Chemical Variations in a Granitic Pluton and Its Surrounding Rocks

Abstract. *New techniques of x-ray fluorescence spectrography have provided, for the first time, abundant data regarding chemical variability of granitic rocks on different scales. The results suggest that current designs of sampling plans for trend surface analysis should be modified; in particular several specimens, preferably drill-cores, may be required at each locality.*

Knowledge of the chemical composition of rocks has been based on inadequate sampling and almost no information is available on variability as a function of scale; recent papers have emphasized the need for reliable quantitative estimates of composition rather than merely qualitative descriptions of rock types. In particular, Whitten (*1*) has posed a series of basic questions ranging from "What sampling program [of a granite body] is necessary to prove homogeneity?" to "Will the same sampling plan be adequate for all variable attributes of a particular complex?" He emphasizes that scale considerations are of the utmost importance and notes the virtual lack of adequate data. Precise estimates of mineralogical composition and variation are often unobtainable for coarse-grained rocks, and chemical analyses have been too costly. The development of the x-ray fluorescence spectrograph as a high precision instrument capable of analyzing all major and minor constituents from a single prepared sample of a rock has enabled us to obtain complete analyses quickly and cheaply.

The chemical variability of one plutonic body is shown in Table 1; this summary is based upon 863 analyses, by x-ray spectrography, of 465 rocks collected according to sampling plans designed to minimize bias and to test variabilities on scales ranging from single specimens of a few hundred grams to outcrop areas of tens of square kilometers. The analytical methods (*2*), petrography, and structural relationships of rock units (*3*) have already been described.

The plutonic complex, Rattlesnake Mountain Pluton, San Bernardino Mountains, California, consists of five main rock types. The most abundant is porphyritic biotite quartz monzonite in the form of a funnel with prominent lobes projecting outward from the flared, upper portion. Concentric screens of hornblende quartz diorite occur within this structure. On the western margin of the pluton there is a narrow transitional zone of diorite followed, in sharp contact, by relatively nonporphyritic biotite quartz monzonite which is apparently typical of the regional "granites" of the San Bernardino Mountains. The eastern margin is bordered in part by hornblende quartz monzonite. The complex is emplaced in a belt of high-grade metamorphic rocks and crops out over an area of 100 km^2.

Every attempt to obtain an unbiased sample involves the collection of specimens according to some definite operational procedure (*4*). Our sampling was stratified in the sense that samples were taken independently from each of five rock units defined by prior geologic mapping and petrographic study (*3*). As indicated in Table 1, only two of the units outcrop on a large scale (> 25 km^2).

Systematic sampling on a grid was used for the small (5 × 5 m) and intermediate (300 × 300 m) scales. The porphyritic biotite quartz monzonite, the main body of the pluton, was sampled with a multistage (nested) plan (*4*). No matter what general plan is adopted, there is still the problem that bias may be introduced when collecting at the outcrop itself. To measure the variance at this level we collected replicate specimens at each locality; and to minimize bias we used a diamond drill (*5*).

Variance due to specimen preparation and analytic methods is determined by carrying two splits of the crushed rock through the entire laboratory procedure. However the analytical precisions quoted for the porphyritic biotite quartz monzonite are based on replicates of only 30 different cores. All the other analyses were made after 1962 with automated equipment linked by teleprocessing to an IBM 7094 computer (*6*); this made it possible to replicate every rock analysis and to obtain, by analysis of variance, the standard deviation (precision) for every run and every element (*7*).

Table 1 gives the mean elemental compositions of rock types and the levels at which variance has been measured. All variations are expressed as

Table 1. Variability of rock types versus sample scale. Abbreviations: n.s., not significant, at *p* 0.05; n.p., scale not present; DF, degrees of freedom. Other values are percent by weight.

Component	Compositions and components of standard deviations (%)							DF
	Na	Mg	Al	Si	K	Ca	Fe	
Porphyritic biotite quartz monzonite: Mean								
	3.36	0.49	8.04	31.4	3.20	1.48	1.60	383
Porphyritic biotite quartz monzonite: Scale								
Analytical	0.04	0.02	0.11	0.3	0.09	0.08	0.11	30
Small	.08	.09	.25	.5	.20	.09	.26	44
Intermed.	.13	n.s.	.22	n.s	.32	.32	n.s.	29
Large	.11	n.s.	.26	n.s.	n.s.	n.s.	n.s.	15
Hornblende quartz monzonite: Mean								
	3.50	0.96	8.32	28.1	3.58	2.75	3.54	48
Hornblende quartz monzonite: Scale								
Analytical	0.04	0.03	0.13	0.4	0.06	0.06	0.11	23
Small	.08	.09	.32	.8	.17	.23	.47	15
Intermed.	.13	n.s.	n.s.	n.s.	.27	n.s.	n.s.	15
Large	n.p.	n.p.	n.p.	n.p.	n.p.	n.p.	n.p.	
Diorite: Mean								
	4.88	1.15	10.30	26.2	1.63	3.01	4.07	96
Diorite: Scale								
Analytical	0.08	0.02	0.09	0.2	0.02	0.02	0.07	47
Small	.10	.04	.16	.3	.03	.08	.08	30
Intermed.	n.s.	.09	n.s.	n.s.	.05	.17	.30	15
Large	n.p.	n.p.	n.p.	n.p.	n.p.	n.p.	n.p.	
Hornblende quartz diorite: Mean								
	2.43	3.17	8.53	24.1	1.10	5.82	7.62	216
Hornblende quartz diorite: Scale								
Analytical	0.05	0.05	0.06	0.2	<0.01	0.07	0.07	107
Small	.15	.23	.33	.8	.12	.31	.53	28
Intermed.	.33	.44	.32	1.0	.20	.69	1.52	28
Large	.44	.73	.40	1.3	.37	1.25	n.s.	48
Biotite quartz monzonite: Mean								
	3.90	0.47	8.56	31.6	2.48	1.49	1.75	120
Biotite quartz monzonite: Scale								
Analytical	0.06	0.01	0.06	0.3	0.07	0.02	0.05	59
Small	.49	.09	.49	.8	.38	.26	.36	29
Intermed.	n.s.	n.s.	n.s.	.9	n.s.	n.s.	n.s.	29
Large	n.p.	n.p.	n.p.	n.p.	n.p.	n.p.	n.p.	

components of standard deviation derived from analyses of variance, element by element, within rock types. For any given component of the variability the figures are estimates of the contribution to the total variance for that level alone; they are not cumulative.

Evaluation of Table 1 shows that elemental variations on the small scale are of great importance and suggests that carefully planned sampling is necessary at the individual locality if higher order variances (that is, larger scales) are to be studied. Rocks are aggregates of minerals, and grain size is a controlling factor in sampling; thus the individual specimens collected must be large enough to be representative rocks, not merely fragments of one of the constituent minerals.

To evaluate the adequacy of specimen size, we proceeded as follows: single diamond drill cores (2 cm in diameter) from one locality were divided into 8-cm segments, and each segment was analyzed; then, in order to simulate the chemistry of cores 16 cm long, adjoining segments were combined and the reduction in variance was computed, this procedure being repeated at increments of 8 cm in core length. As a practical guide, a core length was considered adequate when, for each element, the correlation between adjacent segments was not significant in relation to the variance between segments randomly selected from a core 400 cm long. From a study based on 45 analyses (each for Na, Mg, Al, Si, K, Ca, Ti, Fe) we concluded that a 30-cm core would suffice for the coarsest grained rock included in our investigation.

At the smallest sampling scale, Na, K, Ca, Mg, and Fe are very variable in almost all rock types. In addition, the dependence of elemental variation upon scale of sampling is quite different for neighboring rock units within the complex. Some rock types increase in variability directly and markedly with scale and others show such high variability at the small scale that it is difficult to detect any increase by sampling over larger areas. The implication of these facts for evaluation of compositional trends is clear: without estimates of smaller scale variability it is not possible to judge the significance of larger scale variations. If only one specimen is collected at each locality, no degrees of freedom are available for estimating the variances within a locality. The collection of at least two specimens at each locality permits an estimate of the small scale chemical variability; we compare this with the residual mean sum of squares from the highest degree trend surface considered meaningful. A discussion of the criteria to be used in judging the significance of a trend surface is beyond the scope of this report; the point emphasized is that a direct and independent measure of within-localities variance can and should be made. The figures presented suggest that the variability at the scale of the outcrop (even with cores 30 cm in length) may require the collection of more than two specimens per locality. The particular number depends upon the resolution required for the most variable element which is to be investigated. Data of the nature presented here are available for no other rocks, and hence comparisons with other areas are impossible at the present time.

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References and Notes

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Oscillations of Quasars

Abstract. *Rotation in addition to free gravitational motion can produce oscillations in a large spherical mass of gas. The theory may provide an explanation of the variations of brightness in such objects as 3C273.*

The theory of free gravitational collapse (*1*, *2*) has been applied to the remote radio sources in the universe which have stellar-like optical images ("quasars"). This theory is based on a physically peculiar assumption, namely, that the density and pressure of the contracting material are uniform at each instant of time though they both vary with the time. Without entering into the physical plausibility of this hypothesis (*2*), I should like to describe briefly some of the consequences produced by the combination of free collapse with rotation.

Certain quasars show fluctuation of optical luminosity. The work of H. J. Smith indicates that the quasar 3C273, for example, has a variation of luminosity of period some 13 years. A possible explanation of this phenomenon may lie in alternate expansions and contractions of the whole quasar. A motion of this kind can be deduced from the equations of classical gas-dynamics and Newtonian gravitational theory if the quasar is gaseous and is rotating and if the following assumptions are made. (i) The density and pressure of the gas are uniform at each instant of time but both vary with the time, as they do when there is no rotation. (ii) The velocity components of each element of gas are defined as follows: let r be a coordinate perpendicular to the axis of rotation, let z be a coordinate measured from the diametral plane through the center of the configuration, z being parallel to the axis of rotation, and let ϕ be the azimuthal angle about the axis of rotation; then the three components of velocity of the gas are assumed to be of the form

$$v_r = rf(t), \qquad v_z = zf(t), \qquad v_\phi = r\omega(t),$$

where f and the angular velocity ω are functions of the time alone. (iii) The only force acting is the gravitational self-attraction of the mass of gas, which is governed by Poisson's equation.

The classical equations of motion and of continuity together with Poisson's equation then determine f, ω, and the density as periodic functions of the time. The pressure remains arbitrary. The period of an oscillation is found to be

$$P = \pi\left(1 - \frac{\omega_0^2}{4\pi G\rho_0}\right)^{-3/2}\left(\frac{3}{8\pi G\rho_0}\right)^{1/2}$$

where G is the constant of gravitation and ω_0 and ρ_0 are the angular velocity and the density, respectively, at the moment of greatest extension of the mass of gas.

This oscillatory motion is possible only if $\omega_0^2 < 2\pi G\rho_0$. The configuration always remains spherical and the maxi-