. 11. In the outer core there are rings of 3, 5, 5, and 3 spherons. The inner core consists of three spherons, in the interstices between pairs of rings. The central inner-core spheron has ligancy 12; it is surrounded by an icosahedron formed by the ten spherons of the two middle outer-core rings and the other two inner-core spherons. Each of these two inner-core spherons has ligancy 9; with its neighbors it constitutes a *KM* complex.

I assume that in the process of fission both the mantle and the core undergo splitting. The core could split between the two middle rings, which would result in symmetric fission.

In the process of fission the increasingly prolate deformation of the mantle could cause an equatorial fissure to occur in the mantle, as indicated on the right in Fig. 11. Before the fissure occurs there would be rings of spherons in the mantle between every adjacent pair of rings in the outer core. In the course of the vibration leading to fission the spherons of the mantle would be crowded toward the two poles. At the value of the reaction coordinate corresponding to the configuration shown at the right in Fig. 11 the nu-

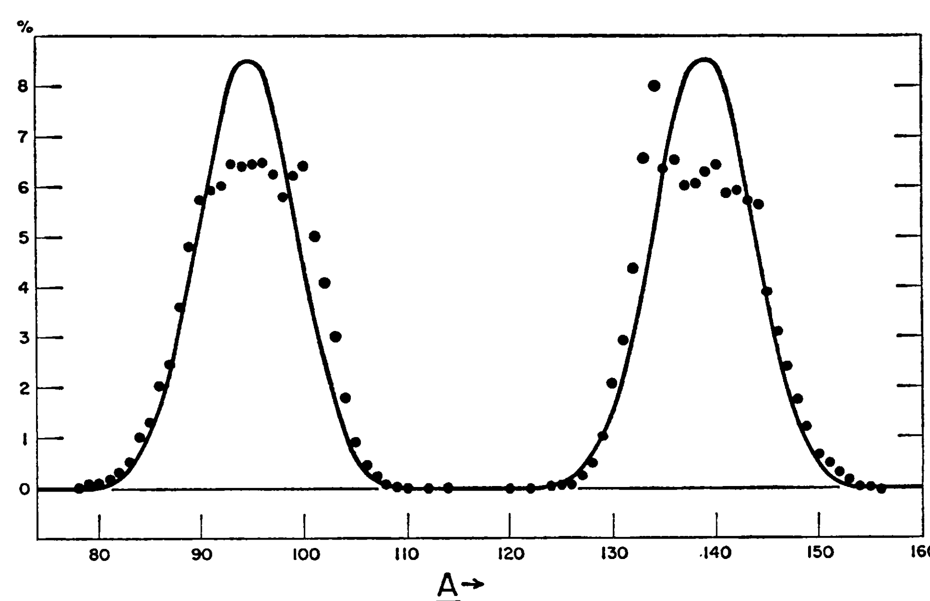

Fig. 13. Yields of nuclei in the fission of U^{236}, as a function of mass number. The points are averages of reported experimental values, and the curve is the result of a simple theoretical calculation described in the text.

cleus might be described as two smaller nuclei in contact; the ten spherons of the middle rings of the core of the original nucleus have become part of the mantles of the two daughter nuclei. This configuration would need only a small additional deformation to reach the saddle point in the energy surface.

The observed width of the distribution functions in mass number of the fission products indicates that the spherons that lie in the plane of the fissure are essentially randomly distributed between the two daughter nuclei, as discussed below for asymmetric fission.

A heavy fissionable nucleus, such as U^{236} (formed by combination of U^{235} and a neutron), undergoes asymmetric fission. This property can be related to the structure assumed by its core in the course of its prolate deformation. In its normal state the nucleus has an inner core of five spherons and an outer core of 20 spherons (Fig. 3), with a moderate prolate deformation, corresponding to the trigonal bipyramidal structure of the inner core (Fig. 10). The instability resulting from Coulomb repulsion of the protons leads to further prolate deformation and to fission, as described in the discussions based on the liquid-drop model. These discussions can be refined by consideration of the change in structure of the core with increasing prolate deformation of the nucleus.

To within its reliability of one or two units, the packing equation applies to ellipsoidal as well as to spherical nuclei. The more highly deformed core of ${}_{92}U_{144}{}^{236}$ assumed in the course of the fission reaction accordingly contains about 25 spherons. The structural principles discussed above lead to the assignment to this core of the structure shown at the left in Fig. 12. The outer core consists of five rings, containing 3, 5, 5, 5, and 3 spherons, with a linear inner core of four spherons, in the interstices of pairs of rings of the outer core. By applying the chart of Fig. 3 to protons, we identify the spherons of the outer core and the two central spherons of the inner core as tritons; the two end spherons of the inner core are dineutrons. The configuration of each end spheron of the inner core and its neighbors is the *KM* configuration, and that of each of the two others with its neighbors is the icosahedral configuration.

The spherons of the mantle, 22 helions and 25 tritons, may be described as forming four rings, in the grooves between pairs of rings of the outer core, and two caps, one at each end. The structure can be represented by the following diagram, where A represents the inner core, B the outer core, and C the mantle.

A:			n^2		t		t		n^2		
B:		$3t$		$5t$		$5t$		$5t$		$3t$	
C:	6α		$5\alpha, 3\frac{1}{2}t$		$9t$		$9t$		$5\alpha, 3\frac{1}{2}t$		6α
					↑						

Plane of fission

In this assignment the assumption is made that the 22 helions of the mantle occupy the end regions of the highly deformed nucleus, as a result of their strong Coulomb repulsion.

The greater stability of the core than of the mantle requires that fission occur along a plane between layers of the core. The number of layers is odd (five); accordingly the fission is not symmetric, as for the lighter fissionable nuclei (with four layers in the core), but is asymmetric.

The proposed mechanism of fission of U^{236} and other heavy fissionable nuclei is illustrated in Fig. 12. The nine tritons initially in the ring in the plane of fission crowd into the two end regions of the mantle, leaving a circular fissure, as shown in the figure. (This crowding is accompanied by the motion of one or more spherons from the poles of the mantle into the end regions of the core.) Each of the two outer-core rings of five spherons that determine the plane of fission may then be described as forming a part of the mantle of a daughter nucleus, and the large deformed nucleus may now be described as a smaller and a larger daughter nucleus in contact over the surface area of five or six spherons.

Eleven of the 25 core spherons of the parent nucleus thus become mantle spherons of the daughters (ten in the two rings of the outer core and one in the inner core), leaving 14 for the cores of the daughters. These cores are seen from Fig. 3 or Eq. 1, however, to contain about 16 spherons, over a range of partitions of the 144 neutrons between the two daughters. The process of fission thus involves the transfer of two (or perhaps three) spherons from the mantle to the two cores. This transfer could take place in three ways (2, 0; 1, 1; 0, 2) for two spherons and in four ways for three spherons. The partition between the two daughters of the nine tritons of the ring initially in the fission plane would be largely influenced by the nature of the two cores. We are thus led to a structural interpretation of alternative channels for the fission reaction, which may account for the observed fine structure in the distribution function of fission products; the observed peaks at $A = 100$ and 134 may be ascribed to the transfer of two spherons to the smaller core, to give fission products with cores of six and ten spherons.

Instead of attempting to introduce such refinements, I shall report the result of a very simple statistical treatment of the fission-product distribution function. The location of the plane of fission for U^{236} shown in the diagram is such as to assign 11α, $16\frac{1}{2}t$, and one dineutron to the lighter of the two daughter nuclei (assuming equal partition for the ten tritons in the fission plane). With one neutron promptly emitted by the light nucleus and 1.5 by the heavy nucleus, the average mass numbers are 94.5 and 139, in close agreement with experiment. The average charge of the light nucleus is 38.5, which is 1.2 larger than for constant charge/mass ratio. This difference has the correct sign but is larger than the observed value (about 0.8). The disproportionately large charge of the light fission products results from their larger share of the mantle.

The partition of the ten tritons in the fission plane is, of course, different for different fission channels. The curve in Fig. 13, corresponding to random distribution of the ten tritons, has been calculated with the de Moivre approximation to the binomial distribution function. The approximation to the experimental points (*21*) suggests that good agreement could be obtained by a more refined calculation involving consideration of the various channels for the fission reaction mentioned above.

The kinetic energy (*22*) of fission products of U^{236}, corrected for ionization defect, has the value 166.9 Mev, which is the Coulomb energy of the charges of pairs of daughter nuclei at the distance 18 f. This value is not in disagreement with the proposed mechanism of fission. The distance between the centers of the nuclei, considered as spheres, is 13 f, calculated with the formula $1.35\ A^{\frac{1}{3}}$ f for the contact radius. Coulomb repulsion causes the nuclei to be distorted, however, and, moreover, the helions, carrying much of the charge, are repelled to the opposite polar regions of the two mantles, thus causing the distance between the two centers of charge to be somewhat larger than 13 f.

There is no structure for an elongated core intermediate between that shown in Fig. 11, with three inner-core spherons, and that shown in Fig. 12, with four. The transition between these structures is calculated by use of Eq. 1, with $n_i = 22$, to occur at $n_t = 69$, that is, at $N = 138$. It is accordingly an expectation from the close-packed-spheron theory that, as observed, ${}_{90}\mathrm{Ac}_{138}{}^{227}$ (formed by bombardment of Re^{226} with 11-Mev protons) gives a three-humped fission product distribution curve (*23*), which has been interpreted (*24*) as showing that both symmetric fission and asymmetric fission occur.

Asymmetric fission is observed in the spontaneous decomposition of ${}_{98}\mathrm{Cf}_{156}{}^{254}$ and other very heavy nuclei. We may ask when the transition to symmetric fission would begin. The next elongated core, in the series represented in Figs. 11 and 12, would contain 31 spherons, and the transition to it should occur for 28 spherons in the core of the undistorted nucleus, that is, at $N = 163$ (calculated with use of Eq. 1). We conclude that ${}_{103}\mathrm{Lw}_{163}{}^{266}$ and adjacent nuclei should show both asymmetric and symmetric fission.

Conclusion and Summary

The close-packed-spheron theory of nuclear structure may be described as a refinement of the shell model and the liquid-drop model in which the geometric consequences of the effectively constant volumes of nucleons (aggregated into spherons) are taken into consideration. The spherons are assigned to concentric layers (mantle, outer core, inner core, innermost core) with use of a packing equation (Eq. 1), and the assignment is related to the principal quantum number of the shell model. The theory has been applied in the discussion of the sequence of subsubshells, magic numbers, the proton-neutron ratio, prolate deformation of nuclei, and symmetric and asymmetric fission.

References and Notes

1. N. Bohr, *Nature* **137**, 344, 351 (1936); ——— and F. Kalckar, *Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd.* **14**, No. 10 (1937); L. Meitner and O. R. Frisch, *Nature* **143**, 239 (1939); N. Bohr and J. A. Wheeler, *Phys. Rev.* **56**, 426 (1939).
2. M. Goeppert Mayer, *Phys. Rev.* **75**, 1969 (1949); O. Haxel, J. H. D. Jensen, H. E. Suess, *ibid.*, p. 1766; *Z. Physik* **128**, 295 (1950).
3. M. Goeppert Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (Wiley, New York, 1955).
4. J. Rainwater, *Phys. Rev.* **79**, 432 (1950); A. Bohr, *Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd.* **26**, No. 14 (1952); A. Bohr and B. R. Mottelson, *ibid.* **27**, No. 16 (1953); K. Alder, A. Bohr, T. Huus, B. R. Mottelson, A. Winther, *Rev. Mod. Phys.* **28**, 523 (1956).
5. J. A. Wheeler, *Phys. Rev.* **52**, 1083 (1937).
6. D. M. Dennison, *ibid.* **96**, 378 (1954).
7. R. K. Sheline and K. Wildermuth, *Nucl. Phys.* **21**, 196 (1960).
8. For example, K. A. Brueckner and J. L. Gammel, *Phys. Rev.* **109**, 1023 (1958); K. S. Masterson, Jr., and A. M. Lockett, *ibid.* **129**, 776 (1963).
9. R. Hofstadter, *Ann. Rev. Nucl. Sci.* **7**, 231 (1957).
10. The name helion is used for the alpha particle: L. Pauling, *Nature* **201**, 61 (1964).
11. W. D. Harkins and E. D. Wilson, *Proc. Nat. Acad. Sci. U.S.* **1**, 276 (1915); L. R. Hofstad and E. Teller, *Phys. Rev.* **54**, 681 (1938); D. M. Dennison (*6*) and many other papers.
12. L. Pauling, *Proc. Nat. Acad. Sci. U.S.*, in press.
13. G. Bergman, J. L. T. Waugh, L. Pauling, *Nature* **169**, 1057 (1952); *Acta Cryst.* **10**, 254 (1957).
14. L. Pauling, *Phys. Rev. Letters*, in press.
15. W. M. Elsasser, *J. Phys. Radium* **4**, 549 (1933); **5**, 389, 635 (1934).
16. L. Pauling, *Nature*, in press.
17. W. D. Harkins, *J. Am. Chem. Soc.* **42**, 1956 (1920).
18. B. R. Mottelson and S. G. Nilsson, *Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrifter* **1**, No. 8 (1959).
19. S. G. Nilsson, *Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd.* **29**, No. 16 (1955).
20. A. W. Fairhall, *Phys. Rev.* **102**, 1335 (1956); **118**, 771 (1960); E. K. Hyde, *The Nuclear Properties of the Heavy Elements. III. Fission Phenomena* (Prentice-Hall, Englewood Cliffs, N.J., 1964).
21. Averages of the values reported by W. H. Walker, *Chalk River Report CRRP-916*, 16 (1960); S. Katcoff, *Nucleonics* **18**, 201 (1960); E. P. Steinberg and L. E. Glendenin, *Proc. U.N. Intern. Conf. Peaceful Uses At. Energy 1st* **7**, 3 (1956); J. A. Petruska, H. G. Thode, R. H. Tomlinson, *Canadian J. Phys.* **33**, 693 (1955); H. Farrar and R. H. Tomlinson, *Nucl. Phys.* **34**, 367 (1962); see *23*, pp. 102, 103.
22. D. C. Brunton and G. C. Hanna, *Canadian J. Res.* **28A**, 190 (1950); D. C. Brunton and W. B. Thompson, *ibid.* **28A**, 498 (1950).
23. R. C. Jensen and A. W. Fairhall, *Phys. Rev.* **109**, 942 (1958).
24. A. W. Fairhall, R. C. Jensen, E. F. Neuzil, Paper P/677, *Proc. U.N. Intern. Conf. Peaceful Uses At. Energy, 2nd* **15** (1958).

Galactose Metabolism and Cell "Sociology"

Galactose, one of the freaks of evolution, furnishes a simple illustration of the extravagances of nature.

Herman M. Kalckar

"Lettre de M. Pasteur à M. Biot [Lille, 11 février 1856]:

". . . Mais, en realité, le sucre de lait modifié par les acides est tout autre que le glucose. Je propose de le nommer *lactose*. On reserverait le nom du sucre de lait ou de lactine pour le sucre cristallisable du lait. . ."

"Le lactose cristallise beaucoup plus facilement que le glucose. . ." [From a communication appearing in the 11 February 1856 issue of *Comptes Rendus*]

The molecular basis for cell-surface patterns governing the "social characteristics" of a cell has become a great chapter in general biology. It began, as did so many other adventures in biology, with Louis Pasteur's discovery of asymmetric molecules and their relevance to the function of the living cell. However, Pasteur's work is involved in a more direct way with the topic discussed here. A concrete example can best illustrate this. Studies on the molecular basis of human blood-group specificity (*1*) have taught us that a number of peculiar cell-surface sugars (like amino sugars, L-fucose, sialic acid) determine this specificity. For instance, the difference between blood groups A and B resides solely in the terminal sugar, *N*-acetylgalactosamine in A and D-galactose in B; otherwise, the chains are identical.

It was Pasteur who early in 1856 first pronounced D-galactose, this peculiar enanthiomorph of glucose, something "*tout autre que le glucose,*" at a time when he began to focus his interest on the study of sugar fermentation. This period of Pasteur's life coincided with circumstances which, according to Dubos (*2*), contrib-

The author is professor of biological chemistry at Harvard Medical School, Boston, Massachusetts, and Henry S. Wellcome Research Biochemist and head of the Biochemical Research Laboratory at Massachusetts General Hospital, Boston.