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High-Field Galvanomagnetic Properties of Metals

The effect a magnetic field has on the electrical resistance helps define the Fermi surfaces of metals.

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What is the shape of the Fermi surface and how does it affect the electrical properties of a metal? This is a question frequently asked in one of the most active fields of solid-state research. In the last 10 years the study of the Fermi surfaces of metals has advanced considerably, partly through development of new experimental techniques and partly through refurbishment of old ones. Two general articles published recently (*1*) show how the electronic structure of a metal is described in terms of the shape of its Fermi surface and also survey the major experimental techniques used to determine the shape. It is our aim in this article to give a more detailed description of the effect a magnetic field has on the electrical properties of metals and to discuss some recent results obtained by the measurement of these properties.

In the late 1930's, measurements of the galvanomagnetic properties (that is, the magnetoresistance and Hall effect) of single crystals of pure metals at liquid helium temperatures and in magnetic fields of several kilo-oersteds indicated that a wealth of information could be obtained about the shape of the Fermi surface and about the carrier scattering processes if only one knew how to interpret the data. The resistance of some metals increases to large values in high magnetic fields, whereas the resistance of others increases very little and approaches a constant value. An even more interesting result is that in some metals the resistance is highly anisotropic and may be increased or decreased by several orders of magnitude simply by changing the direction of the magnetic field by a few degrees.

These complicated experimental results remained a puzzle until 1956, when I. M. Lifshitz and his co-workers (*2*) developed a theory capable of explaining them. Since then, refinements of the theory, coupled with advances in metallurgy and measuring equipment, have made the galvanomagnetic effects a simple yet powerful tool for studying the shape of the Fermi surface.

Theory

A metal is a good electrical conductor because its conduction electrons are free to move through the lattice of positive ions. The position of an individual electron in the metal is completely unknown, but, by the uncertainty principle, its momentum is well defined. The description of the electronic structure of a metal therefore consists of specification of the energy $E(\mathbf{p})$ as a function of the momentum $\mathbf{p}$ of each electron. The momentum may be represented by a point (or state) whose Cartesian coordinates are the components of $\mathbf{p}$ in three perpendicular directions; this is usually referred to as the representation in momentum-space. The exclusion principle permits only two electrons (one of positive and one of negative spin) to have the same momentum, so that, to achieve minimum energy, the momentum states of increasing energy are successively filled until all the electrons are accounted for. The boundary between these filled states and the empty states of higher energy is a surface of constant energy called the Fermi surface. The Fermi surface reflects the *symmetry* of the crystal in position-space, but it still may have a very complicated *shape*, determined by the periodic potential field of the ionic lattice.

To understand the galvanomagnetic properties of metals we must first describe the motion of an electron of charge e and velocity $\mathbf{v}$ moving under the combined influence of the periodic field of the lattice and the Lorentz force. The latter is the force exerted by a magnetic field $\mathbf{B}$ on a moving charge; it may be written,

$$\mathbf{F} = \frac{e}{c} (\mathbf{v} \times \mathbf{B}) \tag{1}$$

where c is the speed of light. The description of the electronic structure of a metal in terms of the shape of its Fermi surface enables us to define the motion in a simple geometric way. Each electron has a constant energy (E) and a constant component of momentum along the magnetic field (p_B), since, from Eq. 1, $\mathbf{F}$ is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$. Thus, to determine the motion of the electrons on the Fermi surface (constant E) in the magnetic field, we take all plane sections (constant p_B)

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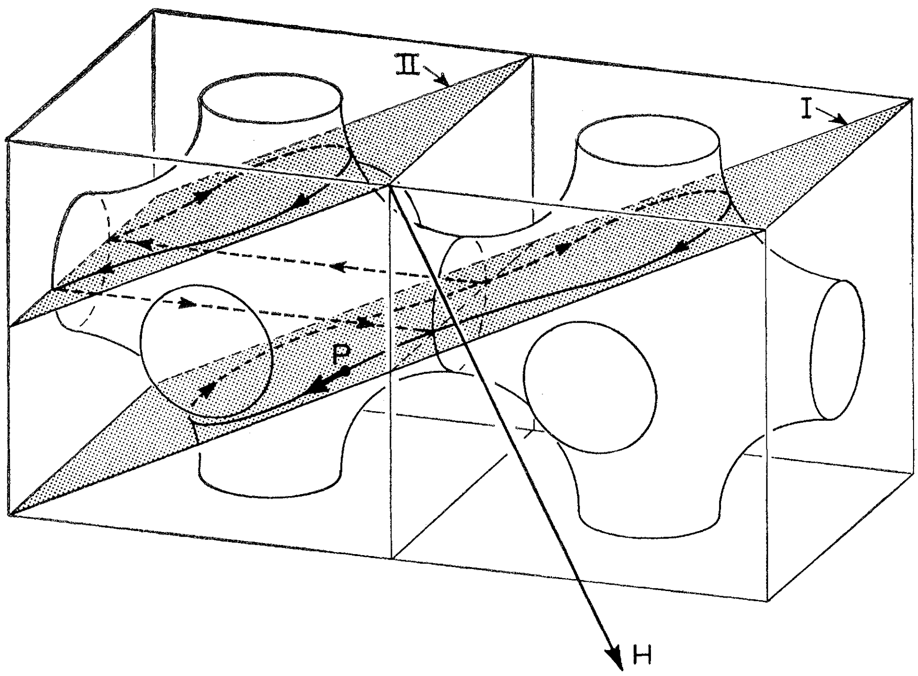

Fig. 1. Possible multiply-connected Fermi surface for a simple cubic metal, showing the construction for a cyclotron orbit passing through point P.

of the surface perpendicular to the field, and the perimeter of each section defines an orbit in momentum-space.

The frequency of the periodic motion around the orbit is known as the cyclotron frequency, ω_c, which may be written,

$$\omega_c = \frac{eB}{m^*c} \quad (2)$$

In Eq. 2, m^* is the cyclotron mass, which depends on the average velocity around the orbit and is usually of the same order of magnitude as the free electron mass. The motion in *position-space* may be more complicated. Corresponding to each orbit in momentum-space, the electron in position-space describes a similar orbit in the plane perpendicular to the field but in addition has a velocity parallel to the field so that the resultant path in position-space is a helix. Fortunately, for most purposes it is sufficient to consider only the motion of the electron in momentum-space.

So far we have not mentioned the scattering of the electrons due to impurities, imperfections, and thermal vibrations of the ionic lattice. These scattering processes, which give rise to the resistance of the metal, vary in a complicated way over the Fermi surface and, together with the shape of surface and the variation of the electron's velocity over it, determine the galvanomagnetic properties of the metal in low magnetic fields. However, the detailed nature of the scattering processes becomes unimportant in magnetic fields so high that all electrons on the Fermi surface execute many orbits before scattering. This is known as the high-field region and can be defined formally by the condition, $\omega_c\tau >> 1$, where τ is the electron scattering (or relaxation) time. The beauty of Lifshitz's theory (2), which treats the galvanomagnetic properties of metals in the high-field region, is that the most striking characteristics of the behavior are governed solely by geometric features of the Fermi surface, and the scattering processes are relatively unimportant.

Before discussing the dependence of the high-field galvanomagnetic properties of a metal on the geometry of its Fermi surface, we consider the various types of orbit. A possible Fermi surface for a simple cubic metal is shown in Fig. 1. It contacts the {100} faces of the Brillouin zone, which is the polyhedron (in this case a simple cube) defined by the set of planes in momentum-space where the wavelength of the electron matches the periodicity of the ionic lattice, and the electron experiences a Bragg reflection. With the field in the direction shown, there exists a closed orbit which starts at P and moves along the intersection of the Fermi surface with plane I until it intersects the (100) zone boundary. The Bragg reflection then reverses the x-component of the momentum so that the representative point is translated through a reciprocal lattice vector onto the ($\bar{1}00$) zone boundary and continues the orbit in plane II, until it translates back into plane I and returns to point P. An equivalent construction is to translate the whole Brillouin zone and Fermi surface through a reciprocal lattice vector and form what is known as the periodically extended zone scheme. This has the advantage in that the whole orbit can now be described in plane I.

Figure 2 shows the open Fermi surface which is formed by periodically extending the surface shown in Fig. 1 in three perpendicular directions and omitting the Brillouin zone boundaries. In Fig. 2a the field is applied along a cube axis, and only closed orbits are possible, whatever the elevation of the plane perpendicular to the field. The electron orbit in Fig. 2a is so described because it surrounds a region of occupied states, and, conversely, the hole orbit surrounds a region of unoccupied states or holes. It is important to realize that the descriptions would be reversed if the surface shown in Fig. 2 were a hole surface—that is, if the regions within the surface were unoccupied states. In polyvalent metals, hole surfaces, which may be open, occur as commonly as electron surfaces.

If the field is rotated away from the cube axis but is still kept in the (100)-plane (Fig. 2b), we have an open orbit, so called because an electron will travel from zone to zone and never return to its original position. If the field is now tilted out of the (100)-plane (Fig. 2d), the open orbit becomes an extended orbit, unless the field is so close to the cube axis that the complicated open orbit shown in Fig. 2c is generated.

The first of the high-field galvanomagnetic properties we consider is the Hall effect, which is a voltage proportional to $\mathbf{J} \times \mathbf{B}$ and perpendicular to both the current $\mathbf{J}$ and to the magnetic field $\mathbf{B}$. This effect arises because the charge carriers tend to be deflected to one side by the Lorentz force (Eq. 1). Charge builds up on the sides of the sample until a transverse electric field $\mathbf{E}_H$ is produced, called the Hall field, which just balances the Lorentz force; it is given by the equation,

$$q\mathbf{E}_H = -\frac{q}{c}[\mathbf{v} \times \mathbf{B}]. \quad (3)$$

The sign of $\mathbf{E}_H$ for a given current density $\mathbf{J}$ depends on the sign of the charge q of the carrier, since

$$\mathbf{J} = Nq\mathbf{v}. \tag{4}$$

Combining Eqs. 3 and 4 we have,

$$\mathbf{E}_H = \frac{1}{Nqc}\mathbf{J} \times \mathbf{B} \tag{5}$$

where N is the number of carriers per unit volume. The Hall coefficient R relating $\mathbf{E}_H$ to $\mathbf{J}$ and $\mathbf{B}$ is then,

$$R = \frac{1}{Nqc} = \frac{\Omega}{nqc} \tag{6}$$

where n is the number of carriers per unit cell of the lattice of volume Ω.

When the Fermi surface of the metal comprises both electron and hole surfaces, Eq. 6 becomes

$$R = -\frac{\Omega}{ec(n_e - n_h)} \tag{7}$$

In Eq. 7, n_e and n_h are the number of electrons and the number of holes, and we have replaced q by $-e$, the electronic charge. Equation 7 expresses the result that electrons and holes produce Hall fields of opposite sign.

Although this simple derivation of the Hall effect is based on a free-electron model, the results are still true in the high-field region for real metals with complicated Fermi surfaces, except when the numbers of electrons and holes are equal ($n_e = n_h$). When this occurs the metal is said to be compensated, and Eq. 7 is no longer applicable. The correct expression for the Hall field is obtained by including the effect of compensation in the conductivity tensor before inverting the conductivity to obtain the resistivity tensor. In a compensated metal the "transverse-even voltage" (*3*), which becomes field-independent and is said to *saturate* in an uncompensated metal, is quadratic in B and dominates the Hall voltage linear in B.

Equality of the numbers of electrons and holes occurs in polyvalent metals more often than one might at first expect. This is because the capacity of a Brillouin zone is just two electrons per unit cell, so that, for any metal with an even number of electrons per cell (*4*), the number of electrons spilling over into higher energy bands may exactly equal the number of holes left vacant in lower energy bands. It can be shown also that, for any metal with an odd number of electrons per unit cell, the net number of carriers per cell, as determined from the Hall coefficient (that is, the algebraic sum of the number of electrons, $-n_e$, and the number of holes, $+n_h$), must be a positive or negative odd integer (*5*).

The other high-field galvanomagnetic property of interest is the transverse magnetoresistance, which is the relative increase in a metal's resistance when the metal is placed in the magnetic field. It is usually expressed as the change in resistance when the metal is placed in the field divided by the resistance in zero field—that is, $[\rho(B) - \rho(0)]/\rho(0) = \Delta\rho/\rho$. In a simple free-electron metal, where all the carriers have the same sign and velocity, there is no magnetoresistance. This is because, in the state of steady current flow, the force due to the Hall field exactly balances the Lorentz force, so that the electrons are not deflected. Thus the very existence of a nonzero magnetoresistance indicates that the electrons in a metal cannot be adequately represented by a free-electron gas.

The field-dependence of the magnetoresistance in the high-field region with no open orbits is radically different for a compensated metal ($n_e = n_h$) and for an uncompensated metal ($n_e \neq n_h$). The reason for this can be seen from Eq. 3. In an uncompensated metal the transverse Hall field and the Lorentz force achieve a balance such that, although the magnetoresistance is nonzero ($\Delta\rho/\rho > 0$) because of the anisotropy of the Fermi velocity and the scattering processes, further increase of the field in the high-field region ceases to increase the resistance, and the magnetoresistance saturates. On the other hand, in a compensated metal the Hall fields cancel each other, because of the equal numbers of carriers of opposite sign. This cancellation leads to a resistance which increases quadratically with increasing field (*6*).

The most useful feature of the galvanomagnetic properties in the high-field region for exploring the shape of the Fermi surface is the fact that these properties are drastically affected by

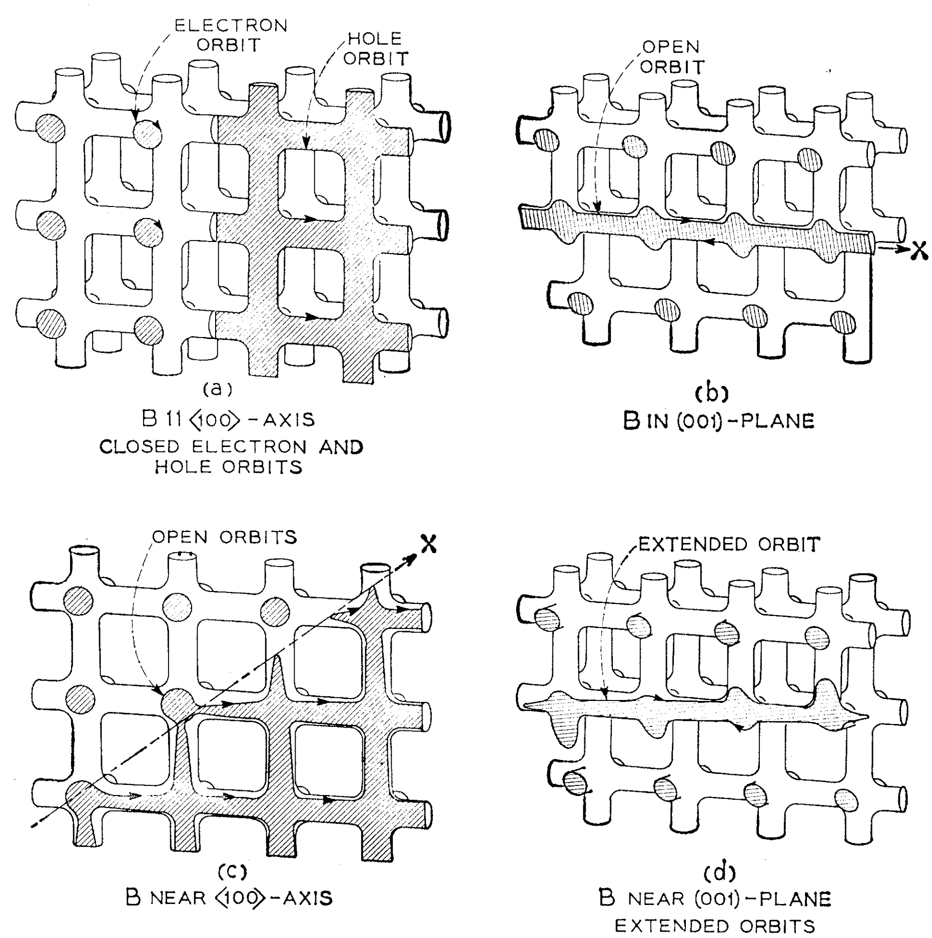

Fig. 2. The periodically extended zone scheme for the Fermi surface illustrated in Fig. 1, showing the possible types of open and closed cyclotron orbits. [From R. G. Chambers, in *The Fermi Surface*, W. A. Harrison and M. B. Webb, Eds. (Wiley, New York, 1960)]

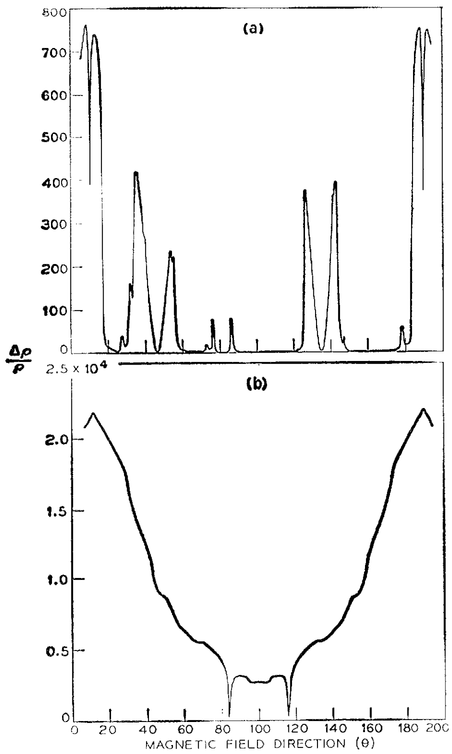

Fig. 3. Typical curves for the magnetoresistance in the high-field region plotted as a function of the magnetic field direction for (*a*) an uncompensated metal (copper) and (*b*) a compensated metal (zinc).

open orbits. If for some field direction any part of the surface permits open orbits (see, for example, Fig. 2, *b* or *c*), these orbits determine the galvanomagnetic properties, and the field-dependence of both the transverse voltage and the magnetoresistance may be quite different from the field-dependence for a field direction which permits no open orbits.

The effect of a set of open orbits can be explained qualitatively by replacing the set of orbits by a cylindrical Fermi surface with its axis along the open direction. The metal then behaves like a two-dimensional conductor with high resistivity along the open direction. If the field **B** is in the *z*-direction and the current is made to flow in the *x-y* plane at an angle α with the open *x*-direction, the expression for the magnetoresistance is of the form,

$$\Delta\rho/\rho = \frac{1}{\rho(0)}\ (C_{xx}B^2\cos^2\alpha + C_{yy}\sin^2\alpha), \quad (8)$$

where C_{xx} *and* C_{yy} are constants. There is also a transverse-even voltage, $C_{xx}B^2 \sin\alpha \cos\alpha$. The dominant term in the electric field is the resultant quadratic term $C_{xx}B^2$, which lies along the open *x*-direction. Thus, the magnetoresistance increases quadratically for all current directions except those perpendicular to the open *x*-direction (α = 90 deg), where it saturates.

It is also possible for a Fermi surface to support open orbits in two different directions for the same field direction. The term, in the electric field, that is quadratic in B cannot be along both open directions at the same time, so it must be zero, and the magnetoresistance saturates for all directions of the current. This situation can be distinguished from that of an uncompensated metal with only closed orbits by the variation of the Hall (or transverse-odd) voltage as $1/B$ rather than as B (the usual variation). A summary is given in Table 1 of the possible situations and the corresponding behavior of the magnetoresistance and the transverse voltages for the case where the magnetic field is perpendicular to the current.

Since open orbits usually appear for relatively small angular ranges of magnetic field direction, their effect is seen as a narrow maximum or minimum (depending upon the metal's state of compensation) in the magnetoresistance when the field is rotated for a given current direction. Figure 3*a* shows a typical curve of the magnetoresistance as a function of the field direction for an uncompensated metal (copper); the

Table 1. The field-dependence of the galvanomagnetic properties of metals in the high-field region.

Type of orbit and state of compensation	Magnetoresistance	Hall voltage (transverse-odd)	Transverse-even voltage
Case 1. All closed and uncompensated ($n_e \neq n_h$)	Saturates	$\frac{\Omega}{n_e - n_h}\ \frac{B}{ec}$	Saturates
Case 2. All closed and compensated ($n_e = n_h$)	$\sim B^2$	$\sim B$	$\sim B^2$
Case 3. Open, 1-direction	$\frac{1}{\rho(0)}\ (C_{xx}B^2\cos^2\alpha + C_{yy}\sin^2\alpha)$	$\sim B$	$C_{xx}B^2\sin\alpha\cos\alpha$
Case 4. Open, 2-directions	Saturates	$\sim 1/B$	Saturates

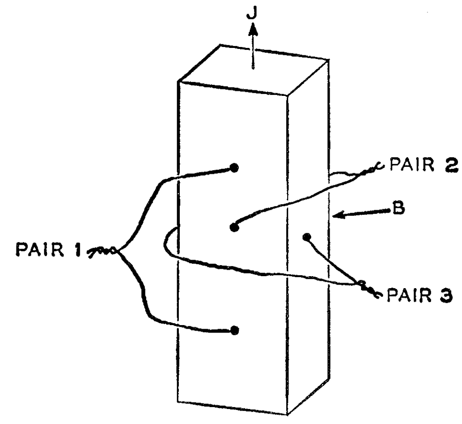

Fig. 4. Arrangement of leads for measuring the three components of potential generated by the current **J** in a magnetic field **B** perpendicular to the current direction.

effects of open orbits are seen as sharp maxima standing out above the background of low saturating magnetoresistance. This is in strong contrast to the behavior of a compensated metal such as zinc (Fig. 3*b*), where the effects of the open orbits are seen as sharp minima falling below the background of high magnetoresistance, where the field-dependence is quadratic.

Experimental Techniques

To achieve the condition $\omega_c\tau >> 1$ for all carriers—the criterion for the high-field region—magnetic fields of several tens of kilo-oersteds are employed to increase ω_c (Eq. 2). The metal sample is also cooled to liquid helium temperatures to reduce thermal scattering of carriers and so increase τ, which is then limited by impurity scattering. Even in available fields of 100 kilo-oersteds the samples must be of high purity to achieve the high-field region for all electrons and holes. Typically, the total impurity content must be less than 100 parts per million (*7*).

Basically, the galvanomagnetic measurements are quite straightforward. The sample is a high-purity single crystal, preferably of rectangular cross section, its length being several times greater than its width. Current leads are attached to the ends of the sample, and three pairs of leads are attached on three different axes of the sample, each axis being at right angles to the other two, as shown in Fig. 4. The potential generated across pair 1 is then a measure of the sample's resistance, and the potentials generated on pairs 2 and 3 are measures of the transverse voltages (Hall voltage and transverse-even voltage). Because the galvanomagnetic properties are very sensitive to the direction of the magnetic field, provision must be made for rigidly clamping the sample in the field and for making small alignment corrections once the sample is in the field and immersed in liquid helium.

In the experimental system used in our laboratory the sample can be tilted to make any required angle with the axis of rotation of the field. Data are obtained by recording the potential across one of the pairs of leads as a function either of field direction, when the field intensity is held constant and the magnet is rotated, or of field intensity, when the field direction is held fixed and the intensity is varied. It is necessary to separate the Hall and the transverse-even voltages and to eliminate spurious voltages, such as contact potentials. Measurements are therefore made for the four combinations of forward and reverse field and forward and reverse current, so that the various components of each potential, odd and even in **B** and **J**, can be separated.

The potentials are amplified by a direct-current amplifier having a sensitivity of about 10^{-9} volt and plotted on the y-axis of an x-y recorder. The x-axis records either the direction or the intensity of the magnetic field, so that continuous rotation or field-dependence curves can be obtained. It is essential that the data be recorded continuously, since it is almost impossible to measure the sharp anistropies exhibited by some metals when the data are recorded point by point. As an aid in data reduction, both axes on the recorder are connected to encoders which record the position of the pen in digital form, and each time the x-axis changes by one unit, an x-y pair is recorded on punched paper tape. The tapes for the four combinations of field and current directions are then processed by a computer, which reduces the raw data and plots the final curves on microfilm. This system reduces the time required to process the data from several hours to less than 1 minute per curve. A schematic diagram of the system is shown in Fig. 5.

To study systematically the topology of the Fermi surface of a given metal by measuring its high-field galvanomagnetic properties, a sample having a nonsymmetrical orientation is first selected, and the transverse magnetoresistance is measured as the field is rotated about the current direction. If the magnetoresistance is generally low and saturates with increasing field, the metal is uncompensated; if sharp maxima are also seen for some field directions, then we know that open orbits occur on one part of the Fermi surface. If the magnetoresistance is generally large and quadratic with increasing field, the metal is compensated; the presence of open orbits is then indicated by sharp minima. Open orbits are more easily missed for this case than for an uncompensated metal, since the magnetoresistance at the associated minima in the magnetoresistance only saturates if the current direction is exactly perpendicular to the open-orbit direction (that is, if $\alpha = 90$ deg in Eq. 8). For both uncompensated and compensated metals, several high-purity samples having different crystallographic orientations must be measured in a similar way for many field directions, either to be sure that there are no open orbits or to map out systematically the directions where the field produces open orbits.

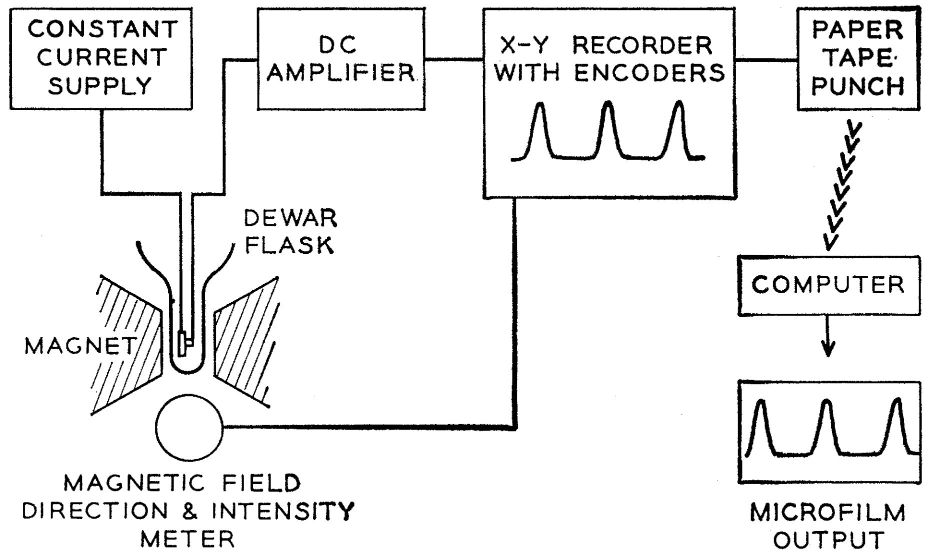

Fig. 5. Schematic diagram of the experimental system for automatic recording of galvanomagnetic data.

Quantitative measurements to determine the exact numerical value of the magnetoresistance are usually not required, since the field-dependence is the important characteristic. But accurate quantitative measurements of the Hall constant in an uncompensated metal provide an invaluable measure of the sign of the carrier (hole or electron). Also, in any metal having an open Fermi surface, the net number of carriers, as measured by the Hall constant, may differ from an integer for some magnetic field directions, and the deviation is a measure of the area of contact of the surface with the face of the Brillouin zone (see Fig. 1). The relative magnitudes of the quadratic voltage corresponding to the magnetoresistance and the transverse-even voltage associated with a set of open orbits show at once the *direction* of the open orbits (see Table 1, case 3). The field-dependence of the Hall voltage is also useful, sometimes, for distinguishing between cases 1 and 4 of Table 1.

Experimental Results

The classic example of the use of high-field galvanomagnetic properties to determine the topology of a Fermi surface is the application of the method to the noble metals (*8*). These investigations put the theoretical work of Lifshitz and his associates on a firm experimental basis. Figure 6 is a stereogram showing the peaks in the magnetoresistance of copper. Each line on the stereogram corresponds to open orbits similar to the open orbit shown in Fig. 2*b*, and each shaded area corresponds to open orbits similar to those shown in Fig. 2*c*. The unshaded areas correspond to field directions where only closed orbits exist.

The observed anisotropy of the magnetoresistance is consistent with the Fermi surface shown in Fig. 7. This surface was first proposed by Pippard (*9*) on the basis of his measurements of the anomalous skin effect, and it has subsequently been confirmed by studies of the de Haas-van Alphen, cyclotron resonance, and magnetoacoustic effects

(*10*). The surface is open when drawn in the extended-zone scheme, and in a three-dimensional model one can see that it supports open orbits in the observed directions. This surface supports open orbits in two directions when B is along a $< 112 >$ direction, and indeed the Hall field is observed to go to zero as $1/B$ (see Table 1, case 4).

Thorough experimental studies of the high-field galvanomagnetic properties of several other nontransition metals have been published and have provided valuable information about the shape of the Fermi surfaces of these metals (*11*). The galvanomagnetic effects are, in addition, contributing significantly to our understanding of the electronic structures of the transition metals, particularly the ferromagnetic metals. These metals occur in the periodic table at the place where the atomic *d*-states are incompletely filled. For several years there has been considerable controversy over the nature of the carriers in the *d*-bands derived from these atomic states. Some authors conjectured that these carriers have such low mobilities that they are effectively localized or bound to the ions, whereas others viewed these carriers as not essentially different from the other conduction electrons, which are free to move through the metal. This was a problem to which the galvanomagnetic effects have supplied a clear and unambiguous answer. Any metal which is compensated has a quadratic magnetoresistance when *all* carriers are in the high-field region (Table 1, case 2). However, if some of the carriers have very low mobilities, they will be in the high-field region only at the highest fields, if at all, so that at lower fields the magnetoresistance will not be quadratic but will saturate as in the case of an uncompensated metal. But a metal of this type can readily be distinguished from a true uncompensated metal by the fact that the Hall coefficient will correspond to a nonintegral number of carriers. Thus, we have a clear test of the relative mobilities of the carriers in a metal.

The first transition metals with high enough purity so that all carriers could achieve the high-field region in fields of a few kilo-oersteds were tungsten and molybdenum. Both metals are in column 6B of the periodic table, and both have body-centered cubic crystal structures and even atomic numbers. The magnetoresistance measurements (*12*) show that both metals lack open orbits and, as expected, are compensated. More recent measurements on very-high-purity tungsten (*13*) show that the field-dependence is accurately quadratic up to 100 kilo-oersteds. This indicates that the number of electrons is almost exactly equal to the number of holes, so that, at least in these two transition metals, all the carriers are free to move through the metal.

Of the other transition metals measured to date, rhenium, platinum, palladium, and iron are found to be compensated and niobium and tantalum to be uncompensated. Only rhenium shows any effects which we can attribute to a difference in mobilities between the different carriers (*14*). At the highest fields, rhenium has a quadratic magnetoresistance, which shows that it is compensated. This is to be expected, despite the odd atomic number, since there are two atoms per unit cell in the hexagonal close-packed structure of rhenium. But at the lower fields there is an initial region of saturation, as shown in Fig. 8. This behavior indicates that at intermediate fields not all the carriers are in the high-field region, n_e effectively does not equal n_h, and the magnetoresistance saturates. At higher fields, $\omega_c\tau$ becomes greater than unity for the remaining carriers, n_e now equals n_h, and the field-dependence becomes quadratic. A simple two-band model has been used to fit the data, and, to match the observed results, a mobility ratio of 20 is required. The important fact is that, even where a difference in mobility is observed, it is ultimately possible to achieve the condition $\omega_c\tau > 1$ for all carriers.

Nickel is a metal for which measurements of the galvanomagnetic properties have proved particularly useful (*6*, *15*). The stereogram of the magnetoresistance peaks for nickel is very similar to that for copper—a result which is surprising in two respects. (i) It shows that part of the Fermi surface of nickel is open and very much like the

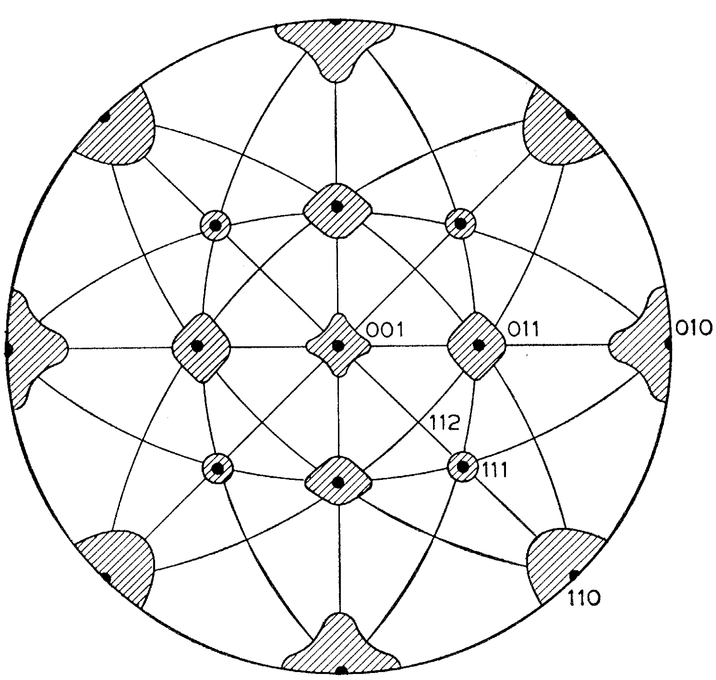

Fig. 6 (left). Stereogram showing the anisotropy of the magnetoresistance of copper.

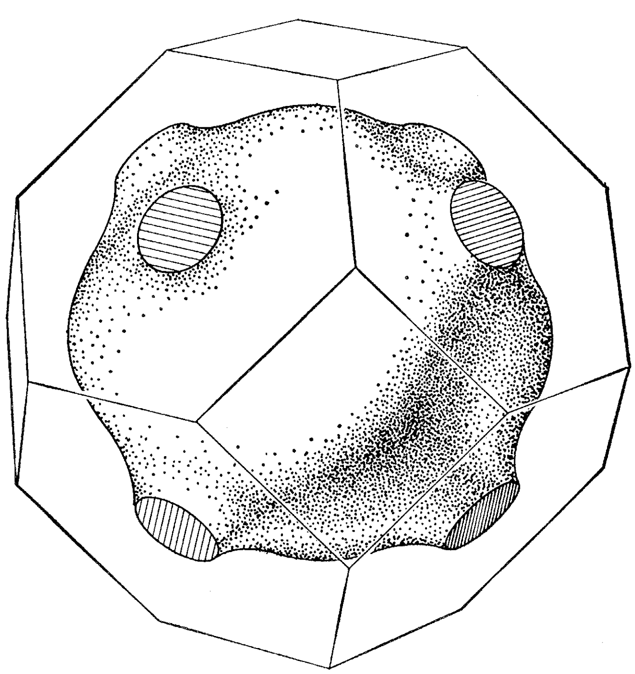
Fig. 7 (right). The Fermi surface of copper.

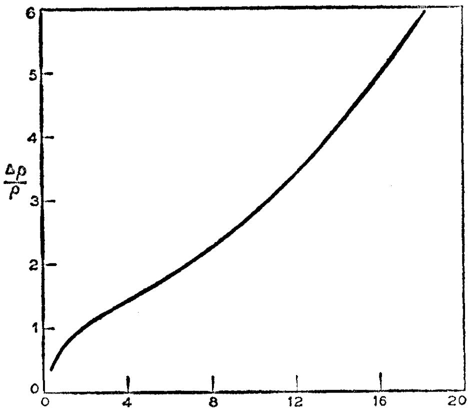

Fig. 8. The field-dependence of the magnetoresistance of rhenium, showing the approach to saturation at intermediate fields which is due to the large ratio between the mobilities of different carriers.

Fermi surface of copper (see Fig. 7), except that the area of contact with the zone boundary is about 10 times smaller in nickel. (ii) Even though its atomic number is even, nickel, like copper, appears to be uncompensated; for a nonsymmetry field direction the magnetoresistance saturates and the Hall coefficient corresponds, within the limits of experimental error, to one electron per atom.

The proposed explanation of the uncompensation of nickel is as follows. We stated earlier the theoretical result that all metals having an even number of electrons per unit cell are compensated, which follows directly from the fact that the capacity of the Brillouin zone is two electrons per unit cell, one of positive and one of negative spin. But it is implicit in this result that electrons having spins of opposite sign have the same energy, and therefore have Fermi surfaces of the same shape. This is certainly true in a nonmagnetic metal, but in nickel the exchange interaction, which is the cause of the ferromagnetism, splits the energies of electrons having spins of opposite sign. It follows that, in a ferromagnetic metal, a Brillouin zone may be completely filled by or emptied of electrons of one spin direction while being partially filled by electrons of opposite spin; or electrons of one spin direction may constitute an electron surface while electrons of opposite spin may constitute a hole surface.

It is appropriate in connection with a ferromagnetic metal to introduce the term *spin zone*. A spin zone is defined as the assembly of quantum states of one sign of spin in a Brillouin zone. If the states in a single spin zone are partially occupied by electrons, the surface in momentum-space separating the occupied and unoccupied regions may have an electron or hole character, depending upon the connectivity of the surface. The capacity of a spin zone is one electron per unit cell. This leads to the result that a ferromagnetic metal having an even number of electrons per cell need not be compensated, though the net number of carriers per cell must still be an integer, as in a nonmagnetic metal.

Illustrating how nickel, with an even number of electrons per cell (28), can be uncompensated, Fig. 9 shows a simple model of its band structure in the form of plots of energy against momentum between the center and the face of the Brillouin zone. The *d*-band is split by the ferromagnetic exchange interaction; the splitting of the *s*-band is not shown, since it is assumed to be much smaller. Nickel has just enough electrons to fill completely all the *d*-bands, so that if n_s^e electrons spill over into the *s*-band there will be an equal number of unoccupied states in one of the *d*-bands. If these unoccupied states constituted hole surfaces for both the up and down spins, nickel would be a compensated metal. This would be the case if nickel were nonmagnetic, since the energies for up and down spins would be equal. But if the energy of the spin-up *d*-band is raised sufficiently by the ferromagnetic exchange interaction, the occupied volume will shrink until it becomes an electron surface containing $n_d^e(\uparrow)$ electrons, while the $n_d^h(\downarrow)$ unoccupied states in the spin-down band will still constitute a hole surface. The equality of the number of electrons in the *s*-band to the number of unoccupied states in the *d*-band then leads to the equation,

$$n_s^e = n_d^h(\downarrow) + [1 - n_d^e(\uparrow)]. \quad (9)$$

Equation 9 can be rearranged to read,

$$(n_s^e + n_d^e) - n_d^h = n_e - n_h = 1, \quad (10)$$

which means, in physical terms, that a metal having this band structure will exhibit the saturating magnetoresistance and linear Hall voltage corresponding to exactly one electron per cell which are observed in nickel.

This schematic band structure is clearly a gross approximation to the true band structure of nickel. But it does contain one feature which is a direct consequence of the experimental

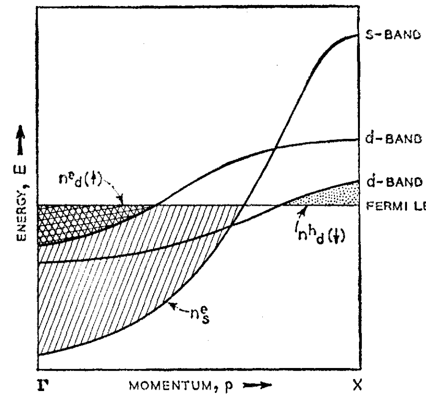

Fig. 9. Schematic representation of the band structure of nickel.

finding that the net number of carriers is one electron per cell—namely, that there is an electron surface in one spin zone of the *d*-bands. More elaborate models of the band structure of nickel have been constructed (*16*) which contain this essential feature and also are consistent with the ferromagnetic and optical properties of nickel.

We have briefly reviewed some of the problems concerning the Fermi surfaces of metals which have been investigated through measurement of the galvanomagnetic properties in the high-field region. There are many more problems yet to be attacked. The transition elements will no doubt yield further surprising results, and work on the rare earth elements and the intermetallic compounds is in the earliest stages. Thus, we can confidently assert that one of the oldest experimental techniques for studying the Fermi surfaces of metals has one of the brightest futures.

References and Notes

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3. The voltage transverse to the current direction consists of (i) a component that changes sign when the magnetic field is reversed, usually referred to as the Hall voltage, and (ii) a component that does not change sign, known as the transverse-even voltage. For a discussion of the latter, see W. A. Reed and J. A. Marcus, *Phys. Rev.* **126**, 1298 (1962).
4. If Z is the atomic number of the metal and s is the number of atoms per unit cell, then sZ is the total number of electrons per cell.
5. E. Fawcett and W. A. Reed, *Phys. Rev.* **131**, 2463 (1963).
6. These results are to be contrasted with the galvanomagnetic properties of metals in the low-field region ($\omega_c\tau << 1$), where the magnetoresistance always has a quadratic field-dependence. The net number of carriers determined from the Hall coefficient in the low-field region can only be an integer by coincidence, since it depends upon both the

relative mobilities and the numbers of the different carriers.
7. The usual test of the quality of a crystal is its residual resistance ratio, that is, the ratio of the resistivity at room temperature to the resistivity at 4.2°K. Although at present there is no way to relate the resistance ratio and purity of one metal to the resistance ratio and purity of another, the ratio is a reliable test between different samples of the same metal. The resistance ratios of the best samples currently available of some of the metals discussed here are, Cu, 8000; Zn, 60,000; Sn, 100,000; Fe, 300; Ni, 2700; Mo, 4000; W, 80,000.
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Chemical and Anatomical Plasticity of Brain

Changes in brain through experience, demanded by learning theories, are found in experiments with rats.

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Here it may be asked whether the organs [of the brain] increase by exercise? This may certainly happen in the brain as well as in the muscles; nay, it seems more than probable, because the blood is carried in greater abundance to the parts which are excited, and nutrition is performed by the blood. In order however, to be able to answer this question positively, we ought to observe the same persons when exercised and when not exercised; or at least observe many persons who are, and many others who are not, exercised during all periods of life.—J. G. SPURZHEIM, 1815 (*1*).

I have shown that the brains of domestic rabbits are considerably reduced in bulk, in comparison with those of the wild rabbit or hare; and this may be attributed to their having been closely confined during many generations, so that they have exerted their intellect, instincts, senses and voluntary movements but little.—CHARLES DARWIN, 1874 (*2*).

One might suppose that cerebral exercise, since it cannot produce new cells (neural cells do not multiply as do muscular cells) carries further than usual the development of protoplasmic expansions and neural collaterals, forcing the establishment of new and more extended intercortical connections.—S. RAMÓN Y CAJAL, 1895 (*3*).

The question is not whether neural events change the status of the tissue in which they occur. The only question which may still be debated is: whether such changes as do undoubtedly occur have the permanence and those other properties which we must attribute to memory-traces. According to our present knowledge the primary effect which nerve impulses produce in ganglionic layers is chemical activity . . .—WOLFGANG KÖHLER, 1938 (*4*).

Thus, the results of our original experiment and the replication strongly support these two general conclusions: (a) Manipulating the environment of animals during the 80 days after weaning can alter significantly the weight of the cerebral cortex, the total ChE [acetylcholinesterase] activity of the brain, and the cortical/subcortical distributions of the specific activity of ChE and of tissue weight. (b) Similar but much greater alterations in the brains of the animals can be accomplished by a program of genetic selection carried out over a few generations.—M. R. ROSENZWEIG, D. KRECH, E. L. BENNETT, and M. C. DIAMOND, 1962 (*5*).

As these quotations show, it has long been speculated that the use of the brain might lead to changes in its size, in the interconnections of its cells, and in its chemical composition. Speculation led to research, and in the last century measurements of the size and weight of brains of men were made in an effort to discover differences that might relate to the degree of intellectual attainment. The first results were encouraging, since men of distinction were usually found to have larger brains than those of inferior intellect. Gradually it was realized, however, that men of different stations in life often differed in health and nutrition as well as in intellect, and that the former factors might affect brain weight. There were also striking exceptions to the general relation—idiots with large brains and geniuses with small brains. The hypothesis of an intrinsic relation between brain size and cerebral exercise or ability was therefore generally abandoned. In its place there were suggestions of more subtle factors involving neural interconnections or chemical changes in the brain. The difficulty of working with such factors discouraged research, and the problem largely reverted to the speculative realm.

In spite of their speculative nature, physical or chemical residuals of experience in the brain continued to be incorporated in most physiological theories of learning. It was generally supposed that changes must occur in the brain in order to account for memory—the registration, the storage, and the retrieval of information. No other hypothesis seemed tenable. So certain were the theoreticians about this that they long ago gave names to these hypothesized changes—they called them "memory traces" or "brain engrams." But, unfortunately, the brain physiologists and anatomists were singularly unsuccessful in finding any solid evidence to justify

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