Book Reviews

Mathematics for the General Reader

Mathematics: The Man-made Universe. An introduction to the spirit of mathematics. Sherman K. Stein. Freeman, San Francisco, 1963. xiv + 316 pp. $6.50.

The Language of Mathematics. Frank Land. Doubleday, Garden City, N.Y., 1963. viii + 264 pp. Illus. $4.95.

Graphs and Their Uses. Oystein Ore. Random House, New York, 1963. viii + 131 pp. Illus. Paper, $1.95.

Mathematics. Samuel Rapport and Helen Wright, Eds. New York University Press, New York, 1963. xvi + 319 pp. Illus. $4.95.

Ten or fifteen years ago most "popular" books in mathematics fell into one of two categories: those that offered "shortcuts" to the development of a mathematical skill and those that were anthologies of more or less trivial puzzles or oddities. Today, concomitant with the revolutionary changes in content and spirit of school mathematics curricula, there are appearing many popular books of an entirely different nature—serious, informative, and carefully designed to communicate mathematical ideas of some depth. These current books belong in a third category; they appeal to the general reader, but they also serve parents who want to keep up with their children and, at the same time, provide "enrichment" materials for high school and college students.

The four books reviewed here fit well into this last category. The first three deal with the substance of mathematics —numbers, geometry, topology, combinatorial theories, and computers—as well as with some applications of mathematics to other areas. The fourth contains essays on mathematics, mathematicians, and the relations of mathematics to the world around us. Stein, Land, and Ore may be read by those with no mathematical background beyond arithmetic, but some acquaintance with elementary algebra and geometry, even of the traditional variety, would be extremely helpful. On the other hand, for a full appreciation of the pieces in the Rapport and Wright anthology, one needs more knowledge of mathematics, such as that which could be gleaned from Stein, for example.

There has been much philosophical controversy among curriculum reformers. Some would stress artistic aspects, including intrinsic structure and "patterns"; for others the primary value of mathematics lies in its potential as a problem-solving tool; still others believe that mathematics should never be divorced from the natural sciences; and there are the worriers who fear that computational and manipulative skills will be lost in the shuffle. In the first three books, one finds overall a good balance of emphasis relative to these aspects, and among the essays in the fourth one finds excellent expositions of virtually all points of view toward mathematics.

Perhaps the most ambitious and broadest of the first three books is that by Stein. The author's attitude is summarized in his preface: "We all find ourselves in a world we never made. Though we get used to the kitchen sink, we do not understand the atoms which compose it. The kitchen sink . . . is a convenient abstraction. Mathematics, on the other hand, is completely the work of man. Each theorem, each proof, is the product of the human mind. In mathematics, all the cards can be put on the table. In this sense, mathematics is concrete, whereas the world is abstract."

The first chapter, "The weaver," leads quickly from the problem of weaving a hatband without a visible seam into some sophisticated mathematics of permutation theory. In similar fashion, the author skillfully connects some naive geometry with deep ideas of topology: "tiling" a rectangle with the theory of electric networks; and the inspector's problem—to cover each section of highway just once—with "memory wheels" and communications theory. Interspersed are some good solid chapters on number theory, rational and real numbers, abstract algebra, set theory, higher dimensional geometry, and many other topics; all are presented in a manner that discloses the elegance of mathematical structure. Yet the author engagingly stresses throughout, the connections with everyday things; he comes quite close to establishing his claim concerning the concreteness of mathematics without gainsaying the abstractness that mathematicians normally ascribe to it.

Land's *The Language of Mathematics* is a British version of a popularization. Its objectives are similar to those of the Stein book, but it is narrower in scope and less attentive to logical niceties. Land, who follows a more classical line than is current in America, places great stress on computation, in an apparent effort to inspire interest in it; also, he seems a bit overzealous in his preoccupation with measured quantities (numbers coupled with units) and how one manipulates them.

Although interconnections among topics are not always brought out and motivations are not always given, there are included some unusual and interesting applications of mathematics. Thus the chapter, "Logs, pianos, and spirals" contains, along with a thorough explanation of logarithms, an exposition of the even-tempered scale, the equiangular spiral with application to a nautilus shell, and Fechner's law. There are chapters on the geometry of conic sections with applications to mechanics, elements of statistics, time and the calendar, and Euclidean and other geometries; throughout, the motif of numerical computation is omnipresent.

By contrast, Ore's little monograph on graphs (in the topological sense) deals with only one corner of mathematics. Diversity is not lacking, however, for this subject has tremendously wide and varied uses, both in and out of mathematics. A graph is, or may be represented by, a set of points (*vertices*), together with segments (*edges*) directed or not, connecting certain pairs of the vertices.

Ore makes this subject come to life, in his typical skillful way, by connecting it with all sorts of things—tournaments, the deep Jordan curve theorem, certain puzzles like the Königsberg

Bridge problem, combinational analysis, genetics, the theory of games, and map coloring. In so doing, he unobtrusively introduces the mathematical bases for classifying graphs, proves many theorems, and even leads the reader to current research in the field.

The treatment draws more on the reader's intuition than on his logical prowess. However, direct attention is given to structural questions in a chapter which shows how the theory of graphs is subsumed under the theory of relations and, hence, under set theory.

In this subject, some results sound sophisticated but are, in reality, trivial; others are easy to state and understand but contain tremendous depth and may be still of undetermined validity. A result of the first type may be illustrated as follows. A graph, thought of as a road map, is called *connected*, if for each two vertices there is a route connecting them. A vertex is called *odd*, if an odd number of edges emanate from it. It is a theorem that in any connected graph, two vertices are connected by a path covering all edges just once exactly when the two vertices are the only odd ones (compare Stein's inspector's problem). To the uninitiated this might be a surprising and forbidding theorem; yet Ore shows the proof to be very simple. An example of the second type of result is the classical four-color map conjecture, whose truth is still in doubt. This conjecture states that in any plane polygonal map, four colors suffice so that no two countries with a common boundary bear the same color. Ore's ninth chapter is devoted to this problem and especially to the ideas, fruitful for other purposes, which have emerged from attacks on it.

All in all, a general reader will find in Ore's book much to excite his imagination and sustain his interest. There are problems for him to solve, with solutions in the appendix; unfortunately the problems are not as numerous as might be desired. Without question, this monograph will shortly become an established classic.

Let us turn finally to the fourth book. As was indicated earlier, it is harder to learn mathematics by reading about it than by reading and studying the subject itself. Nevertheless, it is possible, as Hollis R. Cooley suggests in the foreword to this volume, that some may find in general readings inspiration that will lead them into the subject itself. Certainly the editors could not have selected their materials more wisely to accomplish this purpose.

There are biographical-historical pieces on Archimedes, Newton, Gauss, Einstein, and Galois. These amply demonstrate, as the editors intended, that "mathematicians, though extraordinary in their abilities, are people like other human beings and not practitioners of black magic." No one can help but be saddened by the story of Galois, whose genius was cut short by death at 20, but who in his short tragic life built the foundations, still standing today, for most modern algebra.

Included are philosophical essays by Whitehead, Hardy, and Poincaré and excerpts on the substance of mathematics by such experts as George Gamow, Tobias Dantzig, and Richard Courant. Here the reader will find attitudes of practitioners toward their subject, insights into the methods of reasoning and the objectives of mathematicians, as well as some glimpses of the subject matter itself. Thus, a rather good descriptive introduction to topology is provided by Richard Courant and Herbert Robbins; the Poincaré introspective analysis of mathematical discovery is intriguing because of its novelty and the fact that to this day so little is known about the subject.

Finally, the scientific and technological aspects of mathematics are amply described in a section entitled "Mathematics and the world around us." Eminently qualified authors, including E. T. Bell, Albert Einstein, and Morris Kline, cover connections with astronomy, physics, biology, probability, and computing machines. One is led to understand that mathematics is not just a convenient scientific tool but is in fact the substance of scientific theories. Moreover, one sees clearly that mathematics and the sciences are inextricably interwoven, and that no cleavage between them could be effected without adversely affecting both.

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Chemistry

Topics in Organic Chemistry. Louis F. Fieser and Mary Fieser. Chapman and Hall, London; Reinhold, New York, 1963. xii + 668 pp. Illus. $10.

It is always a pleasure to have a new book by the talented Fiesers. Few, if any, authors are able to write as vividly and interestingly on such a variety of topics. This latest book consists of (i) a number of chapters which had to be left out of their monumental *Advanced Organic Chemistry* and (ii) a new section entitled "Supplements to advanced organic chemistry."

The first section covers a wide range of specialized topics: polynuclear hydrocarbons, heterocyclic compounds, alkaloids, terpenoids, steroids, vitamins, chemotherapy, synthetic polymers, and dyes. Although these chapters are unfailingly interesting, informative, and enjoyable reading, I would like to make a few minor criticisms. Stereochemical formulations could have been used even more liberally whenever they were known—for example, aspidospermine (p. 150) and eremophilone (p. 186). A few minor errors are unavoidable: the stereochemistry of the carbomethoxyl of yohimbine is incorrectly indicated (p. 151), a double bond is missing from the ibogamine formula (p. 150), carotol is listed as cartol (p. 187) and M. F. Carroll as M. F. Carol (p. 215), and β-phenoxyphenylpropionic acid (footnote 2, p. 258) should be β-phenoxybenzoylpropionic acid. I also wish that a clearer mechanism had been indicated for the phenyldihydrothebaine rearrangement on page 138. These are small faults in a very useful survey of many areas.

The second section is more controversial, but certainly very stimulating. It consists of reactions, syntheses, and the like, which have appeared since the publication of *Advanced Organic Chemistry* and which the Fiesers have chosen as interesting illustrations or extensions of the "Advanced" topics. There is, of course, bound to be disagreement about the relative value of various materials selected for treatment in this section; but even though some important advances must surely have been omitted, at least there is no doubt that what is included is, for the most part, very stimulating. It is certainly up-to-date. This carries with it a certain danger that importance may sometimes lose to novelty. A case in point is that of listing the structure of the attractant of the female cockroach (1963) on page 596, despite the extremely shaky evidence on which it is based.

All in all, this is an outstanding book in which students and researchers alike will find much to excite their curiosity and stimulate their imagination.

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