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A Law for Loudness Discrimination

Abstract. A law for loudness is proposed which implies detection and averaging in a general nonlinear device with excitation by a complex signal-noise wave form. This law provides a mechanistic explanation for subjective intensity in the general time varying case. Mathematical solutions to the general law may be obtained in certain elementary cases, the well-known power-law characteristic being an example. A modified Weber law is derived which is in good agreement with experiments at both small and moderate stimulus intensities.

Loudness. The Weber fraction $\Delta S/S$ is the just-noticeable difference in the stimulus intensity ΔS relative to total intensity S. The Weber law is: $\Delta S/S$ is independent of S. The law sometimes fails when S is large. It generally fails when S is small in the case where unavoidable noise, both external and internal to the sensory system, becomes important (*1*).

The subjective measure for stimulus intensity may be termed L, which stands for loudness but is not restricted to the auditory sense modality. Measure L is fundamental. An accurate expression for the Weber fraction depends upon the adequacy of this measure.

A modified Weber fraction may be defined as $\Delta L/L$, from which a modified Weber law may be deduced as

$\Delta L/L$ is independent of S

S. S. Stevens credits the idea behind this modified Weber law to Brentano (*2, 3*). Brentano's hypothesis will be further examined after a generalized expression for L is developed.

Laws for subjective intensity have been proposed by many people. Most recently S. S. Stevens (*2, 3*) has proposed a power law for subjective intensity. These proposed laws as well as the Weber fraction and Brentano's hypothesis follow a similar convention in that they employ mean-square measure for stimuli. The loudness function proposed here is *not* based on mean-square measure. The distinct break with tradition in this regard has led to the present use of the symbol L for subjective intensity instead of the commonly employed symbol ψ. The rather significant consequences of the present formulation will be commented upon later.

Let $s(t)$ be a time varying signal with mean-square value S and $n(t)$ be additive, statistically independent interference with mean-square value N. A general "loudness function" is proposed as (*4*)

$$L = \mathrm{Av} \sum_{k=0}^{k=\infty} C_k |f(t)|^{n+k} \cong \mathrm{Av}\, C_0 |f(t)|^n \quad (1)$$

where $f(t) = s(t) + n(t)$ is the gross stimulus temporal wave form, where "Av" denotes a suitable time average with an appropriate weighting function, and where C_k are coefficients which are not necessarily independent of time or other parameters (such as the frequency of the stimulus). The approximation of Eq. 1 applies when $|f(t)|$ is small. Since the exponent n need not be an integer, the proposed loudness function is more general than a Taylor series.

The definition of Eq. 1 allows time varying phenomena, such as occur in adaptation and fatigue, to be represented. For a finite averaging time and for $f(t)$ a random variable (in part), loudness in Eq. 1 is also a random variable; randomness in a subjective measure correlates with observed human behavior. Perhaps of most significance is that Eq. 1 can be modeled with practical electronic circuits. Not only does this permit L to be evaluated in cases which are not mathematically tractable, but it means that *a mechanistic interpretation for subjective intensity has been achieved.* Such a direct modeling can be quite realistic in the physiological sense if, for example, part of the circuitry uses pulse generators in the emulation of sensory cells.

If an infinite averaging time and constant coefficients C_k are assumed, mathematical solutions can be obtained for certain special stimulus wave forms, although mechanistic interpretation is thereby hindered. When $f(t)$ consists of independent Gaussian signal and noise components, a generalization of the Stevens law results as (*4*)

$$L = \sum_{k=0}^{k=\infty} C_k K_k (1 + S/N)^{(n+k)/2} \cong C_0 K_0 (1 + S/N)^{n/2} \quad (2)$$

where the $C_k K_k$ are constants and where the approximation is for the case when $|f(t)|$ is small.

When $f(t)$ consists of a sine wave imbedded in Gaussian noise, L may be expressed in terms of confluent hypergeometric functions; in spite of less familiar function notation, quantitative values remain similar to those given by the simpler Gaussian signal case. Use

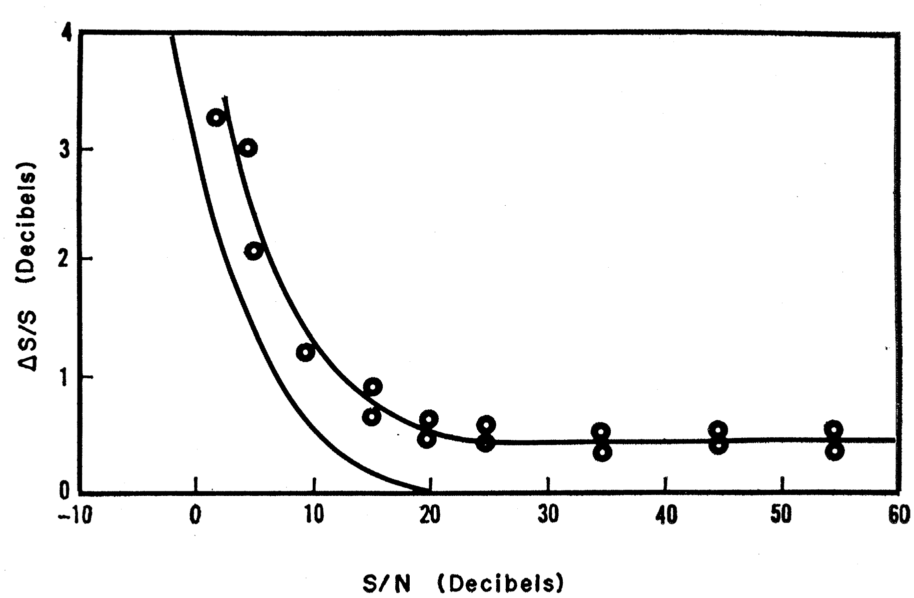

Fig. 1. A comparison of the experimental and theoretical values for the just-noticeable difference in stimulus intensity relative to total intensity (for loudness). The lower curve is plotted from Eq. 6. The circles represent Miller's experimentally determined values. The upper curve is obtained from linear translation of the lower curve.

of an infinite averaging time is valid in predicting an ensemble-average value for L provided that the actual animal averaging time may be presumed to be long compared with temporal details in $f(t)$.

A new Weber law. The modified Weber law may be obtained by using the approximation $\Delta L \cong dL$ for infinite averaging time. From

$$\Delta L/L = \frac{\Delta L/\Delta S}{L}\Delta S \cong \frac{dL/dS}{L}\Delta S \quad (3)$$

and using the approximation of Eq. 2, there results

$$\Delta L/L \cong \frac{(n/2)(\Delta S/N)}{1+S/N} \quad (4)$$

Observe that $\Delta L/L$ becomes directly proportional to $\Delta S/S$ for large S as in the classic Weber law. Let $\Delta S'$ and S' apply for large S and let ΔS and S signify general values. From the modified Weber law (ΔL is independent of S) there may be formed

$$(\Delta L/L)_{\text{General } S} = (\Delta L/L)_{\text{Large } S} = S' \quad (5)$$

from which

$$\frac{\Delta S/S}{\Delta S'/S'} \cong 1 + \frac{1}{S/N} \quad (6)$$

Equation 6 is plotted in Fig. 1; a particular $\Delta S'/S'$ is assumed and a slight shift along the S/N axis has been made in order to account for uncertainties associated with the measurement of noise. The theoretical result compares with empirical data obtained for hearing by Miller (*5*). The corresponding result for the sine wave signal is similar.

Note in Fig. 1 that the point where the theoretical curve is 3 decibels above its large S/N plateau requires signal stimulus mean-square value S to be equal to the mean-square value of noise N. If the external signal is not corrupted by noise, this particular value for S appears to measure the noise internal to the sensory system.

The noise implied in the foregoing derivation is assumed to be intermingled with the external stimulus. If the noise is totally internal, results remain valid provided that equivalent external noise can be defined. There is evidence that neural noise is in part stimulus-dependent and hence can not be determined in terms of an externally applied equivalent; to this extent, the curve of Fig. 1 is an approximation (*6*).

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Angular Displacement of the Visual Feedback of Motion

Abstract. The hypothesis is that such displacement produces no disturbance of behavior within a limited "normal" range, but that a breakdown angle can be found beyond this range which will produce disturbance in motion in proportion to the magnitude of the feedback displacement. Results with two tasks specifically support the hypothesis that limiting angles of neurogeometric control of motion and variation in performance occur with varying magnitudes of angular displacement of feedback.

Although angular displacement of vision has been studied heretofore as an example of perceptual disorientation with experimental spectacles, the significance of such study for critical behavioral theory and nervous system function has not been considered. The sensory feedback theory of the neurogeometric organization of brain function and behavior (*1*) assumes that neural detection of angular displacement of the movement-produced stimulus feedback represents a primary process in the brain's regulation of different component movements of reciprocal tremor, posture, transport, and manipulation, as well as their integration in behavior.

Figure 1*a* illustrates what we mean by angular displacement of the sensory feedback of human motions, as achieved via closed-circuit television. The subject, instead of viewing his own motions directly, views his movements, as in drawing and writing, in a television monitor. The television camera provides a substitute visual image of the subject's movements, which is transmitted to him as a normally or angularly displaced sensory feedback image. If the camera is moved to different points in space relative to the performance field, vision is, in effect, changed from its normal locus. The magnitude and direction of this angular displacement of visual feedback of any motion may be quantitatively controlled.

The particular hypothesis of neurogeometric theory under test in this study assumes that angular displacement of the locus of vision differentially affects various patterned movements of the human individual. That is, a coordination exists between vision and each movement component of the body such that there exists for each movement pattern a limited and critical angle of displacement of its sensory feedback. Within this angle patterned movement may be performed as an organized response without interference. Visual displacement beyond this critical angle causes movement (motor performance) to break down. The degree of both this disruption and the recovery of movement control with learning is assumed to be a function of the displacement angle.

The general method of angularly displacing the visual feedback of motion is described in connection with Fig. 1*a*. The figure shows how this method was applied to angular displacement of the visual feedback of two behavior patterns used in this study—drawing circles and tracing a visual gated maze.

In this experiment, 24 female subjects practiced the two experimental tasks of circle drawing and maze tracing for 9 days under four conditions: 0, 20, 40, and 60 degrees of angular visual displacement in the horizontal plane of the performance field. The vertical direction of the television camera corresponded to the normal angle of the line of sight in the vertical plane of the performance field. On the 10th day performance was measured at eight angles of horizontal visual displacement: 0, 10, 20, 30, 40, 50, 60, and 70 degrees. An appropriate design was used in order to obtain control of experimental physical variables and sequence of observations.

The instructions to the subjects in the circle drawing task were crucial. They were told to draw as "perfect" a circle as possible on the easel, regardless of what appeared on the television display, noting that what they drew and what they saw might not correspond. Thus, visual cues were pitted against kinesthetic cues. In the maze task, subjects were told to trace between the gates of the maze as accurately as possible.

The data of the measures of performance at the eight angles of displacement during the test period are summarized in Fig. 1, *b* and *c*. The curve for maze tracing (Fig. 1*b*) shows that