The second consideration, which smacks of "political conservatism," is nonetheless founded on our Constitution—that is, that only problems found difficult of solution on local levels may be referred to the central government. The federal government should fill clear and present needs, where local authorities—state governments, universities—are not able to supply the requisites.

Perhaps the future of the intramural programs could be most satisfactorily sought in this second consideration. These programs could be directed toward problems where the strength of the federal government can fill a need beyond the capacity of weaker institutions. Let us do what we can, and let Uncle do what he must for all our benefit. There should be no competition, but rather complementation.

GORDON E. GREEN
School of Dentistry, Temple University, Philadelphia, Pennsylvania

Emotional Perils of Mathematics

People are turned aside from being mathematicians—by which I mean "pure" mathematicians—far more by temperament than by any intellectual problems. There are certain emotional difficulties which are intrinsic to the mathematical life, and only a few people are able to live with them all their lives.

First of all, the mathematician must be capable of total involvement in a specific problem. To do mathematics, you must immerse yourself completely in a situation, studying it from all aspects, toying with it day and night, and devoting every scrap of available energy to understanding it. You can permit yourself occasional breaks, and probably should; nevertheless the state of immersion must go on for somewhat extended periods, usually several days or weeks.

Second, the mathematician must risk frustration. Most of the time, in fact, he finds himself, after weeks or months of ceaseless searching, with exactly nothing: no results, no ideas, no energy. Since some of this time, at least, has been spent in total involvement, the resulting frustration is very nearly total. Certainly it seriously affects his attitude toward all other affairs. This factor is a more important hindrance than any other, I believe; to risk total frustration, and to be almost certain to lose, is a psychological problem of the first rank.

Next, even the most successful mathematician suffers from lack of appreciation. Naturally his family and his friends have no feeling for the significance of his accomplishments, but it is even worse than this. Other mathematicians don't appreciate the blood, sweat, and tears that have gone into a result that appears simple, straightforward, almost trivial. Mathematical terminology is designed to eliminate extraneous things and focus on fundamental processes, but the method of finding results is far different from these fundamental processes. Mathematical writing doesn't permit any indication of the labor behind the results.

Finally, the mathematician must face the fact that he will almost certainly be dissatisfied with himself. This is partly because he is running head-on into problems which are too vast ever to be solved completely. More important, it is because he knows that his own contributions actually have little significance. The history of mathematics makes plain that all the general outlines and most of the major results have been obtained by a few geniuses who are not the ordinary run of mathematicians. These few big men make the long strides forward, then the lesser lights come scurrying in to fill the chinks, make generalizations, and find some new applications; meanwhile the giants are making further strides.

Furthermore, these giants always appear at an early age—most major mathematical advances have been made by people who were not yet forty—so it is hard to tell yourself that you are one of these geniuses lying undiscovered. Maybe it is important for someone to fill in the little gaps and to make the generalizations, and it is probably necessary to create an atmosphere of mathematical thought so that the geniuses can find themselves and thrive. But no run-of-the-mill mathematician expects in his heart to prove a major theorem himself.

I wonder how much of this psychological difficulty is present in other scholarly fields. I suspect that no other field suffers so acutely from all four problems. The experimental sciences in particular, I think, are pretty well preserved from the second and third difficulties. An experimentalist can perform an experiment and, at the end, will have a set of data; and these data at least will indicate that such-and-such either is or is not significant. He knows before he starts the experiment that, except for equipment failure, he will finally have *something*. He is not faced with nearly certain frustration. Furthermore, publication standards permit experimentalists to describe details of procedures followed and difficulties encountered.

I also think the experimentalist has a reasonable hope for personal satisfaction. Experimental advances are frequently made by unknowns; in fact, there aren't many experimentalists in history who have consistently made important discoveries, if we don't count those who have been lucky enough to head active research organizations for long periods.

Whether other speculative disciplines are immune from the four emotional problems I've outlined isn't clear to me. But I feel that differing standards of precision may ease the problem of frustration, in the sense that it is often possible in these other fields to hide the fact that you don't have anything to say. A mathematician who says nothing in an obscure manner is usually caught quickly—but, alas, not always.

DONALD R. WEIDMAN
Department of Mathematics, Boston College, Chestnut Hill, Massachusetts

Force-Free Body: A Thought Experiment

Hanson's interesting essay, "Galileo's discoveries in dynamics" (29 Jan., p. 471), stresses the point that, even in a purely conceptual universe, a particle can never be totally free from unbalanced external force; therefore the law of uniform rectilinear motion (the law of inertia) can never, even in principle, be tested. This conclusion results from the evident need for a measuring rod, a clock, and an observer as the minimum furniture in an otherwise bare conceptual universe in order to demonstrate uniform rectilinear motion, but these material bodies exert an unbalanced gravitational force on the particle under test.

One *can*, however, conceive of a universe in which the test body is totally force-free while its velocity is measured as a function of time. Let all the matter in this universe be disposed in concentric, uniform, spherical shells surrounding the test body. Then the test body is force-free as long as it stays within the otherwise empty space inside the smallest shell.

The only remaining conceptual problem consists of making the measuring instruments into spherical shells. (For example, optical lenses cannot be employed without disturbing the spherical symmetry.) The following arrangement seems to meet the requirement. The inner shell is thin and black, except for pinholes at two or more widely separated locations. A transparent substance of equal density fills the pinholes so the gravitational uniformity of the spherical shell is not disturbed. The next shell is transparent and thick (it could even be empty space) and is surrounded by a layer of photographic emulsion. The test body contains a clock which controls the periodic flashing of an isotropic light. (Uniform radiation in all directions is necessary so that the test body experiences zero net-radiation force.) Thus, if the motion of the test body relative to the shells is in a straight line at constant speed, equally spaced rectilinear (after due correction for the geometrical optics involved) images will appear in the photographic emulsion behind each pinhole. After the experiment is completed, the observer comes in from infinity and determines the positions of the images.

Under the conditions of this "thought experiment" the test body is rigorously force-free while its velocity is recorded, thereby refuting Hanson's statement that "the counterfactual character of Galileo's law stands not merely as an observation that no bodies *are* found to be force-free but, rather, as a consequence of there being no body whose motion is uniform and rectilinear which could possibly be force-free. Any alternative crushes the gravitational cornerstone of mechanics. Appraisals of the law's logical status are pierced by this point. The law thus refers to entities *not* such that, although never observed, they remain observable but, rather, entities that are unobservable in physical principle."

MARVIN M. MUELLER
307 Manhattan Loop,
Los Alamos, New Mexico

The ingenious thought experiment suggested by Mueller is doubly admirable: it is provocative and instructive intrinsically, and it indicates the need for greater clarity in the exposition of my work on Galileo. It was not my intention to deny the possibility of conceiving of a force-free body in general, but rather only in the kind of universe we actually inhabit. The concept makes sense; how else could I meaningfully deny that it has any application in this world? My point was that we could imagine nothing which would give the concept any application whatever in *this* world, in Galileo's.

Mueller shrewdly delineates another universe within which all the constituent matter is arranged in spherical shells, nullifying thereby any unbalanced gravitational force acting on a test body within the innermost shell. Of this thoughtful theoretical thrust, however, three things can be said, I think.

1) Mueller has changed the conditions of the thought experiment fashioned in my paper. This I invited him to do by failing to indicate that my construction was bounded by the known properties of this universe, given which one could not even imagine a body as force-free.

2) Archimedes cried out for a fixed platform from which he could lever away the world. He did not deny that from a second platform his own original "thought platform" could itself be levered away. Mueller has pointed out that the assumptions we make in studying the dynamics of this world could themselves be demonstrated in some other world; but this does not establish such another world to be assumption-free. Consider Mueller's own presuppositions: (i) that light corpuscles (such as those emitted from his test body) traverse Euclidean straight lines, and (ii) that these corpuscles traverse equal areas in equal times. Without these assumptions Mueller, coming "in from infinity," could not infer from equally spaced rectilinear images in the emulsion an inertial motion in the test body. (Notice the phrase "after due correction for the geometrical optics involved"; his assumption of rectilinear, uniform trajectories for all light corpuscles is obvious here.)

3) In Galileo's world Mueller's assumptions are unnecessary. The kinematical properties of photons can be proved, not just presupposed, by demonstrating that light corpuscles traverse paths *that would be traversed by force-free bodies*, if they existed. Mueller's think-tank employs a different springboard; he demonstrates that force-free material particles (in his spherical universe) traverse paths that *would be traversed by light corpuscles.* He assumes what Galileo could show; Galileo assumed what Mueller can show. But whoever undertakes to show anything whatever must assume something or other—that is part of the logic of proof. Galileo and Mueller both feel it makes sense to think of "force-free bodies." But, given Galileo's other commitments, it does not represent a genuine possibility in this universe, as I tried to show. Mueller transforms the idea into a genuine possibility by changing his universe.

N. R. HANSON
Department of Philosophy, Yale University, New Haven, Connecticut

Numerical Analysis: Pure or Applied Mathematics?

The exchange of views (Letters, 16 July, p. 243) subsequent to Hamming's article "Numerical analysis vs. mathematics" (23 April, p. 473) raises a number of interesting points on the relation between science and mathematics and whether numerical analysis should be viewed as being closer in spirit, outlook, or working methods to one or the other. In my view, all parties to the dispute have come close to, but seem to have missed, what may well be some essential aspects of the relations between the three fields (or two, if one wishes to assign numerical analysis to one of the others).

An important distinction between science and mathematics, which is appreciated by all concerned, is the empirical content of the former, that is, the appeal to experiment, and the purely formal structure of the latter. One might make a comparison with language; the rules of grammar or syntax permit us to distinguish between grammatical and ungrammatical sentences, but they can make no statements about the world of phenomena and everyday objects which we use the language to discuss. Experience, observation, or experiment can be viewed as providing the semantic content, or making meaningful the vocabulary that is used to discuss the world of science, while