

ers may hesitate upon encountering such terms as *secateurs*, *rubber bungs*, and *nail varnish*. These should prove to be minor distractions, however, in view of the book's overall usefulness as a laboratory reference source.

The breadth of information embraced by the 17 chapters can perhaps be best summarized by the following list of contents—(chapter 1) general laboratory apparatus and techniques; (chapters 2, 3, and 4) histology, including preserving, fixing, embedding, cutting, mounting, and staining; (chapter 5) special histological techniques such as maceration, whole mounts, fossil peels, and autoradiography with stripping film; (chapter 6) some useful tests for biologically important substances; (chapter 7) care of house plants and aquaria; (chapter 8) museum and herbarium work—for example, techniques of liquid and dry preservation; (chapter 9) culture and sterilization techniques and specifications for approximately 30 useful media; (chapter 10) inoculation and culture techniques; (chapter 11) demonstration and measurement of growth; (chapter 13) plant-water relations; (chapter 14) manometry; (chapter 15) paper and column chromatography; (chapter 16) distillation; and (chapter 17) analysis of soils, water, and plant materials.

Each chapter is followed by a list of references or a bibliography, a very valuable feature. Of less use to readers in the United States are two short sections that follow chapter 17, which provide supply sources (all of which are in the United Kingdom) for various forms of plant material and sample examination questions (two to five questions per chapter) for use by those who seek certification as laboratory technicians in England.

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Geometry

Regular Figures. L. Fejes Tóth. Pergamon, London; Macmillan, New York, 1964. xii + 339 pp. Illus. \$12.

This book seems to have everything that could be desired in a mathematical monograph—a pleasant style, careful explanation of all technical terms used, a great variety of topics with a single unifying idea, a good bibliography and

index, and many beautiful illustrations, including four plates and a pocketful of stereoscopic anaglyphs. Although the work is mostly concerned with geometry, it has connections with art, crystallography, biology, city planning, and the standardization of industrial products.

Chapter 1, "Plane ornaments," includes the first complete and readable proof in English that there are exactly 17 essentially different wallpaper patterns, a fact that was utilized unconsciously by the Moors in decorating the Alhambra. It was first proved by E. S. Fedorov in 1891 and has been rediscovered as a mathematical theorem at least three times since then. This chapter ends with an almost incredible tour de force due to H. Voderberg—a systematic (but completely asymmetric) tessellation of the Euclidean plane with congruent tiles, each of which is an irregular enneagon (9-gon).

Using the notation of L. Schläfli (1814–1895), Fejes Tóth defines the regular tessellation $\{p, q\}$ as consisting of equal regular p -gons, q round each vertex. In chapter 2 he uses the same notation for a regular *spherical* tessellation, and proves that such a pattern exists for every pair of positive integers (except $p = q = 1$) satisfying the inequality

$$\frac{1}{p} + \frac{1}{q} > \frac{1}{2}.$$

For instance, $\{2, 1\}$ has one face (a digon), one edge (a semicircle such as a meridian), and two vertices (the north and south poles). In this connection, he should perhaps have mentioned the beautifully illustrated paper by H. Emde, "Homogeneous Polytope" [*Math. Revs.* **21**, 1105 (1960)].

In chapter 3, reversing the above inequality, Fejes Tóth obtains the infinite family of hyperbolic (non-Euclidean) tessellations $\{p, q\}$. As background for this discussion, he gives a clear but concise account of the theory of inversion, the invariance of cross ratio, and the two conformal models of the hyperbolic plane. On page 97 the drawings of $\{3, \infty\}$ and $\{\infty, 3\}$ are particularly striking. (Some readers may be puzzled by the unusual notation $\sin^{-1}\beta$ for $\operatorname{cosec} \beta$ on page 93.) It is interesting to be reminded of the contrast between the successful career of Lobachevsky and the tragic life of Bolyai, "unhappy in his marriage, broken in health . . . hated and cast out by the philistines of a small town

owing to his uncompromising straightforwardness, separated from mathematical life, books and periodicals, but fully aware of the significance of his epoch-making discovery and in full possession of his sound judgment."

Chapter 4 deals with the most important kinds of polyhedra—regular, semiregular, and so on. Fedorov's five parallelohedra are constructed by the method of B. N. Delaunay, who "succeeded in giving a complete enumeration of their 4-dimensional analogues (whose number of types turned out to be 52)." This remark leads naturally to the treatment of regular polytopes in chapter 5, where the discovery of n -dimensional geometries (by Grassmann and Schläfli) is described as enabling us "to create an infinite set of new universes, the laws of which are within our reach, though we can never set foot in them."

Despite the high quality of these first five chapters, it is in the remaining five that the author reveals the full scope of his mathematical ingenuity. Here we see many examples of the extremal problems for which he and other Hungarian geometers are justly famous. The spaces considered are the same (and in the same order) as in chapters 1–5. For instance, chapter 7, on spherical figures, deals with such problems as distributing n points on a sphere so that the minimal distance between pairs is maximized. This "problem of Tammes" is introduced on page 226 with a charming description of its botanical origin. Its difficulty is made evident by the fact that, despite the best efforts of the world's geometers for 35 years, it has only been solved for $n \leq 12$ and for $n = 24$ (the last case by R. M. Robinson in 1961).

We have another example on page 253—two particularly fine drawings to illustrate the theorem that, if the hyperbolic plane is packed or covered by equal circles (of finite or infinite radius), then the packing density $\leq 3/\pi$ and the covering density $\geq 2\sqrt{3}/\pi$.

Among the few misprints that I noted, only the following are serious— $\{3, 4, 4\}$ for $\{3, 4, 3\}$ on page 140, line 7; $(2n-1)/2$ for $(n-1)/2$ on page 202, line 5; R for $(RR_1R_2)^2$ at the end of page 290; and "consequent" for "consistent" on page 306, line 14.

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