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Stability of Lakes near the Temperature of Maximum Density

Abstract. *In deep lakes that are near the temperature of maximum density, stability is greatest when the decrease of temperature with depth is about one-half of the decrease in temperature of maximum density with depth. Observed temperature decreases in lakes under such conditions tend to follow the curve that produces greatest stability.*

It was shown (*1*) that a simple equation of state for water, valid at moderate pressures and near the temperature of maximum density, could be derived from compressibility data; the latter were computed from sonic velocities. The equation of state yielded a rate of decrease of temperature of maximum density with increase of pressure of 0.021°C/bar.

If the equation of state is valid, then other inferences drawn from the equation should be consistent with observation. My purpose is to propose three corollaries and to show that they explain observed temperature profiles in deep lakes near maximum density. These corollaries are:

I) If a column of water is mechanically stable, the temperature-depth curve must change its sign of slope where it crosses the line of maximum density.

II) A lake that is at the temperature of maximum density throughout is not mechanically stable.

III) If the surface of a lake is at maximum density, stability is greatest when the change of temperature with depth is one half of the change in temperature of maximum density.

The equation of state previously derived is

$$v = v_0[1 - p(a + b\theta) + c\theta^2] \qquad (1)$$

where v is specific volume, v_0 is specific volume at 4°C and atmospheric pressure, p is "gauge" or "lake" pressure, θ is $(4.00\text{-}T)$°C, a is 49.458×10^{-6}/bar, b is 0.327×10^{-6}/bar°C, and c is $7.8 \times 10^{-6}/(°C)^2$. It was shown by partial differentiation of Eq. 1 that the line of maximum density has a slope of $b/2c$ or 0.021°C/bar. Thus, if θ' is defined as the temperature of maximum density,

$$\theta' = (b/2c)p \qquad (2)$$

In general, an equation of state is $v = f\ (p,\theta)$. From differential calculus

$$\mathrm{d}v = \left(\frac{\partial v}{\partial p}\right)_\theta \mathrm{d}p + \left(\frac{\partial v}{\partial \theta}\right)_p \mathrm{d}\theta$$

or

$$\frac{\mathrm{d}v}{\mathrm{d}p} - \left(\frac{\partial v}{\partial p}\right)_\theta = \left(\frac{\partial v}{\partial \theta}\right)_p \frac{\mathrm{d}\theta}{\mathrm{d}p} \qquad (3)$$

∂ and d being partial and total derivatives, respectively. The left side of Eq. 3 provides a standard of stability, for $\mathrm{d}v/\mathrm{d}p$ is an observed change of specific volume with pressure, or depth, in a lake, and $(\partial v/\partial p)_\theta$ is the corresponding isothermal change. The latter is related to the isentropic derivative by the equation

$$\left(\frac{\partial v}{\partial p}\right)_\theta = \left(\frac{\partial v}{\partial p}\right)_S - \frac{T}{c_p}\left(\frac{\partial v}{\partial T}\right)_p^2$$

whose last term becomes vanishingly small near the temperature of maximum density (S denotes entropy). This equation shows that there is no internal heating or cooling when water at the temperature of maximum density is compressed or expanded adiabatically; therefore the adiabatic and isothermal processes are identical.

An observed temperature profile, $\mathrm{d}v/\mathrm{d}p$, is stable if, and only if,

$$\frac{\mathrm{d}v}{\mathrm{d}p} - \left(\frac{\partial v}{\partial p}\right)_\theta < 0$$

That is, for stability a small displacement must induce a restoring force. Therefore, from Eq. 3, stability also implies that

$$\left(\frac{\partial v}{\partial \theta}\right)_p \frac{\mathrm{d}\theta}{\mathrm{d}p} < 0 \qquad (4)$$

Corollary I is proved in the following manner. Partial differentiation of Eq. 1 gives

$$\left(\frac{\partial v}{\partial \theta}\right)_p = v_0(-bp + 2c\theta)$$

But from Eq. 2, $bp = 2c\theta'$, so that

$$\left(\frac{\partial v}{\partial \theta}\right)_p = 2cv_0(\theta - \theta')$$

Equation 3 then becomes

$$\frac{\mathrm{d}v}{\mathrm{d}p} - \left(\frac{\partial v}{\partial p}\right)_\theta = 2cv_0(\theta - \theta')(\mathrm{d}\theta/\mathrm{d}p)$$

or, in temperatures Celsius for easier visualizing,

$$\frac{\mathrm{d}v}{\mathrm{d}p} - \left(\frac{\partial v}{\partial p}\right)_T = 2cv_0(T - T')\frac{\mathrm{d}T}{\mathrm{d}p} \qquad (5)$$

Then from Eqs. 4 and 5 the column of water is stable if, and only if,

$$2cv_0(T - T')\frac{\mathrm{d}T}{\mathrm{d}p} < 0 \qquad (6)$$

If the temperature profile crosses the line of maximum density so that $(T - T')$ reverses sign, then, by Eq. 6, $\mathrm{d}T/\mathrm{d}p$ must also reverse sign to maintain stability. Thus corollary I is proved.

If a lake is isothermal, so that in Eq. 5

$$\frac{\mathrm{d}T}{\mathrm{d}p} = 0,$$

the condition of Eq. 6 is not satisfied and the lake is not stable. Physically, a vertical displacement of a volume of water in an isothermal (adiabatic) lake profile would not result in a return force. Likewise, if a lake is at maximum density,

$$(T - T') = 0;$$

nor is the condition of Eq. 6 satisfied, and a lake at maximum density does not possess stability. In this case, the lack of a return force is strictly true only for a small vertical displacement. However, the absence of such a return force is equivalent to an absence of stability, and corollary II is therefore proved.

Corollary III is proved by defining a family of linear temperature profiles passing through 4°C at atmospheric pressure, and with the equation

$$\theta = (rb/2c)p \qquad (7)$$

Note that when the parameter $r = 0$ the profile is isothermal, whereas when $r = 1$ the profile is at maximum density. Let ϕ represent the right side of Eq. 3, so that

$$\phi = \left(\frac{\partial v}{\partial \theta}\right)_p \frac{\mathrm{d}\theta}{\mathrm{d}p}$$

Maximum stability occurs when the value of ϕ is minimum. Again by partial differentiation of Eq. 1

$$\left(\frac{\partial v}{\partial \theta}\right)_p = v_0(-bp + 2c\theta)$$

Therefore

$$\phi = v_0(-bp + 2c\theta)(rb/2c)$$

By substituting $2c\theta$ from Eq. 7,

$$\phi = [(v_0b^2p)/2c](r^2 - r) \quad (8)$$

In Eq. 8, ϕ has the minimum value when the parameter r has the value ½. The temperature profile with the greatest stability has the equation

$$\theta = (b/4c)p$$

The rate of decrease of temperature with pressure, $b/4c$, is about 0.010°C/bar. This is one-half of the decrease of temperature of maximum density with pressure; corollary III is therefore proved.

It has been reported (*1*) that among the temperature profiles reported by Strøm (*2*), those that cross the line of maximum density tend to change sign of slope where they cross the line. No reported temperature profile coincides with the computed line of maximum density. Finally, Strøm's "envelope," which tends to parallel all his reported profiles, has a slope of about 0.011°C/bar, which is one-half the slope of the line of maximum density and, as we have shown, is the line of greatest stability; thus all three of the proposed corollaries are consistent with Strøm's data.

My conclusions are also supported by Johnson's observations on Great Bear Lake (*3*). He noted that the temperature profile of 26 July 1963 reversed its sign of slope at about the depth at which it crossed the computed line of maximum density. The observed decrease in temperature with depth in Great Bear Lake was about 0.011°C/bar, which is about the temperature profile with the greatest stability according to corollary III. I conclude that data reported by Strøm and Johnson are in agreement with the line of maximum density previously computed by me at 0.021°C/bar (*1*).

HENNING EKLUND

*U.S. Public Health Service,
433 West Van Buren Street,
Chicago, Illinois 60607*

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16 October 1964

Cytopathic Effect of Rubella Virus in a Rabbit-Cornea Cell Line

Abstract. *A rabbit-cornea cell line has been found susceptible to three strains of RK-13–adapted rubella virus. The lesions are very pronounced and appear within a week.*

Weller and Neva first described cytopathic effects of rubella virus in continuously cultured primary human amnion cells (*1*). The changes were, however, slow to appear and difficult to detect. McCarthy *et al.* overcame this difficulty; they used a rabbit-kidney cell line (RK-13) which was sensitive to rubella virus and which showed upon infection very characteristic microfoci formation (*2*). The search for microfoci in inoculated cultures is, however, occasionally difficult and laborious. Cytopathic effect of rubella virus involving the entire cell sheet was reported by Günalp in a line of green-monkey kidney cells (GMK, AH-1) where changes appear in about 2 weeks unless the virus has been previously adapted to the cell line (*3*).

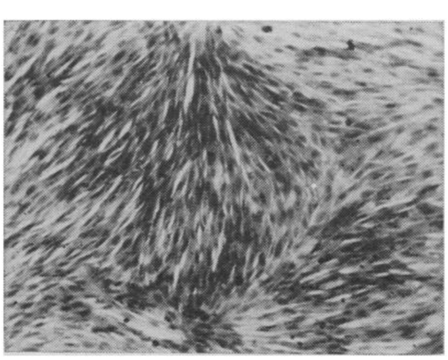

Fig. 1. Uninoculated rabbit-cornea cells (× 83).

I now report the cytopathic effect of rubella virus in a rabbit-cornea cell line established some years ago at this institute by M. Volkert.

The following virus strains were used in the experiment: "Judith" supplied by A. Svedmyr, Stockholm; "RV" supplied by J. L. Sever, Bethesda, Maryland; and "Orta" supplied by J. L. Melnick, Houston, Texas. The three virus strains were adapted to RK-13 cells in this laboratory. For comparison, the virus strains were titrated simultaneously in the rabbit-cornea cell line and in RK-13 cells originally supplied by G. M. Schiff, Bethesda. The growth medium for the rabbit-cornea cells was Earle's solution supplemented with 10 percent of calf serum, and containing (per liter) lactalbumin hydrolyzate, 1.7 g; yeast extract "Difco," 0.57 g; sodium bicarbonate, 0.84 g; streptomycin sulfate, 0.05 g; and penicillin, 200,000 international units. The RK-13 cells were grown in Medium 199 supplemented with 5 percent calf serum and containing (per liter) sodium bicarbonate, 1.12 g; streptomycin sulfate, 0.05 g; and penicillin, 200,000 international units. Both cell lines were grown at 37°C. The maintenance medium for both cell lines was Medium 199 with sodium bicarbonate (2.24 g/liter); penicillin, streptomycin, and 1 percent of inactivated calf serum. After the medium was changed the tubes were inoculated with rubella virus and rotated at 34°C.

When uninoculated, the rabbit-cornea cell line (Fig. 1) forms a monolayer of densely packed oblongated cells with a fibroblast-like appearance.

In cultures inoculated with rubella virus (Fig. 2) the cytopathic effect usually is discernible 4 days after inoculation. On the 7th day an extensive degeneration of the cell sheet is seen which on the 8th to 10th day reaches total degeneration with the cells falling off the glass.

In cells removed by the collodion-membrane method (*4*) and stained by hematoxylin and eosin the first manifestations of the cytopathic effect is vacuolation of the cytoplasm at the poles of the cell. Next a rounding of the nucleus from the normal oval to a circular shape is seen after a retraction of the cytoplasm around the altered nucleus. Further, the nucleus becomes more dense, and the cytoplasm develops an "ink-spot" appearance with pear-shaped drops radiating from a

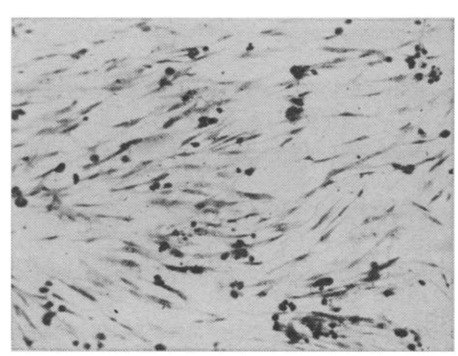

Fig. 2. Rabbit-cornea cells inoculated with 100 $TCID_{50}$ rubella virus, strain Judith; 7th day after inoculation (× 83).