components of standard deviation derived from analyses of variance, element by element, within rock types. For any given component of the variability the figures are estimates of the contribution to the total variance for that level alone; they are not cumulative.

Evaluation of Table 1 shows that elemental variations on the small scale are of great importance and suggests that carefully planned sampling is necessary at the individual locality if higher order variances (that is, larger scales) are to be studied. Rocks are aggregates of minerals, and grain size is a controlling factor in sampling; thus the individual specimens collected must be large enough to be representative rocks, not merely fragments of one of the constituent minerals.

To evaluate the adequacy of specimen size, we proceeded as follows: single diamond drill cores (2 cm in diameter) from one locality were divided into 8-cm segments, and each segment was analyzed; then, in order to simulate the chemistry of cores 16 cm long, adjoining segments were combined and the reduction in variance was computed, this procedure being repeated at increments of 8 cm in core length. As a practical guide, a core length was considered adequate when, for each element, the correlation between adjacent segments was not significant in relation to the variance between segments randomly selected from a core 400 cm long. From a study based on 45 analyses (each for Na, Mg, Al, Si, K, Ca, Ti, Fe) we concluded that a 30-cm core would suffice for the coarsest grained rock included in our investigation.

At the smallest sampling scale, Na, K, Ca, Mg, and Fe are very variable in almost all rock types. In addition, the dependence of elemental variation upon scale of sampling is quite different for neighboring rock units within the complex. Some rock types increase in variability directly and markedly with scale and others show such high variability at the small scale that it is difficult to detect any increase by sampling over larger areas. The implication of these facts for evaluation of compositional trends is clear: without estimates of smaller scale variability it is not possible to judge the significance of larger scale variations. If only one specimen is collected at each locality, no degrees of freedom are available for estimating the variances within a locality. The collection of at least two specimens at each locality permits an estimate of the small scale chemical variability; we compare this with the residual mean sum of squares from the highest degree trend surface considered meaningful. A discussion of the criteria to be used in judging the significance of a trend surface is beyond the scope of this report; the point emphasized is that a direct and independent measure of within-localities variance can and should be made. The figures presented suggest that the variability at the scale of the outcrop (even with cores 30 cm in length) may require the collection of more than two specimens per locality. The particular number depends upon the resolution required for the most variable element which is to be investigated. Data of the nature presented here are available for no other rocks, and hence comparisons with other areas are impossible at the present time.

A. K. BAIRD, D. B. MCINTYRE
E. E. WELDAY, K. W. MADLEM
Department of Geology,
Pomona College, Claremont, California

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Oscillations of Quasars

Abstract. *Rotation in addition to free gravitational motion can produce oscillations in a large spherical mass of gas. The theory may provide an explanation of the variations of brightness in such objects as 3C273.*

The theory of free gravitational collapse (*1*, *2*) has been applied to the remote radio sources in the universe which have stellar-like optical images ("quasars"). This theory is based on a physically peculiar assumption, namely, that the density and pressure of the contracting material are uniform at each instant of time though they both vary with the time. Without entering into the physical plausibility of this hypothesis (*2*), I should like to describe briefly some of the consequences produced by the combination of free collapse with rotation.

Certain quasars show fluctuation of optical luminosity. The work of H. J. Smith indicates that the quasar 3C273, for example, has a variation of luminosity of period some 13 years. A possible explanation of this phenomenon may lie in alternate expansions and contractions of the whole quasar. A motion of this kind can be deduced from the equations of classical gas-dynamics and Newtonian gravitational theory if the quasar is gaseous and is rotating and if the following assumptions are made. (i) The density and pressure of the gas are uniform at each instant of time but both vary with the time, as they do when there is no rotation. (ii) The velocity components of each element of gas are defined as follows: let r be a coordinate perpendicular to the axis of rotation, let z be a coordinate measured from the diametral plane through the center of the configuration, z being parallel to the axis of rotation, and let ϕ be the azimuthal angle about the axis of rotation; then the three components of velocity of the gas are assumed to be of the form

$$v_r = rf(t), \qquad v_z = zf(t), \qquad v_\phi = r\omega\,(t),$$

where f and the angular velocity ω are functions of the time alone. (iii) The only force acting is the gravitational self-attraction of the mass of gas, which is governed by Poisson's equation.

The classical equations of motion and of continuity together with Poisson's equation then determine f, ω, and the density as periodic functions of the time. The pressure remains arbitrary. The period of an oscillation is found to be

$$P = \pi\left(1 - \frac{\omega_0^2}{4\pi G\rho_0}\right)^{-3/2}\left(\frac{3}{8\pi G\rho_0}\right)^{1/2}$$

where G is the constant of gravitation and ω_0 and ρ_0 are the angular velocity and the density, respectively, at the moment of greatest extension of the mass of gas.

This oscillatory motion is possible only if $\omega_0^2 < 2\pi G\rho_0$. The configuration always remains spherical and the maximum radius may be denoted by A. The minimum radius is then

$$A_m = \frac{\omega_0^2 A}{4\pi G\rho_0 - \omega_0^2}.$$

A period P of about 13 years is obtained if

$$\rho_0 = 1.5 \times 10^{-10} \text{ g/cm}^3,$$
$$\omega_0^2 = 1.5 \times 10^{-17} \text{ (rad/sec)}^2,$$

though, of course, certain other combinations of ρ_0 and of ω_0 would lead to the same period. The mass of a quasar is suspected to be of the order of 10^8 solar masses and, with the value of ρ_0 already quoted, it follows that A, A_m in light-years are given by

$$A = 7.2 \times 10^{-2}, \qquad A_m = 9.76 \times 10^{-3}.$$

These are small radii, as would be expected if all parts of the quasar's surface appear to a distant observer to vary in brightness simultaneously. Since the Schwarzschild gravitational radius of the configuration, $2GM/c^2$, is 3.1×10^{-5} light-years, the minimum radius is only some three hundred times the gravitational radius. These small dimensions, and the intense gravitational fields accompanying them, might lead to the conclusion that general relativity ought to be employed instead of classical theory. It is not necessary to do this, however, because in the corresponding problem of free gravitational collapse without rotation it can be shown that general relativity and classical mechanics lead to essentially the same conclusions. The absence of a pressure gradient is a more serious limitation which further study will no doubt remove.

G. C. McVittie
University of Illinois Observatory, Urbana

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High-Voltage Laue X-ray Photography of Large Single Crystals

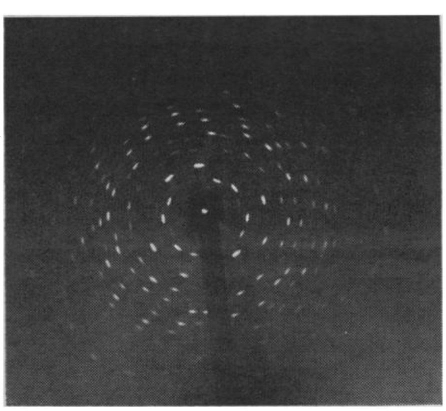
Abstract. *The perfection of relatively large single crystals can be tested by Laue transmission x-ray diffraction photographs by using much higher voltages for the x-ray generation than are used in conventional diffraction experiments. The technique can be used for determining the orientation spread of subcrystal units and for studying the loss of primary extinction due to occluded chemical impurities. An advantage of this method is its possible application to crystals inside polycrystalline or glass containers.*

Crystals that are large compared with the penetration of conventional electron or x-ray beams cannot be readily studied by diffraction techniques. Such large crystals can be subjected to a "surface" diffraction examination or to destructive studies by sectioning; these procedures are always tedious and usually unsatisfactory. Techniques in which more penetrating radiation is used may greatly simplify the study of large crystals. The application of neutron diffraction (*1*) for such purposes has already been discussed. The work described here illustrates the possibilities of high-voltage x-rays, up to about 200 kv, whose penetration depth is more than an order of magnitude greater than the x-rays used conventionally in diffraction experiments. The use of high-voltage Laue photography has been mentioned (*2–4*).

At accelerating voltages above those corresponding to the K-lines of the heaviest elements, there are no convenient monochromatic radiation sources of adequate brilliance for diffraction. Thus it is necessary to revert to the Laue technique, in which a stationary crystal is exposed to collimated polychromatic radiation. An advantage of high-voltage Laue photographs is that during exposure the specimen can be contained in a furnace or vacuum vessel made of polycrystalline or glassy material. Transmission Laue photographs taken with high-voltage x-rays (Fig. 1) are comparable in detail with conventional Laue pictures. They could be used for identification, orientation, or preliminary crystal perfection studies.

The collimation system (*2*) was placed as close as possible to a fine-focus (0.5 mm) x-ray tube. Exposures were of the order of 10 minutes on Ilford G x-ray film. The crystal specimen was adjusted so that the x-ray beam was perpendicular to the plane or axis of elongation of the specimens. The tangential elongation (Fig. 1) of the spots is largely due to beam divergence which can be measured by direct photography of the beam at different distances from the slit. For the slit used here, the incident-beam divergence from the central ray is about half of one degree, dependent slightly on wavelength. The radial dimension of the Laue spots depends on the size of the x-ray tube focal spot (the 0.5-mm microfocus tube was used for all illustrations), the specimen thickness traversed by the x-rays (*2*), and the divergence of the x-ray beam. To minimize spot broadening caused by divergence (*5*), the distance between the specimen and the film (measured along the diffracted ray) should equal the distance of the specimen from the effective point of divergence.

The increase in radial length of the spots, resulting from the greater thick-



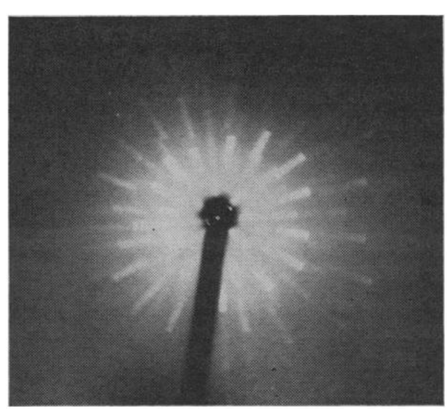


Fig. 1 (left). Laue photograph of a 0.5-mm thick germanium crystal cut perpendicular to the [111] direction; 150 kv x-rays parallel with [111], 8 ma, 10 minutes, 6 cm between slit and film. Fig. 2 (right). Laue photograph of a 2-cm thick aluminum crystal; 150 kv, 5 ma, 10 minutes, 6 cm between slit and film.