critical for differentiating Kope from Point Pleasant, but it is diagnostic in distinguishing the Kope from beds above it. The smoothed curve of bedding indexes (log C', Fig. 1) shows how sharp and persistent the difference between the two formations is.

The name Kope is taken from Kope Hollow, north of Levanna, Ohio, and largely on the Russellville 7.5-minute quadrangle in Brown County, Ohio. No continuous section of the entire formation is known, but the sections in Fig. 1 yield a composite thickness of 73.5 m and may be considered cotypical sections; the sections in Fig. 1 include both boundaries but are matched solely on lithic qualities. Many incomplete exposures in the area between Levanna (log A, Fig. 1) and Maysville (logs C and C', Fig. 1) make it clear that the top of the formation slopes southeast from about 224 m above sea level at Levanna to about 209 m at Maysville.

In summary, the Kope formation is a conspicuous unit of Upper Ordovician shale and interbedded limestone in the Maysville area of Kentucky and Ohio; it lies between the Point Pleasant formation and shaly limestones equivalent to the Fairmount and McMillan formations of the Cincinnati section. The upper boundary of the Point Pleasant is conspicuous on clastic ratio logs (log A, Fig. 1) and is equally so on outcrop if adjacent beds are exposed. The upper boundary of the Kope is not sharp but is limited to a thin interval on both clastic-ratio and bedding-index logs. It is readily recognized in the field because lithic and bedding characters of the Kope and overlying beds are persistent for many tens of feet away from the contact interval (*7*).

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Table 1. Clastic ratios (CR)* of Kope and bounding units.

Formation	Range	Mean
Supra-Kope beds	0.25–1.3 (mostly less than 1)	0.5
Kope: upper 3/5	2.0–∞	3.8
Kope: lower 2/5	1.3–11.0	2.5
Point Pleasant	0.0–2.0 (mostly close to 1)	1.0

* CR = (thickness of shale + mudstone) /thickness of limestone. It is computed for successive 0.9-m units.

Table 2. Bedding indexes (BI)* of Kope and bounding units.

Formation	Range	Mean
Supra-Kope beds	300–830	570
Kope: upper 3/5	100–400	230
Kope: lower 2/5	100–500	300
Point Pleasant	200–500†	300

* BI = (No. of beds per 0.9-m unit × 100)/3.
† Always higher than basal Kope.

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Libration of Pluto-Neptune

Abstract. *Numerical integration of the orbits of the five outer planets over 120,000 years reveals that the distance between Pluto and Neptune at the closest approaches oscillates within a narrow range. The distance is never much less than the aphelion distance of Pluto from the orbit of Neptune. The near commensurability in the periods of Pluto and Neptune and the eccentricity of Pluto's orbit are responsible for the libratory motion.*

Because the radius of perihelion of Pluto is less than the radius of the orbit of Neptune, the usual Laplace expansion of the disturbing potential diverges. There is also the possibility of an indefinitely close approach. It was therefore proposed by Brouwer (*1*) that the orbit characteristics of Pluto be explored by numerical integration of the equation of motion of the five outer planets over an extended period of time. This has now been done at our laboratory, and almost immediately a remarkable regularity of the orbit of Pluto was revealed.

The integrations were performed on the Naval Ordnance Research Calculator (NORC), a 13-place computer. We used the data of Eckert *et al.* (*2*), and the method of integration that they employed. The integrations, which required about 100 hours of computer time, were run backward from the present to more than 120,000 years ago. This length of time represents some 750 revolutions for Neptune and approximately 500 revolutions for Pluto.

Some remarks are in order on the numerical strength of the results. Double precision accumulation of the first and second sums was used to enhance the precision of the computation. The energy and angular momentum were monitored and never differed from the value at epoch by more than two parts in 10^{10}. This corresponds to a theoretical standard deviation of 10^{-5} radians in the phase of Jupiter and of 10^{-10} radians in the phase of Pluto at the end of the integration. Methods of error analysis are given in (*2*) and (*3*).

It may be recalled that the orbit of Neptune is nearly circular with a radius of 30 AU (astronomical units), while the orbit of Pluto is rather eccentric, with a radius of perihelion of just under 30 AU and a radius of aphelion of 50 AU. The mass of Neptune is about 20 times that of Pluto. The orbit of Pluto is inclined 15 degrees to that of Neptune with the perihelion of Pluto 114 degrees in advance of the ascending node.

In the course of the integrations, the geometrical relations between Pluto and Neptune were observed. A libration of the relative positions of the two planets at the close approaches was observed which has a period of about 19,670 years. It is best described geometrically by considering the path of Pluto in a frame centered at the Sun and rotating with Neptune. This is shown in Fig. 1 for one synodic cycle. In that time Pluto passes through two perihelia and two aphelia. The path is almost periodic. At one extremum of the libration, Neptune is positioned with respect to the path as shown by one of the N's in Fig. 1. At the other extremum, it is positioned at the other N. In the course of the libration, Neptune moves with respect to the synodic path back and forth on the 76-degree arc between the two N's. The loops in the path of Pluto are due to the eccentricity of the orbit and to the consequently varying angular rate about the Sun. It is noted that in each synodic period there are three minima of the distance between Pluto and Neptune. Two are when Pluto is near each of the loops at the perihelia and the third is near one of the aphelia. In the course of the librations, the minima near the perihelia

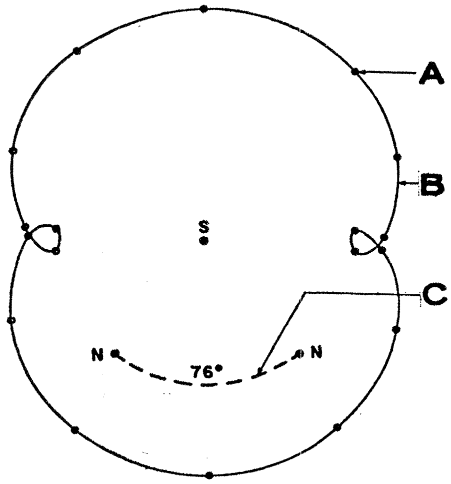

Fig. 1. Path of Pluto about the Sun (S) and Neptune (N) at extrema of libration. (*A*) Solid circles at steps of 10,000 days. (*B*) Synodic period of path, 500 years. (*C*) Period of libration, 20,000 years.

oscillate between 25 and 53 AU and the minimum near the aphelion oscillates from 18 AU at the extrema to almost 22 AU when Neptune is in conjunction at aphelion. The absolute minimum distance between the two planets, 18 AU, is then governed more by the radius of aphelion of Pluto than by the radius of perihelion.

The mechanism of the libration can be appreciated if one considers the attraction of Neptune on Pluto resolved along and transverse to the radius SP from the Sun to Pluto. If this attraction is averaged over a synodic period, the mean radial component can be seen from Fig. 1 to have twice the frequency of the libration. The mean transverse component on the other hand has the frequency of the libration and so must be the cause. At the perihelion loop closer to Neptune, the transverse component is large both because the attraction of Neptune is relatively large and because it makes a large angle with SP. Furthermore, from the solid circles at steps of about 27 years on the path, it is seen that Pluto dwells at each loop for about 25 percent of the synodic period. Thus the transverse attraction of Neptune on the nearer loop dominates the libratory motion. According to whether the nearer loop leads or lags behind Neptune, Pluto loses or gains energy, its mean motion increases or decreases, and in both cases the nearer loop accelerates its librational velocity away from Neptune. In effect, Neptune drives the nearer loop away.

The orbital periods are strictly commensurate at the extrema of the librations. This occurs at just two instants in each libration cycle, and so strict commensurability will almost never be found. This corresponds to the Kirkwood gaps in the asteroids. The amplitude of the libration is governed partly by the initial conditions. If the periods are initially too far from the 3 : 2 commensurability, the acceleration of the loops will not suffice to prevent circulation.

The explanation of the mechanism of the libration is possibly applicable also to asteroids near resonance with Jupiter and to satellites of Jupiter and Saturn (*4*). The number of loops, or incipient loops, though, could be different. There would be three for the 4 : 3 commensurability of the periods of Hyperion-Titan.

In the more analytical language of celestial mechanics, the Pluto-Neptune libration is characterized by an oscillation of the angle $\delta = 3l_P - 2(l_N + \pi_N - \pi_P) - 180°$, where l and π are the mean anomaly and the longitude of perihelion, and P and N are Pluto and Neptune. The period of the libration is about 19,670 years and the amplitude is 76°.

Because of the libration about the commensurability ratio, the closest approach of Pluto to Neptune is locked in near aphelion and the minimum distance between the bodies is approximately 18 AU. Any radical disturbance due to an unusually close approach, which would have to be nearer perihelion, is therefore ruled out.

The libration amplitude is in some error due to the relatively very short span of observations and also is subject to modulation as the orbital elements change. Reasonable secular changes of the elements considerably in excess of those seen in these 120,000 years could hardly break up the libration. Therefore, considering the small radius of perihelion, the orbit of Pluto displays a truly wonderful degree of stability.

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Oxygen Isotope Fractionation between Coexisting Calcite and Dolomite

Abstract. *The oxygen isotopic composition of calcite from carbonate rocks of the lower unit of the Flagstaff formation (Paleocene) exposed along the western margin of the Wasatch Plateau, Utah, is about 7 per mil lower than that of coexisting dolomite, suggesting that isotopic equilibration of these two minerals can occur at relatively low temperatures. Unlike recent isotopic evidence presented by Degens and Epstein, the data do not preclude a primary chemical origin for the dolomite.*

The origin of dolomite, ubiquitous in the rocks comprising the geological record, has been investigated intensively for many decades. Ingerson (*1*), in a recent review of the dolomite problem, stated that the origin and mechanism of formation of the fine-grained, so-called "primary" dolostones remain one of the most interesting and puzzling problems of sedimentary geochemistry. The major problem concerns the question of whether or not dolomite, a stable phase at temperatures and pressures found at the earth's surface (*2*), can crystallize from sea, lake, or river water as a direct chemical precipitate. According to Ingerson, it has never been demonstrated unequivocally that dolomite has ever precipitated in nature directly from solution; nor has dolomite ever been formed in the laboratory under conditions similar to normal marine, lagoonal, or lacustrine environments. To support the arguments for and against the primary origin of at least some dolomite, a wealth of petrographic, stratigraphic, and chemical evidence, all inconclusive, has been